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Dept of Speech, Music and Hearing

ACOUSTICS FOR VIOLIN AND GUITAR MAKERS

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Chapter II: Resonance and Resonators

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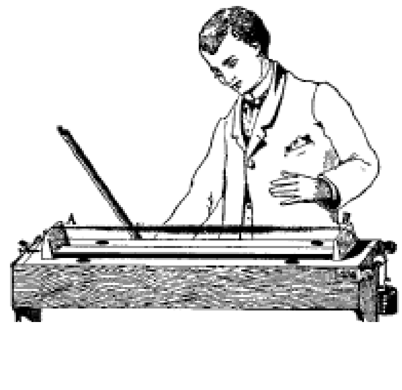

Index of chapters

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ACOUSTICS FOR VIOLIN AND GUITAR MAKERS

Chapter 2 – Fundamentals of Acoustics
RESONANCE AND RESONATORS

Chapter 2.
FUNDAMENTALS OF ACOUSTICS - RESONANCE AND RESONATORS
First part: RESONANCE

INTRODUCTION

In chapter 1, I presented the fundamental properties of sound and how these properties can be measured. Fundamental hearing sensations were connected to measurable sound properties. In this, the second chapter the concept of RESONANCE and of RESONATORS will be introduced. Resonators are fundamental building blocks of the sound generating systems such as the violin and the guitar. The chapter starts with introducing the resonance and how a resonance is described. Thereafter vibration sensitivity (technical term mobility) of resonances is discussed and finally how mechanical and acoustical measures of a resonator are related.

2.1. RESONANCE

A RESONATOR or a system of resonators gives one or several RESONANCES. The blown tone of a bottle is the result of a resonance and the bottle is the resonator. A practical property to measure a resonator is its frequency response (vibration sensitivity, techn term mobility). The response curve gives a picture of each resonance, i.e. its FREQUENCY (the peak frequency), its resonance LEVEL response (the peak height for the selected driving) and its BANDWIDTH (the peak width). These properties are related to the mechanical properties of the resonator; the MASS (weight), STIFFNESS (spring) and FRICTION. Often a resonator consists of a vibrating area, such as a violin top plate. Then the distribution of the vibrations are important. Positions of maximal vibrations are called ANTINODES and the positions of no vibrations are called NODES. Furthermore the decay (the reverberance) of a resonance can be important. The relations between these different measures and properties will be explained in this chapter.

Let us first look at the guitar tone displayed in Fig. 1.9. The time history of the tone is smooth except the sharp wiggles soon after the attack. The tone is not made up by a single partial but a spectrum of partials, the levels of which slowly change. The frequency separation between the partials is constant and equals the frequency of the fundamental (the first partial). It is the level of the partials that give the guitar tone its character, the timbre.

But why do we obtain a decaying spectrum of partials at these very frequencies? Let us cautiously move towards the answer by looking at a vibrating string.

AN EXAMPLE OF RESONANCE: THE VIBRATING STRING

A large scale picture of the vibrating string can be presented by means of a long rubber band with one end fixed to a wall. The rubber band is held stretched by a hand at its free end. If the free end is slowly moved up and down, the whole band moves up and down in time with the hand. If the beat tempo, the frequency, is increased the string tends to swing out more between the fixed end and the hand end. At a specific frequency the band will swing up and down in a large bend between wall and hand, but still only requires small hand movements. The vibrations at the bends are maximum. If the beat frequency is increased further the bend will diminish and it is difficult to make the rubber band vibrate visibly. With still further increase of the beat frequency the rubber band will divide itself into two bends. At a specific beat frequency the vibrations at the bends are maximum. With further increased beat frequency the bends diminish and thereafter the rubber band will divide itself into three bends, etc.

Let us repeat the same experiment with better control and use a small electrical vibrator attached to a rubber band with its other end fixed to a wall (a similar demonstration can be made using a string of a musical instrument). An electrical tone generator (oscillator) provides the electrical driving signal. The magnitude and the frequency of the driving signal are easy to adjust with high accuracy. The vibrator is replacing the hand. At low frequencies the rubber band again moves in phase with the motion of the vibrator, but with increasing frequency it starts bending out more and more in the middle. At a specific frequency, the first resonant frequency the bend becomes maximum. The position of maximum motion is called an ANTINODE. With further increase of the frequency the bend will first vanish and thereafter at the second resonant frequency, two maxima of motion will come out and the rubber band does not move in the middle. We have now two ANTINODES and in between a NODE. If we continue to increase the frequency the two antinodes vanish and at a higher frequency we will find three bends, antinodes, and between the antinodes, nodes. Further increase of the frequency will show vibrations with four, five, six etc. antinodes with three, four, five etc. nodes. We shall find that the maxima of vibrations are at at fixed frequencies. By measuring the frequencies we will find that the frequencies are 2, 3, 4, 5 times the frequency with maximum vibrations with only one bend.

The experiment has shown that certain things happen at specific frequencies (the resonant frequencies) - the rubber band vibrations become maximum at specific positions (antinodes) and are zero at other positions (nodes).

THE ORIGIN OF A RESONANCE

What is behind these peculiarities and what has that to do with the guitar tone? It is RESONANCES. The vibrating rubber band and also the vibrating string have resonances. But before I explain what is happening to the guitar string, let me explain the origin of a resonance, how its properties are described and are measured.

The RESONANCE is an acoustical building brick of fundamental importance. To describe a resonance we need to answer the following questions:
At what frequency is a resonance, the resonant frequency?
How limited is this resonance in frequency, the bandwidth?
How easily are the vibrations to excite, the vibration sensitivity at resonant frequency?
How are the vibrations distributed, the modes of vibration (or operating deflection shapes)?

A simple resonator is a small ball attached to a string of rubber bands, c.f., Fig. 2.1a. When the finger is slowly moved up and down the ball moves in the same way up and down (in time, in phase with the hand). If the beating frequency of the finger is increased the up and down motion of the ball increases more and more. At a certain beating frequency the ball motion becomes maximum. Further increase of the frequency will make the ball motion smaller and finally at high frequencies only the finger will move.

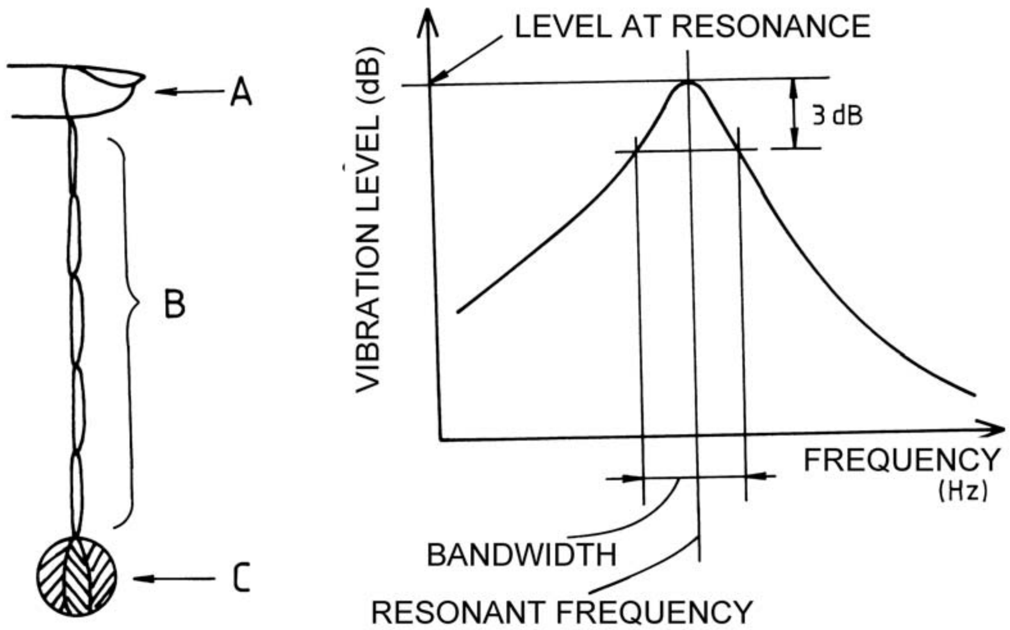


Figure 2.1. A simple resonance: a) a simple resonator - a ball (C) attached to a rubber band (B) and hung on a finger (A), and b) vibration sensitivity curve of the resonance - resonance frequency (RF), peak height (RL) and bandwidth (B measured 3 dB below the peak maximum).

