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ACOUSTICS FOR VIOLIN AND GUITAR MAKERS

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Chapter IV:
Properties of the Violin and the Guitar String

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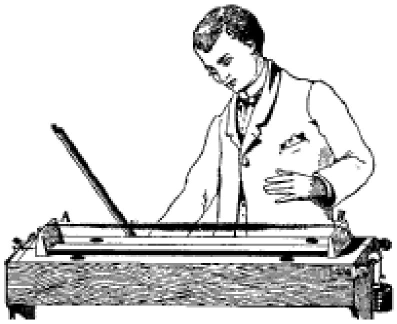

Index of chapters

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ACOUSTICS FOR VIOLIN AND GUITAR MAKERS

Chapter 4 – Applied Acoustics
PROPERTIES OF THE VIOLIN AND THE GUITAR STRING

Part 1: FUNDAMENTALS OF STRINGS

Part 2: PLAYED STRINGS AND SCALES

Chapter 4.
APPLIED INSTRUMENT ACOUSTICS - PROPERTIES OF THE VIOLIN AND THE GUITAR STRING
First part: FUNDAMENTALS OF STRINGS

INTRODUCTION

In the first part of this chapter the acoustical fundamentals of the string is presented. First fundamental string theory is introduced, resonances, vibration sensitivity and their relations to mechanical properties of the string. Thereafter the properties of string materials are given and finally the effects of winding on a string.

4.1 FUNDAMENTAL STRING THEORY (resonance, resonant frequency, bandwidth, nodes, antinodes, vibration sensitivity (mobility), tension, mass, specific specific vibration sensitivity (specific mobility)).

ACOUSTICAL PROPERTIES

The acoustical properties of a resonator can be measured in terms of vibration sensitivity (mobility). For the simple resonator a resonance curve is obtained with a peak like in Fig. 4.1. The peak and the shape of the complete curve is fully determined by three measures, the position, the height, and the width of the peak i.e., the RESONANT FREQUENCY, the PEAK LEVEL, and the BANDWIDTH respectively, see Fig. 4.1. Often another level measure is interesting, i.e. the specific vibration sensitivity (specific mobility). The specific vibration sensitivity can often be used as a material constant. The peak level can be calculated from the resonant frequency, the bandwidth, and the specific vibration sensitivity. The bandwidth is also a measure of how long it takes a constant driving force to bring the system into equilibrium. Thus the bandwidth predicts the length of the starting transient.

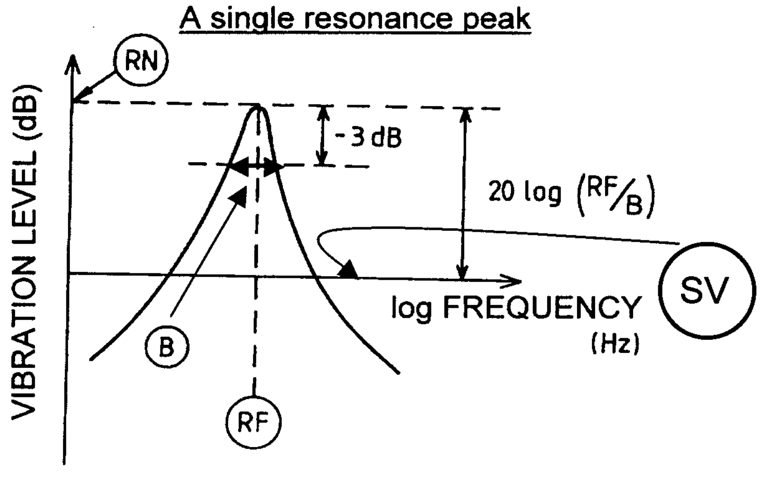

Figure 4.1. Acoustical properties of a resonance: vibration sensitivity response with resonance frequency RF, bandwidth B, peak level RN and specific vibration sensitivity SV.

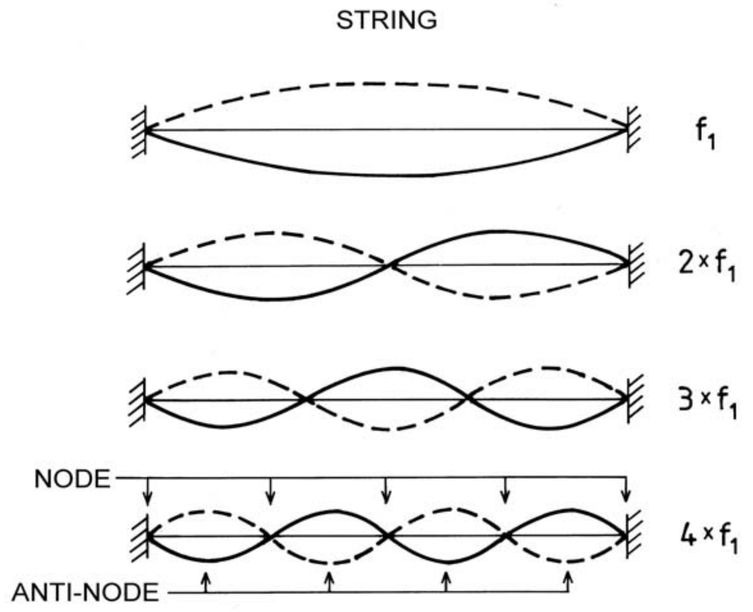

Figure 4.2. Resonances of the string: modes of vibration, their nodes and antinodes

The string has several resonances, see Fig. 4.2. The vibrations are distributed in specific ways. For the ideal string the resonant frequencies (the partials) are 2x, 3x, 4x, etc. the frequency of the lowest resonance, the fundamental tone. At each resonance the string vibrates in a different mode, see Fig. 4.2. In the lowest resonance the string has one antinode, and nodes at its ends, in the second resonance it has two antinodes with nodes in the middle and at the ends, etc. The vibration sensitivity curve of a string displays a number of peaks, see Fig. 4.3. Each peak has its own frequency, the resonant frequency, its own bandwidth, and its own peak height. The peak height indicates the strength of each string tone (partial, resonance) and the bandwidth indicates its reverberation time. The specific vibration sensitivity is a measure of how the string will cooperate with the instrument.

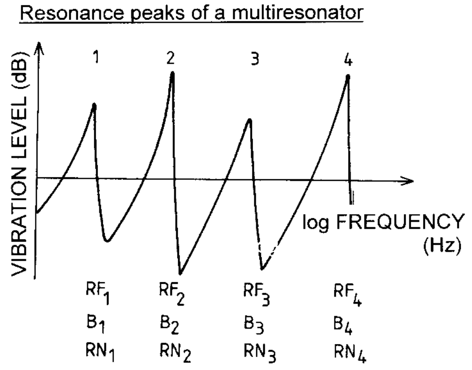

Figure 4.3. Resonances of the string: every resonance peak has its own frequency, bandwidth and peak height.

SPECIFIC VIBRATION SENSITIVITY - DRIVING FORCE

The specific vibration sensitivity is a fundamental measure of how sensitive a resonator, a string or a plate is to vibration forces. But, if one wants to study the sensitivity to vibrations in detail also the resonance properties such as frequency and bandwidth must be included.

The specific vibration sensitivity (and the resonant properties) is a measure how vibrations are transferred, from a string to a violin for instance.

FIGURE. 4.4. Force transfer between two resonator systems a) stiffness controlled and b) mass controlled

Example 4.1: If you are standing on a thick plank and want to bend a board in the ceiling (for nailing it, see Fig. 4.4) then
a) it is easy if the plank is stiff and the board soft, but
b) it is difficult if the plank is soft and the board is stiff.

Thus it is not only how strong you are but also the strength of the support you are pushing from that is important. If you push and pull repeatedly in the board in case
a) the board will bend much and the plank little and
b) the board will bend little and the plank much. (compare with the formula for specific vibration sensitivity - the stiffness corresponds to the spring in Fig. 2.8).

Example 4.2: If you are pushing a small dinghy away from a yacht then the dinghy is given a considerable motion but the yacht a small one. If you push and pull alternatively the dinghy moves much and the yacht little. The motions are independent of whether you are in the dinghy or in the yacht (compare with the formula for specific vibration sensitivity - the weights of the boats corresponds to the mass in Fig. 2.8).

We can see a certain relation between the motions in the two cases, board-plank and dinghy-yacht, i.e., vibrations are efficiently transferred from a stiff plank with low vibration sensitivity to a soft board with high vibration sensitivity. The same relation applies to the two boats. From a heavy boat with low vibration sensitivity vibrations are efficiently transferred to a light boat with high vibration sensitivity. In the opposite direction the transfer works inefficiently; it is difficult to transfer vibrations from an object with high vibration sensitivity to one with low vibration sensitivity. The transfer of vibrations is dependent on both the force applied and the ratios of the mobilities of the two objects. Let us accept with the sketched background that the specific vibration sensitivity of the transmitter and the receiver determines the vibration transfer.

Example 4.3. The output connectors of the amplifier should have the same labelling as the loudspeaker - 4 ohms to 4 ohms or 8 ohms to 8 ohms.

Example 4.4: For a car motor to work efficiently an adaptation must be used - the gear box.

Example 4.5: A string has a high specific vibration sensitivity and a violin or a guitar has a low one - thus the string will vibrate much and the violin or the guitar little. But the closer to equality the ratio of string vibration sensitivity to the violin or guitar vibration sensitivity are, the more sound comes out of the instrument. If the mobilities are equal a "wolf tone" is obtained.