The size of the ball vibrations (upand down motion) follow the vibration sensitivity curve plotted in Fig. 2.1b. The curve shows low vibration sensitivity at low and high frequencies. In between there is a peak, a resonance peak giving the resonant frequency (the peak frequency), the peak height giving the vibration sensitivity at the resonance and the peak width giving the bandwidth. The bandwidth of resonances vary much from a fraction of a Hz for the string to tenths of Hz for body resonances.

The resonator consists of a mass (weight) - the ball - and a spring - the rubber band. The turbulence in the air surrounding the ball tries to brake its motion and provides friction. If the stiffness of the rubber band is increased the resonant frequency is increased. If the mass (weight) of the ball is increased the resonant frequency is decreased. If the friction (against the air) is increased by a thin plate under the ball, the width of the resonance peak is increased.

It is difficult to move a finger up and down maintaining the same beat size and beat frequency. Therefore an arrangement as shown in Fig. 2.2 may be used. A loudspeaker gives a strong tone. By turning a knob on a tone generator the tone frequency is changed without changing the strength of the tone. The plate with attached rubber band is hung over the loudspeaker and the strong loudspeaker tone sets the plate into vibration. The frequency is slowly changed and at the same time the magnitude of the plate vibration is measured. The vibration size is plotted for each frequency and thereby the vibration sensitivity of the ball-rubber band resonator is obtained (in the real world this very experiment may be hard to conduct though – a suitable combination loudspeaker, plate and rubberband may be hard to find).

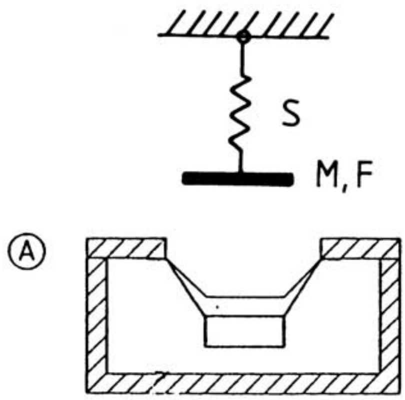


Figure 2.2. Principle set-up for measuring the vibration sensitivity.

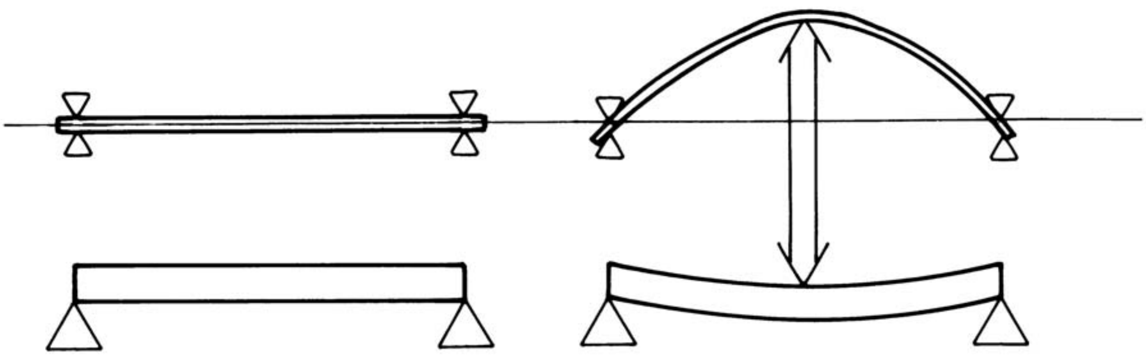


Figure 2.3. Specific vibration sensitivity and stiffness - bending with a force between a thick and a thin plank (applies to low frequencies).

2.2. VIBRATION SENSITIVITY

VIBRATION SENSITIVITY (techn term mobility) is a measure of how easily vibrations can be started and maintained. If one holds a swing, and pulls and pushes it to and fro with the same force then the swing starts to swing, i.e., it starts to vibrate. How much the swing vibrates is a measure of the vibration sensitivity. More mathematically expressed the is the ratio between vibration velocity and the driving force. Mathematical formulas describing the vibration sensitiivty (mobility) are given in the appendix of this chapter. At a specific push and pull frequency the swing vibration become maximum, i.e., the swing is vibrating at its resonant frequency.

The SPECIFIC VIBRATION SENSITIVITY (specific mobility) is a measure on how easily a resonator (a swing, a string or a violin back plate) can be set into vibration, when its resonant properties are fully eliminated.

Example 2.1. What is the specific vibration sensitivity of and how much does a plank bend, which is ten times stiffer compared to a second plank? The picture is drawn for static forces, but applies also to vibration forces below the resonant frequency of the system. Here only stiffness is involved, which is only applicable for low frequencies and the static case. The same force is applied in the two cases. By using formulas (see appendix) one finds that the stiffer plank is bent 1/10 of the second one. The difference in specific vibration sensitivity is squareroot of 1/10, which is approximately 1/3 and corresponds to -10 dB. Rule of thumb - 10 times stiffer results in a 10 dB lower level, c.f., Fig. 2.3. The relation is also true for vibration forces at frequencies well below the resonant frequency of the system.

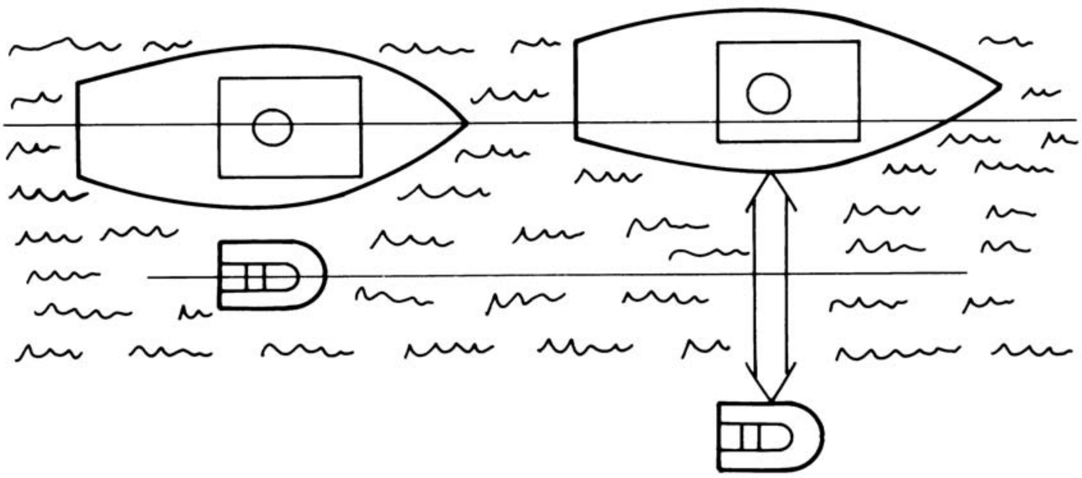


Figure 2.4. Specific vibration sensitivity and mass (weight) - displacement by a force between a big motorboat and a small rubber boat (applicable at high frequencies, c.f., example 2.2).

Example 2.2. What is the specific vibration sensitivity and how much less does the ten times heavier motorboat move than the smaller rubberboat if pushed? The picture is drawn for a static force but applies to vibration forces above the resonant frequency of the system. A little calculation shows that the heavier boat obtains a motion of 1/10 of the smaller one. Furthermore the specific vibration sensitivity becomes the squareroot of 1/10, i.e. 1/3 corresponding to 10 dB lower specific vibration sensitivity for the heavy boat car compared to the small one. The relation is also true for vibration forces at frequencies well above the resonant frequency of the system

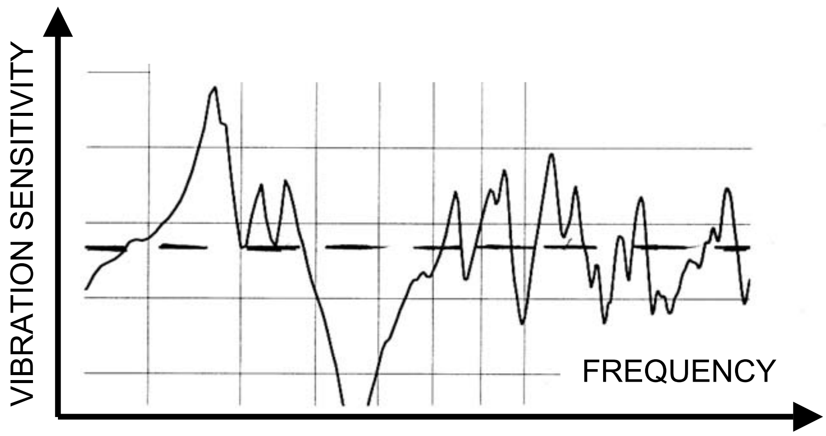


Fig 2.5 The specific vibration sensitivity, the dashed line, can be low but resonance properties of the instrument body may increase the total vibration sensitivity, the full line, to give large interferance with the string motion and cause a “wolf tone”.

The results of examples 2.1 and 2.2 are "reasonable" - a stiff spring or a heavy mass (weight) is more difficult to set into vibration, and the specific mobilities are lower. Complications are added in the range of a resonant frequency. The resonant properties are superimposed, c.f., the formula predict larger vibrations close to the resonant frequency.

The specific vibration sensitivity also gives an average (geometric mean) measure of how two vibrating systems may cooperate. If the specific vibration sensitivity is large (a string for instance) in one of the systems and small in the other (the body of a musical instrument), then the two systems will work fairly independently of each other, c.f. the strings on a violin or a guitar. If the specific vibration sensitivity with the superimposed resonance properties (c.f. Fig 2.5) are about the same of a string and of an instrument body, then the two systems will effect each other, c.f., the wolf tone of the cello.

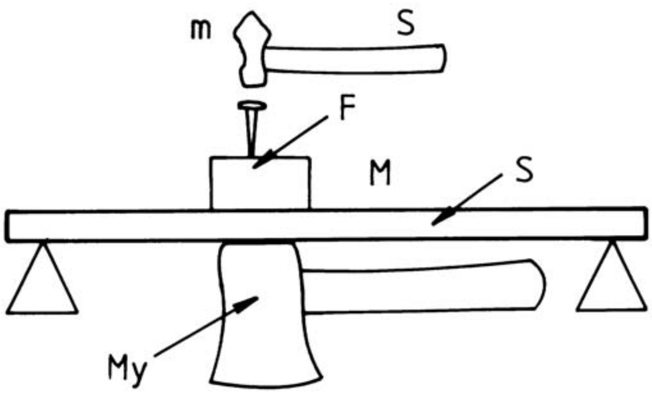


Figure 2.6 A two-resonator system as example on vibration sensitivity and specific vibration sensitivity. Nailing a small block of wood to a bending board by means of a small hammer with bending handle and an axe as extra mass-support.

In common life one meets these practical problems such as nailing in weak boards, c.f., Fig. 2.6. The thin board is set into vibration, it will vibrate at its resonant frequency. The vibration sensitivity of the board can be decreased by adding mass with an axe, which greatly simplifies the nailing. The force from the hammer is transmitted by friction between the nail and the block to the board. The mass of the small hammer head together with the stiffness of the handle gives a second resonance. This resoance is much damped by the holding hand. In somewhat similar ways can the function of musical instruments be modelled as seen by the eyes of the physicist, although the musical instruments are much more complicated. The string of the guitar replaces the small hammer, the bridge replaces the block and nail, and the body replaces the board.

A resonance is determined by the spring and the mass. At a specific frequency, the resonant frequency, the spring effect and the mass effect are of the same size. They counteract and the vibration sensitivity becomes maximimum. The effects of very large friction is also calculated.

2.3. THE MECHANICAL AND ACOUSTICAL MEASURES OF THE RESONATOR

One very interesting question is: How much are the acoustical measures changed if one of the mechanical measures is changed? The changes we shall study correspond to doubling of the stiffness, mass and friction, c.f., Fig. 2.7.

In the case shown in Fig 2.7a only the mass is changed (for simplicity two rubber bands and two weights are used as standard). If the mass is doubled (four weights) the resonance frequency decreases half an octave (from 500 to 350 Hz, i.e. 1/1.4 times). If the mass is halved (one weight) the resonance frequency is increased half an octave (from 500 to 700 Hz, i.e. 1.4 times). The peak heights (the levels) are approximately the same and the levels of these curves are the same at low frequencies (100 Hz for instance). At high frequencies the level (the vibration sensitivity) is larger for the smaller mass.

If only the stiffness of our resonator is changed, then the following happens, see Fig. 2.7b. For doubled stiffness (four rubber bands) the resonance frequency increases half an octave and for half the stiffness (one rubber band) the resonance frequency decreases half an octave. For high frequencies the level (the vibration sensitivity) is the same but for low frequencies it is higher for the smaller stiffness. The peak heights are little influenced by the stiffness changes.

Thus we have found that the effect on the resonance frequency by a doubling of the mass is equivalent to halving the stiffness and vice versa. There is, however, one large difference. The doubling of the mass decreases the vibration sensitivity at high frequencies only, while halving of the stiffness increases the vibration sensitivity at low frequencies only.

Finally if the friction is doubled and halved as indicated in Fig. 2.7c the peak height is decreased and increased by 6 dB, respectively. In addition the effect of very large friction, bandwidth 500 Hz being equal to the resonant frequency is calculated. For such large losses (large bandwidth) no clear resonance peak is obtained.

Suggested experiment: Start with three similar bars, use one as reference, add and take away a small weight from the second. Add and take away a stiffening rib from the third. Tap and listen - is the tap tone pitch (frequency) changing as one should expect. Further experiments can be made by thinning one of the bars, c.f., chapter 5.

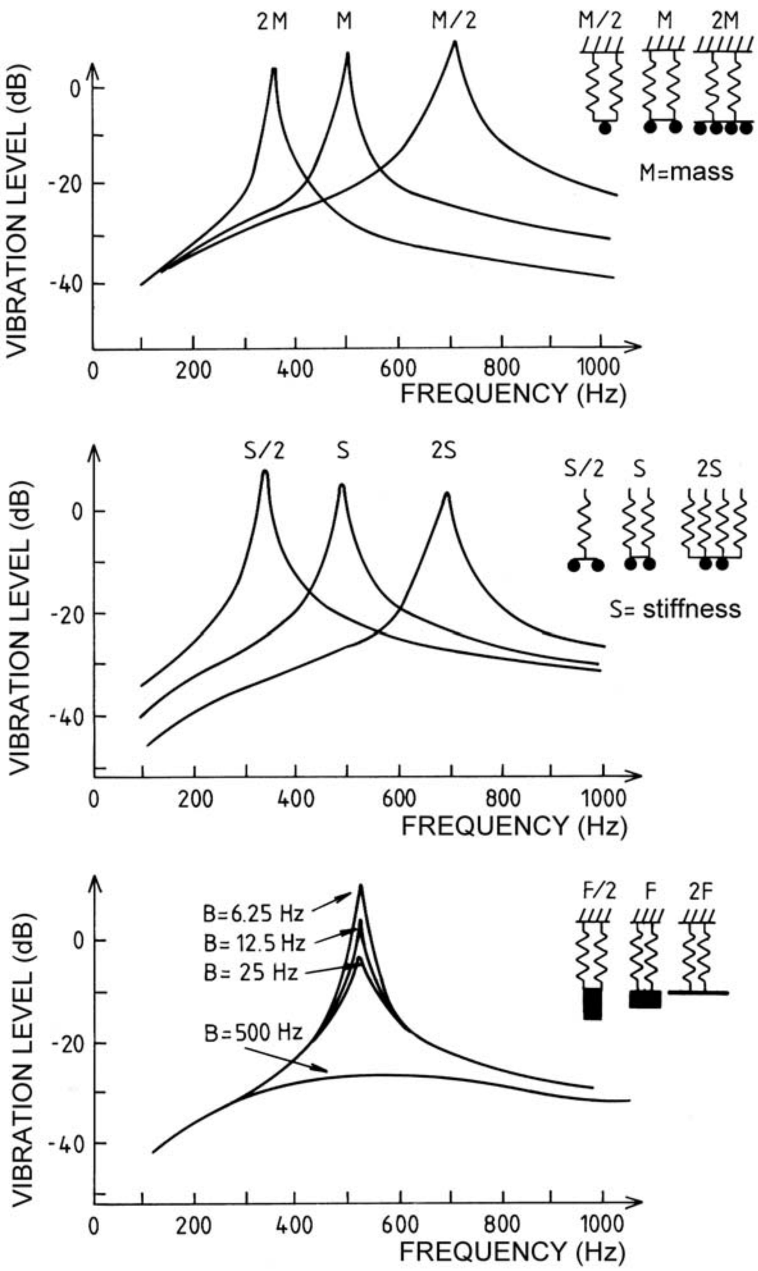
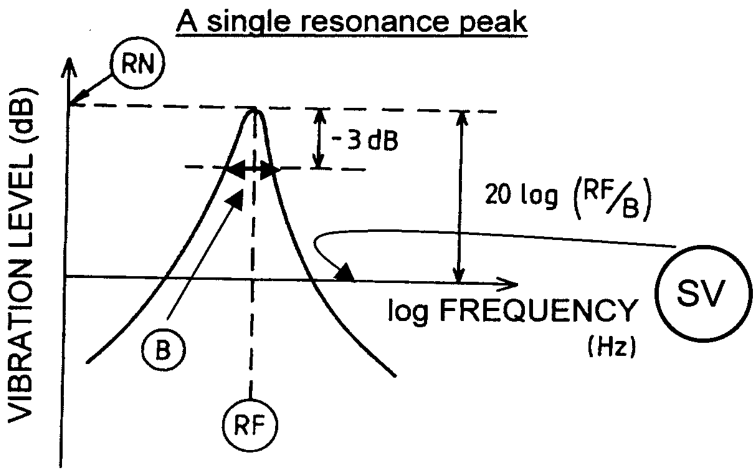


Figure 2.7. Vibration sensitivity with different mechanical properties.



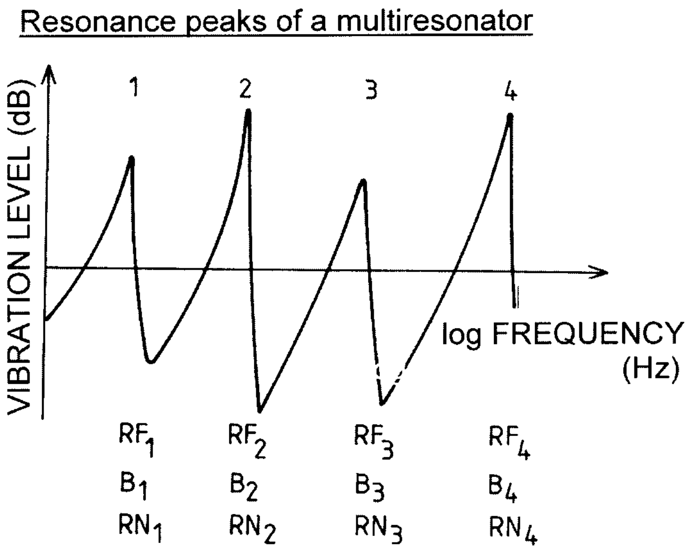


Figure 2.8. Vibration sensitivity for a vibrating system consisting of a single resonance (upper frame) with the resonant frequency RF, the height of the resonance peak RL, and the bandwidth B, and for a vibrating system consisting of several resonances (lower frame) where each resonance has it own resonance frequency, height and bandwidth.

ACOUSTICAL PROPERTIES OF A RESONANCE. MULTIRESONATOR SYSTEMS.

The properties of a resonance can be described by its vibration sensitivity curve i.e. a curve of the vibration sensitivity for every frequency. Usually such a curve is called a resonance curve or a frequency response curve, c.f., Fig. 2.8a. The frequency at the peak is the RESONANT FREQUENCY. The height of the peak is the PEAK LEVEL The width of the peak 3 dB below maximum is the BANDWIDTH (often the ratio of the resonant frequency divided by the bandwidth is given, the so called Q-factor). The acoustical properties of a resonance are described by the three measures RESONANT FREQUENCY, PEAK LEVEL and BANDWIDTH (Q-factor).

The mechanical properties of a resonator are:
1. STIFFNESS such as the stiffness of a spring or the springiness of a rubber band.
2. MASS such as the weight of a ball.
3. FRICTION such as the air friction when the ball is moving.

The different mechanical properties influence the resonance curve differently as has been shown in Fig. 2.7. More mass (weight) gives lower resonance frequency and lower levels at high frequencies, and higher stiffness a high resonance frequency and lower levels at low frequencies. The mass and the stiffness do not influence the bandwidth. The friction does however. Little friction gives a high level and narrow bandwidth and a high friction gives a low level and a large bandwidth. The friction influences only the resonance curve in the neighbourhood of the resonance frequency. To build up the vibrations at a resonance demands a specific time, which gives a specific starting characteristic. In the same way it takes some time for the resonance vibrations to vanish, which gives an ending characteristic. Both the starting and the ending time are set by the bandwidth of the resonance peak.

The resonance curve for a single resonator can be measured and looks as in Fig. 2.8a, and gives a measure of the resonant frequency, the bandwidths and the level. The resonance curve for a multiple resonator system such as the guitar can look like the resonance curve 2.8b. Each peak corresponds to a resonance and has its own resonant frequency, bandwidth, and level as each resonance is made up its own stiffness, mass and friction. In general all three measures are different for the different resonances.

2.4 SUMMARY: RESONANCE

In this first part of this chapter the concept of resonance has been introduced. The mechanical properties of a resonance system such as mass (weight), stiffness (springiness) and friction have been related to the acoustical properties of the resonance system, such as resonant frequency, bandwidth and level. The duration for starting and ending characteristics (transients) is determined by the bandwidth of the resonance. Furthermore a method to measure the acoustical properties of a resonance system has been sketched.

2.5 KEY WORDS:

Resonance (eigenmode), resonant frequency, bandwidth and level, vibration sensitivity, starting time (start duration), ending time (end duration), mass (weight), stiffness (springiness), and friction.

CHAPTER 2.
Second part: RESONATORS

INTRODUCTION

In this part properties of resonators used in string instruments will be introduced. First simple resonators as the hole-volume (the Helmholtz) resonator and the string will be presented. Thereafter properties of complex resonators as the bar and the plate. Finally a simple way to measure the properties of plate resonantors as well as further complications as shape and arching will be introduced.

2.6. SIMPLE RESONATORS

THE HOLE-VOLUME RESONATOR (RESONANT FREQUENCY)

The simplest resonator encountered in musical instruments is an enclosed air volume with a hole. Often this resonance is called the Helmholtz resonance The resonator is called a Helmholtz resonator. The air volume works as a spring (stiffness) and the mass (the weight) is the air plug in the sound hole. Except for the constructional difference it works as the ball-rubber band resonator, c.f. Fig. 2.1. A large volume and a hole with small cross section gives an air tone of low frequency.

A simple example of a hole-volume resonator is a bottle, which is set into vibration by blowing over the neck opening. The air volume in the bottle works as the spring and the air plug in the bottle neck is the mass, see Fig. 2.9. When one gently blows over the neck opening the air plug is set into vibration and co-operates with the spring of the air volume in such a way that the air plug will vibrate in and out. These vibrations give the "bottle tone" one hears. In the musical instruments such as the guitar and the violin the inner air volume of the sound box is the spring and the air plugs in the sound holes are the vibrating masses. The resonant tone is often called the air tone.

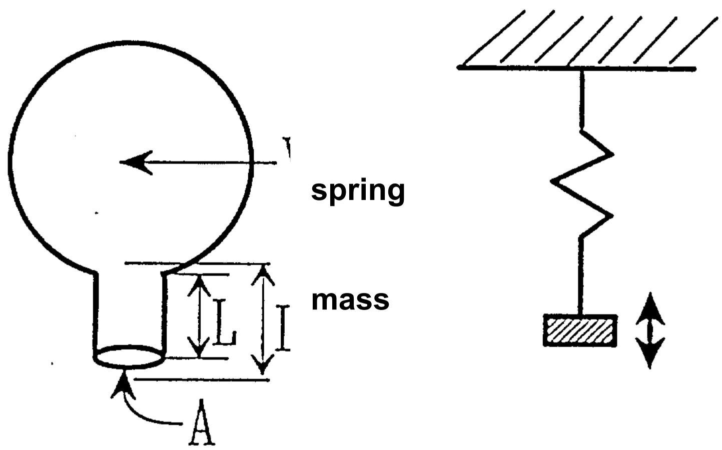


Figure 2.9. A simple resonator - hole-volume resonator (the Helmholtz resonator) - a bottle and its mechanical analogue.

The resonance frequency is determined by (area A/volume V see appendix), i.e. large sound holes and a small corpus volume give a high resonant frequency while small holes and a large volume give a low resonant frequency (formulas see appendix). The shape of the air volumes and sound holes are generally so complex that an accurate calculation of the air resonance frequency is difficult. For practical purposes it is simplest to measure the resonant frequency. The formulas suggests how changes of the size of volume and hole area alter the resonance frequency.

THE STRING RESONATOR (FREQUENCIES - NODES - ANTINODES).

Let us as the next example study the resonances of a stretched string. The vibrations may look like the upper part of Fig. 2.10. We have seen this earlier, see section 2.1. We have also seen that the string may vibrate as in the lower parts, i.e. the string has not one resonance but several. The resonances occur at different frequencies and the vibration modes look different for each resonance. The vibration mode must now be included in our description of a resonance. For all resonances sketched there are no vibrations at the end fastenings. In the first resonance (topmost) the string has its maximum of vibration in the middle of the string, i.e. there is an antinode in the middle. In the second resonance (the second topmost part) there are no vibrations in the middle, i.e. there is a node in the middle of the string. The maximum vibrations are one quarter string length from the ends, i.e. there are two antinodes at these positions. In the third resonance (the next lower part) there are two positions of no motion (excluding the fastening points), i.e.nodes and three positions of maximal vibrations, i.e. antinodes. In the fourth mode there are three interior positions of no vibration and four positions of maximum vibrations. At resonance the string divides itself into "subparts" by a number of nodes with antinodes in between.

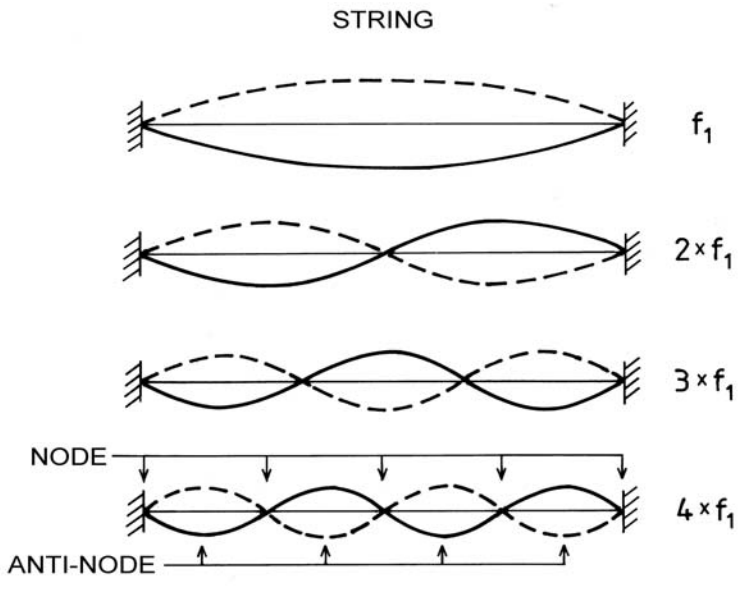


Figure 2.10. Resonances of a stretched string.

We have thus made a simple summary of a string's way to vibrate at resonance, i.e. the maxima of vibrations, ANTINODES, and the minima of vibration, NODES, for the four lowest resonances. Often the vibration modes of an object are described by giving the nodes (nodal lines). Another way, which may be more informative is to use the antinodes. In Fig. 2.10. we see that the first resonance has one antinode, the second two, the third three…. the 17:th resonance 17 antinodes, etc.. Note that the number of nodes also increases as the resonance number increases. The vibration modes are thus fully determined by the fastening and the length of the string. The frequency is determined by the string length (l), the string mass (weight) and the string tension. Relations between resonance frequencies, positions of nodes and antinodes are presented in Table 2.1.

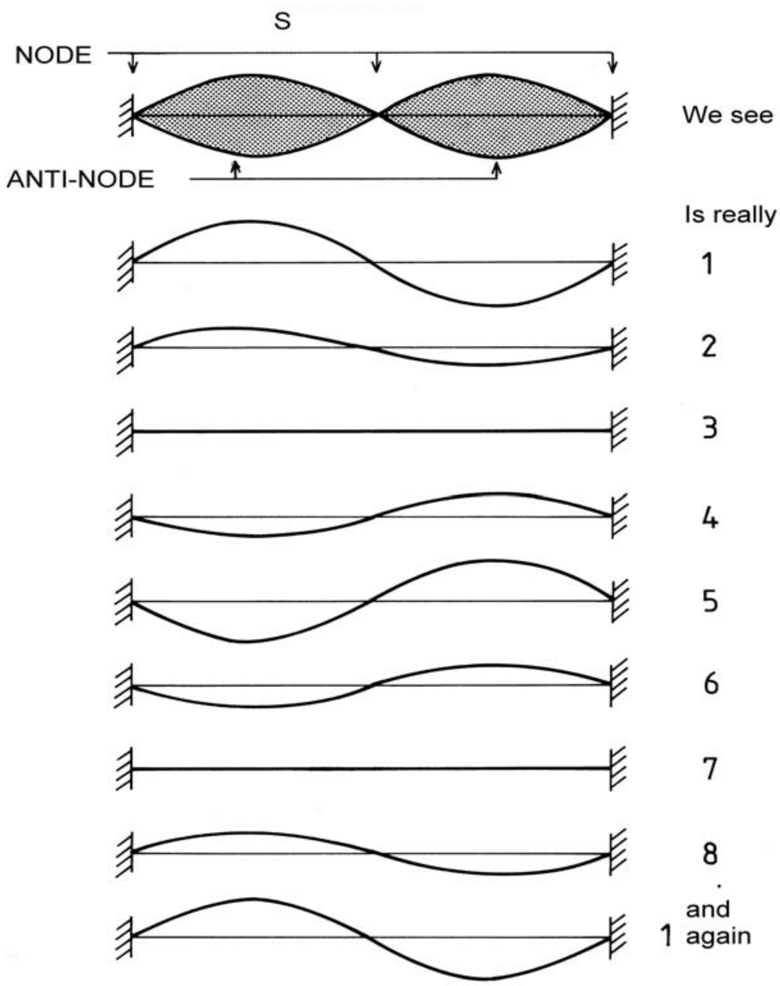


Figure 2.11. The vibrations of the second string resonance - the motion seen by the eye (top plot) and vibration shapes at consecutive instants (c.f., snapshots).

Table 2.1. The vibrating string - resonance frequencies, positions of nodes and antinodes.

If the frequency of resonance no	1	is f1	
then the frequency of resoance no	2	f2 = 2 x f1	
	3	f3 = 3 x f1	
	....		
	17	f17 = 17 x f1	etc

i.e. the frequency of the n:th resonance equals n times the first one.
If the length of the string is L then the nodes are at positions

for resonance no	1 position	0		L	
	2	0 L/2		L	
	3	0 L/3 2L/3		L	
	....				
	17	0 L/17 2L/17 ……....		L	etc.

The antinodes are halfway between the nodes i.e.

for resonance no 1	position L/2	
2	L/4 3L/4	
3	L/6 3L/6 5L/6	
	....	
17	L/34, 3L/34, 5L/34, ... …………33L/34	etc.

The vibration shapes sketched in Fig. 2.9 are the extreme positions of the string during vibration.. The extreme positions of the second resonance looks like the upmost frame in Fig. 2.11. But the vibration shape is changing all the time. In the lower frames of Fig. 2.11, the real vibration shapes are sketched for consecutive instants (snapshots of string shapes).

A resonance occurs at its own fixed frequency, when the resonator is driven (excited) at just that very frequency. At the pluck of a guitar string or at the hammer blow of the piano string, all the string resonances are set into vibration at once. The different string resonances will behave differently depending on their bandwidths for instance. Note the difference in the starting of the vibrations, plucking means pulling the string aside and then be left to vibrate freely (decay). In the piano the string is given a short push and is thereafter left to vibrate (decay) freely.

2.7 COMPLEX RESONATORS

We have so far studied fairly simple resonators, the properties of which can be described by exact mathematical formulas. We shall now continue with more complex resonators for which no exact mathematical formulas can be derived.

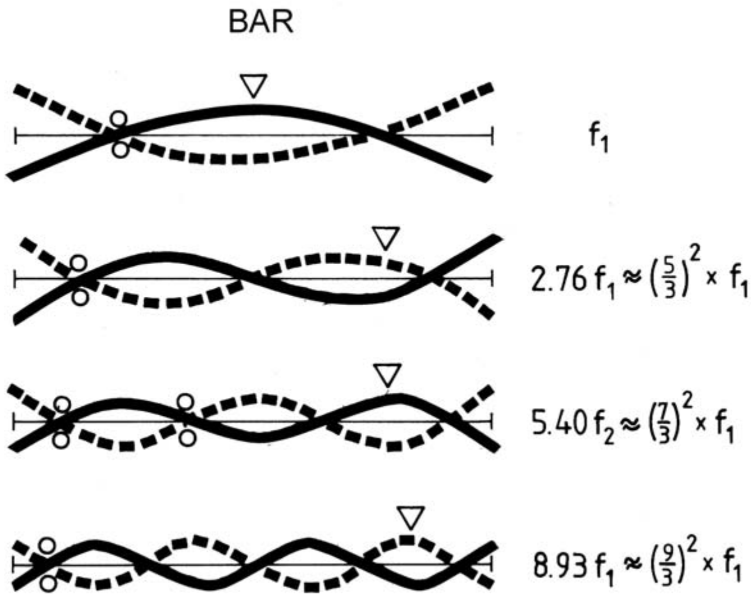


Figure 2.12. Vibration modes for resonances of a bar with both ends free.

THE BAR RESONATOR (FREQUENCIES, NODES, ANTINODES, VIBRATION MODES AND TAPPING TESTS). Let us so turn our attention to the bar resonator and its resonances. We shall treat the free bar, i.e. a bar not fastened at the ends or elsewhere.

The bar has many resonances. The vibration modes are sketched in Fig. 2.12. In the first resonance, the bar has two nodes, not at the ends but 22.4 % of the bar length from its ends. The vibrations are maximum at the ends and in the middle (about equal size), i.e. these are the positions of antinodes. For the second resonance there are three nodes at 13.2 %, from the ends and in the middle. The antinodes are at the ends and between the nodes. The following resonances have an increasing number of nodes, and the antinodes are in between and at the ends. Note that there are simple relations between the order number of the resonance and its number of nodes and antinodes. Resonant frequencies and positions of nodes are given in Table 2.2. Note that the relations between resonant frequencies and nodal positions are not as simple as for the string resonator. Resonant frequencies, position of nodes and position of antinodes can, however, be described with accurate (but complicated) mathematical formulas.

The lowest resonant frequency is determined by the stiffness, by the mass (weight) and by the length. A little work with the formulas shows that the resonant frequency is proportional to the thickness of the bar (and thus to its mass too).

What practical use follows from this theoretical backing? One good thing is that resonance frequencies can be obtained with tap tone testing. To do so one needs to know where to expect nodes and antinodes. For a bar it is simple. The length of the bar determines the position of nodes.

Table 2.2. Resonant frequencies and node positions of a free bar.

Resonance	Frequency	Nodes at
no 1	f1	22.4% 77.6%
2	f2=2.76 x f1 (≈5/3)2)	13.2% 50% 86.8%
3	f3=5.40 x f1 (≈7/3)2	9.4% 35.6% 64.4% 90.6%
4	f4=8.93 x f1 (≈9/3)2	7.3% 27.7% 50% 72.3% 92.7%

The antinodes are at the ends and in the middle between the nodal lines

Take for instance resonance no 1 for a 50 cm long bar. Hold the bar lightly at 22.4% from one end, i.e. 50 x (22.4 / 100) = 11.2 cm from the end. The bar will not vibrate at this position for the first resonance and this is a good position for holding. Tap at the antinode halfway between the ends, and the optimum way of listening to resonance no 1 has been chosen. For further instructions, see paragraph 2.8 Measurement of resonances in bars and plates.

The bandwidth of a resonance determines how long the tone can be heard. The pitch and the duration of the tap tone give measures of the frequency and the bandwidth of the resonance. In principle a scratching at the antinode can also be used. For a sufficiently sharp resonance, a weak tone with the pitch corresponding to that of the resonance is heard. A sharper resonance will give a better defined "scratch tone".

2.8. MEASUREMENT OF RESONANCES IN BARS, PLATES, AND SHELLS

TAPPING TESTS FOR RESONANCES (EIGENTONES).

The resonances of a bar (and a plate) can be sought and identified by applying the following rules:

1. HOLD lightly at a NODE. One should always hold at a node!
2. TAP at an ANTINODE.
3. To suppress a disturbing resonance, tap at the node of the disturbing resonance.
4. The author has found it convenient to hold the bar (or the plate) between the first finger and thumb of the left hand as lightly as possible. The left arm is held over the head with the bar hanging freely just outside the right ear. The tapping is done with the right hand, a finger tip, a knuckle or a nail (the higher resonance frequency the harder "hammer head" should be used). The tapping point is just outside the ear and it is shifted relative to the bar by moving the left arm up or down. The holding point is shifted by letting the bar slide a small amount between the first finger and thumb. It is often suitable to lean slightly forwards to ensure that the bar is hanging freely (nothing is allowed to touch the bar but the left hand first finger and thumb).

The best way is, however, obtained with the so called Chladni method and a loudspeaker, c.f. Fig. 2.2. A loud tone of the loudspeaker sets the bar into vibration and small particles, such as coarse saw dust is sprinkled over the bar. The nodal lines are found in the following way:

1. the loudspeaker is placed under an expected antinode
2. the supports in the form of small pieces of foam plastic are placed under two expected nodal lines
3. the saw dust is sprinkled over the bar
4. the frequency of the loudspeaker tone is adjusted to a frequency at which the particles have the largest motion
5. the positions are sought where the saw dust collects, i.e., the positions of nodal lines

6. the measurements are optimised by repositioning the supports at nodal lines and the loudspeaker under an antinode.

The frequency of maximum vibration equals the resonance frequency and the lines where the saw dust collects are nodes (nodal lines). The positions where the saw dust starts moving are the antinodes. A coarse measure of the vibration sensitivity peak level can be obtained by watching how much the volume control of the amplifier must be turned up to start moving the saw dust at antinodes (a low volume means a strong resonance).

THE PLATE RESONATOR (FREQUENCIES AND NODAL LINES).

A plate has two main directions in which it may bend and therefore resonances are found in two directions. The nodal line patterns can for a free plate in principle be ordered as

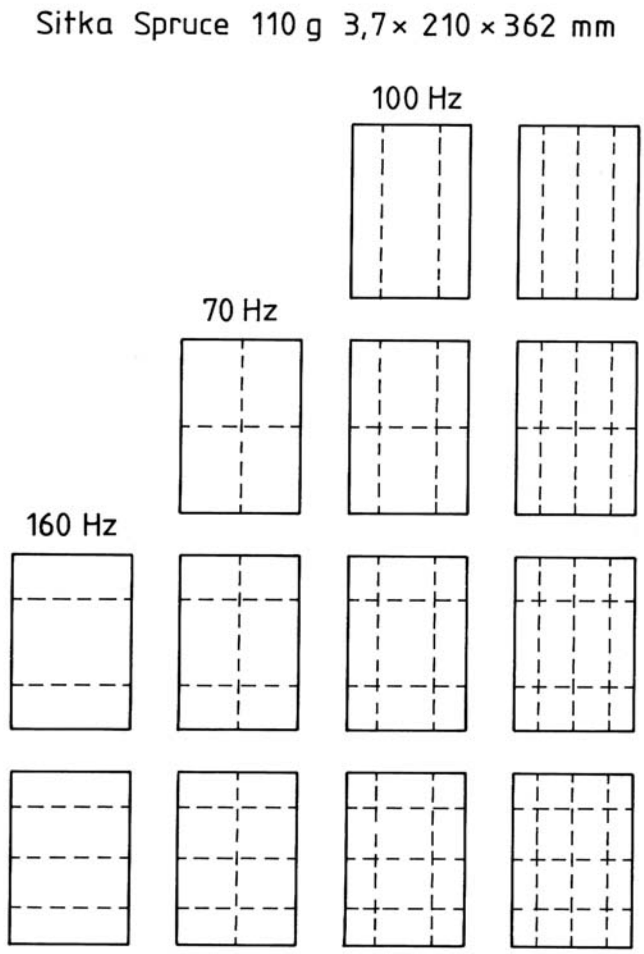


Figure 2.13. Nodal lines (broken lines) for a rectangular plate with free edges (frequencies for a Sitka Spruce plate 110 g, 362 x 210 x 3.7 mm).

in Fig. 2.13. In the first row the resonances with zero vertical nodal lines are placed (two resonances with two and three horizontal nodal lines are sketched - compare the bar Fig. 2.12). In the second row the resonance with one vertical nodal line is placed, etc. In the first line there are resonances with zero horizontal nodal lines placed, in the second line with one horizontal nodal line, in the third line those with two horizontal nodal lines, etc. . Higher resonances have nodal lines which are combinations of the simplest ones. The antinodes are centred between the nodes in the inner part and along the edges. The resonance frequencies are determined by the thickness, length, width, and mass of the plate.

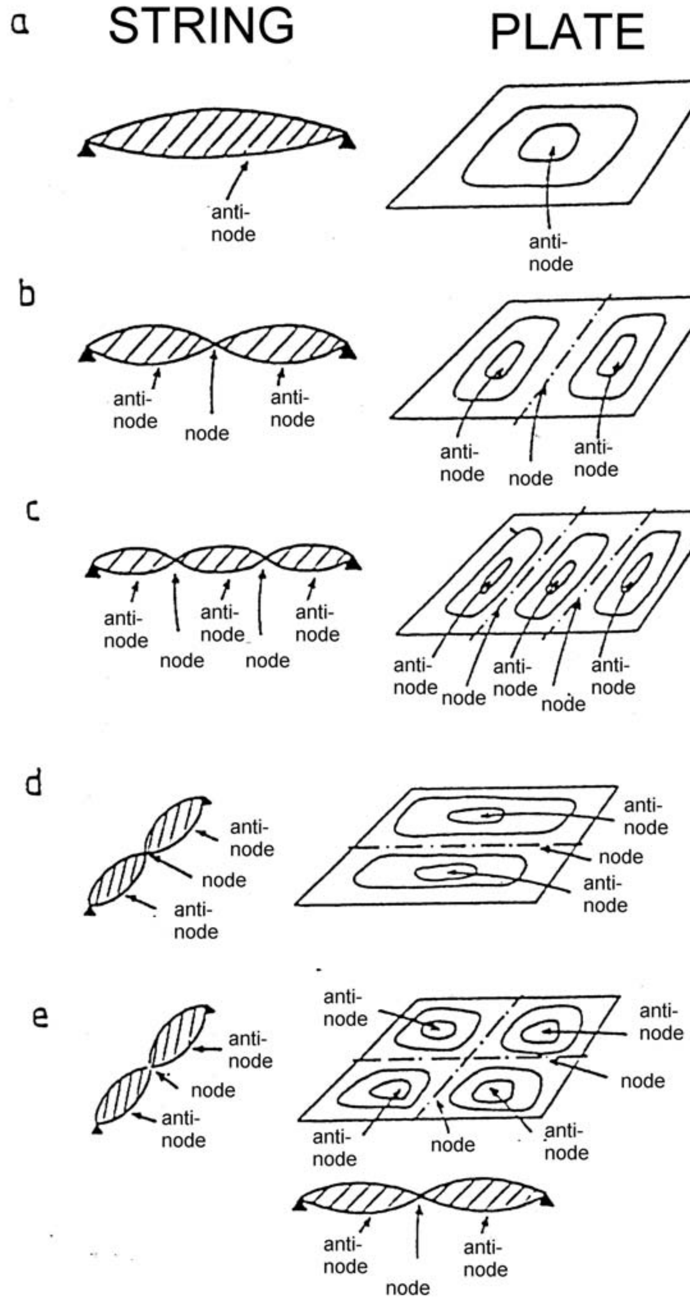


Figure 2.14. Vibration modes for a plate fastened along its edge compared to those of a string. Nodal lines along the edges and inside (broken-dotted lines). Lines of equal vibration are also plotted (full lines, bent and closed).

In Fig. 2.13 three measured resonances are marked together with their nodal lines and frequencies. A complication should be mentioned. When a plate is bent in one direction it will by itself bend in other ways at the same time. When the plate is bent down at the short edges of Fig. 2.13 the longer edges will bend slightly upwards. This means that the nodal line patterns sketched in Fig. 2.13 and 2.14 look somewhat different in the real world, especially the high frequency ones.

THE INFLUENCE OF FASTENING AND THE SHAPE OF THE PLATES

The plates are, however, often fastened along the edges, and such a rectangular plate will give vibration modes as sketched in Fig. 2.14. The magnitude of the vibrations have been marked with lines of equal vibration. Observe the close analogy with the string in two directions.

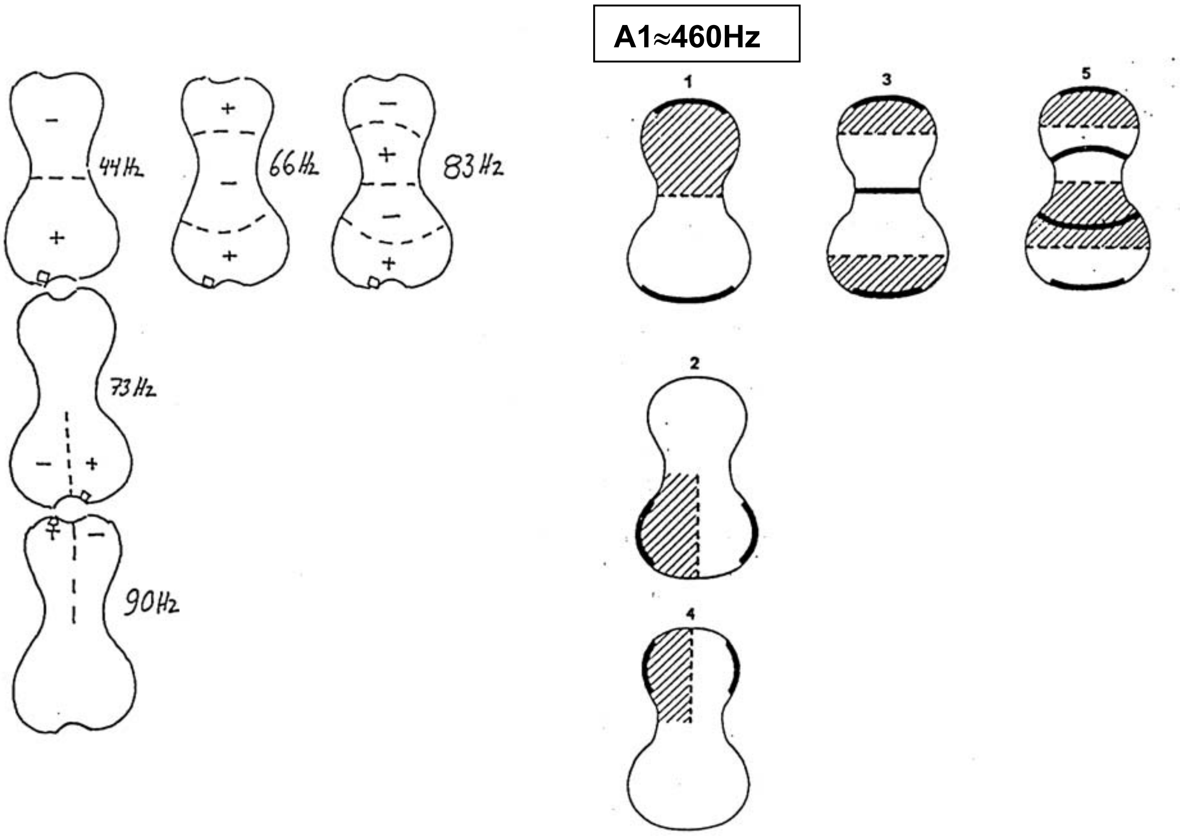


Figure 2.15. (left) Vibration modes of a violin shaped rubber membrane - the first seven modes with nodal lines and antinodes indicated with a plus or minus sign , and (right) air modes of the violin body, A1 being the first..

The frequency and not only the vibration modes are much dependant on the fastening along the edges (or at the ends). Say that the first resonance frequency of a bar with free ends is 100 Hz. If the ends are clamped the same first resonance frequency is still obtained, but the nodal lines are moved to the ends. If the ends are fastened with hinges the frequency is lowered to 44 Hz. In the violin and the guitar the plate fastening is be somewhere between clamped and hinged.

The shape of the fastening also influences frequencies, nodal lines and antinodes. The influence of the nodal lines for a rubber membrane stretched over a set of ribs are shown in Fig. 2.15. The rubber membrane was set into vibration by a small vibrator and small cork fragments marked lines of no motion at the different resonance frequencies, i.e. nodal lines. It can be seen that the "waist" of the violin divides the membrane in two partly independent areas. The same kind of nodal patterns are found in the upper and lower part, c.f., 73 Hz and 90 Hz, and 106 and 127 Hz, respectively. The resonances of the aircavity show, similar patterns, see fig. 2.14 (right) . Note that resonance 1 (A1) is close to the main resonances of a violin.

ARCHING (SHELLS)

A violin top is not a flat plate but a slightly arched plate. Effects of the arching can be tried in a simple way by means of a playing card, c.f. Fig. 2.16. The card is held arched between the thumb and pointer. If the distance between the two fingers is varied the card will bend in and out in the middle. If one with the second hand increases or decreases the arch height of the card one feels with the first hand that the fingers on the sides of the card move together. The arching couples motions perpendicular to the card surface to motions in the plane of the card, especially along the edges.

An arched plate is in physics called a shell and not a plate. To test the influence of arching a series of simple experiments were made. A rectangular spruce plate, 3 mm thick, 215 mm wide and 290 mm long (fibres along the plate and annular rings perpendicular to the surface) was selected. First the plate was bent by means of a string-loop across the plate at each of the nodal lines marked in Fig 2.13 (160 Hz). A wedge was pushed between each string and the plate giving a 6 mm arch (H, in Fig 2.16) and the the resonant frequency increased 50 % (increased stiffness) compared to flat. By locking the motion of the edges with a light clamp at each nodal line the resonant frequency increased another 50 %. Secondly the plate was bent by two string-loops along the nodal lines marked in Fig 2.13 (100Hz) and two wedges. The resonant frequency increased 60 %. Experiments with locking the edges by clamps introduced a new complication. The static clamping force gave a large influence on the dynamical properties. The experiments show that the arching has a large influence. In the top and back plates of the violin the arching is larger than in the experimental plate.

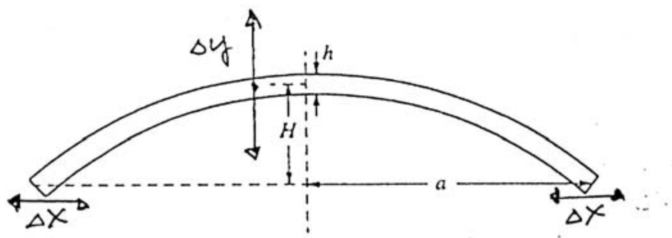


Figure 2.16. In an arched plate (arch height H) vibrations Δy result also in vibrations Δx.

2.9. SUMMARY:RESONATORS

In this part four kinds of resonators have been described: the hole-volume resonator (the Helmholtz resonator), the stretched string, the bar and the plate. The resonant frequency of the hole-volume resonator is determined by the size of hole and volume - large hole and small volume give high resonant frequency. The resonant frequencies of the string are determined by the length, the mass (the weight) and the tension of the string - a short string with high tension and small mass results in high resonant frequencies. The frequency of the second, third, etc. resonances are two, three etc. times the frequency of the first resonance. The resonant frequencies of the bar and the plate are determined by the length, the width, thickness and mass. A short bar, and a short and narrow plate give high resonant frequencies. A thick and light bar and plate will have high resonant frequencies. In general there are no simple relations between the different resonant frequencies of the bar and plate, respectively. The effects of different fastenings have been introduced. Furthermore the vibration at resonance have been described by nodal lines and antinodes. Finally a simple way has been described on how to test resonant frequencies of bars and plates by tapping and listening. Furthermore a somewhat more advanced method by Chladni patterns, giving vibration patters as well as resonant frequencies. For the violin top and back the arching increases the stiffness of a plate in perpendicular to the arching.

2.10 KEYWORDS:

Resonator, resonance, resonant frequency, bandwidth, decay time, vibration modes (vibration patterns), nodes, antinodes, hole-volume resonator, string resonator, bar resonator, plate resonator, free ends, hinged ends and clamped ends.

2.11 APPENDIX

The resonant frequency of the hole-volume resonator is in principle $\sqrt{A/V}$ where A is area of hole and V is volume of cavity. The length with end corrections l_{eff} are considerably larger than the length l for thin-walled musical instrument, and resonant frequency $= (c/2\pi)\sqrt{A/(l_{eff}V)}$.

Formulas and some numerical values on the relation between acoustics properties and mechanical properties of a resonator

In mathematical language the mobility (vibration sensitivity) for a resonance

(vibration velocity/ vibration force) =

$$= (2\pi f/\sqrt{SM}) \times (1/\sqrt{(B/f_o)^2 + (f/f_o - f_o/f)^2} =$$

=(specific mobility) × (resonance properties) (c.f. Fig. 2.5)

In the formulas S is the spring of the resonator, M is its mass and B its bandwidth, f_0 is the resonance frequency $=(1/2\pi)\sqrt{S/M}$ and f is the frequency of evaluation.

Examples:

1a) If the mass is doubled or halved, what happens to the resonance frequency?
A little calculation shows that the resonance frequency is lowered or increased approximately 6 semitone steps (from 500 to 350 or 500 to 700 Hz).

1b) If the stiffness is doubled or halved, what happens to the resonance frequency?
A little calculation shows that the the results are opposite to those in 1a, i.e. the resonance frequency is lowered or increased approximately 6 semitone steps (500 to 700 Hz or 500 to 350 Hz).

2a) What is the time for 60 dB decay (the reverberation time) for the bandwidths 500, 25, 12.5 and 6.25 Hz?
With some calculations it can be shown to be 0.0002, 0.088, 0.176, and 0.352 seconds, respectively.

2b) How much higher is the resonance peak higher level than the specific vibration sensitivity at 500 Hz for the three bandwidths?
Again a little calculation shows the peak level is 0, 26, 32, and 38 dB above the specific vibration sensitivity, respectively.