MECHANICAL PROPERTIES

The properties of the simple resonator are determined by its mechanical properties, i.e. the mass (the weight), the stiffness and the total friction. For the multiresonator system like the string, material constants are obtained by the mass (weight) per meter of the string, the string tension (the stiffness), and the internal friction.

PRACTICAL FORMULAS FOR SMALL CHANGES

In the practical cases one chooses between different strings and thus it is interesting to estimate how different mechanical properties influence the main acoustical properties. Two simple formulas can be derived:

the resonant frequency change (in percent) = - length change (in percent) +
1/2 x tension change (in percent) - 1/2 x mass change (in percent).

the specific vibration sensitivity change (in percent) = - 1/2 x tension change (in percent) - 1/2 x mass change (in percent)

The formulas point out two very important facts which are presented in the examples below:

Example 4.6. What is the consequence of the relation between the resonant frequency and mechanical properties like weight per meter and string tension?

In most musical instruments the fundamental frequency and the string length are set by the tuning of the instrument and its construction. Only the string tension and the mass per meter of the string can be selected. Therefore the ratio between the tension and the mass per meter must be constant in order to keep the fundamental frequency as required. The practical formula says the same in another way - if the mass is increased by 10 % the tension also must be increased by 10 %. Result: for many practical applications the string mass per meter is the fundamental measure.

Example 4.7.: What is the consequence of the relations between specific vibration sensitivity and mechanical properties?

Again we start from the fact that most musical instruments have a given string length and tuning. This means that only the string tension and the string weight per meter can be chosen. The tuning condition sets a specific ratio of tension to weight. The formula for the specific vibration sensitivity says that the specific vibration sensitivity is proportional to 1/weight and again the weight per meter is the most important measure for the string. Result: it is sufficient to know the weight per meter, thereby the specific vibration sensitivity of the string is given.

Example 4.8.: If we start from normal conditions and thereafter change a mechanical property at a time, how are the acoustical properties changed?

a) if the string tension is increased 2 %
=> the frequency is increased 1 %, i.e., from 440 to 444 Hz
the specific vibration sensitivity decreases 1% (the sound level increases 0.1 dB)
b) if the weight is increased 10% but the tuning is kept
=> the string tension must be increased 10% and the specific vibration sensitivity is lowered 20 % (the sound level increases 2 dB)
c) if the string length is increased 5 % but the tuning is kept.
=> the string tension must be increased 10% and the vibration sensitivity decreases 5 % (the sound level is increased 0.5 dB)

Example 4.9.: Say that a resonance (partial) of a guitar string decays 10 dB in a second. What is the bandwidth of the string resonance?
A little calculation shows that this reverberation corresponds to a bandwidth B of 0.3 Hz, which is a very sharp resonance peak.

4.2 HOMOGENEOUS STRINGS (mass, tension, tensile strength, elasticity modulus, typical string tension, thickness, inharmonicity, and partials)

A common wish is to have a string with strong tone and a high fundamental frequency. How can this be obtained? In principle the string should be made short, heavy and tightened up to a high tension. But the heavy string favours tuning to low frequencies and the light one to high frequencies and therefore a compromise must be made.

In the theoretical introduction we have already shown that the weight of the string is the most important and "automatically" determines the string tension. It is therefore interesting to know the density (weight per cubic meter) for materials, see table 4.1. For string materials we are in the heavy range, see table 4.2 for typical string materials.

Table 4.1. Densities of common materials

Air	1.2	kg/m3
Water	1 000	"
Lead	11 000	"

Table 4.2. Density of typical string material

Steel	7 700	kg/m3
Gut	1 300	"
Silk	1 300	"
Nylon	1 200	"

Example 4.10. How much thicker should a gut string be to give the same tension as a steel string for the same pitch?
A little calculation on weight, tension and fundamental frequency shows that the gut string must be 2.5 times thicker.

Example 4.11. How much do we loose in specific vibration sensitivity when we use a gut string as thick as a steel string which we tune to the same frequency?
The density for steel is 6 times that of gut which means that the gut string tension becomes 1/6 of the steel string tension. This means that the specific vibration sensitivity increases 16 dB, and the sound level decreases 16 dB (this is a very large decrease).

The strength of a string is tested by tensile tests. In a measurement apparatus the string tension (load) is increased and the resulting lengthening (strain) is measured. The tension is increased until the string breaks. Thereby a load-strain diagram is obtained, see Fig. 4.5a. The first part of the curve is a straight line. If the pulling force is disconnected within a certain range (up to the limit **P**) the string regains its original length. The strain within this range is elastic. At the limit **P** the elastic strain turns into a plastic strain and a remaining lengthening is left after the tension has been disconnected, se Fig. 4.5b. If the string is stretched until it breaks, the fracture limit **B**, a measure is obtained of the maximum tension possible.

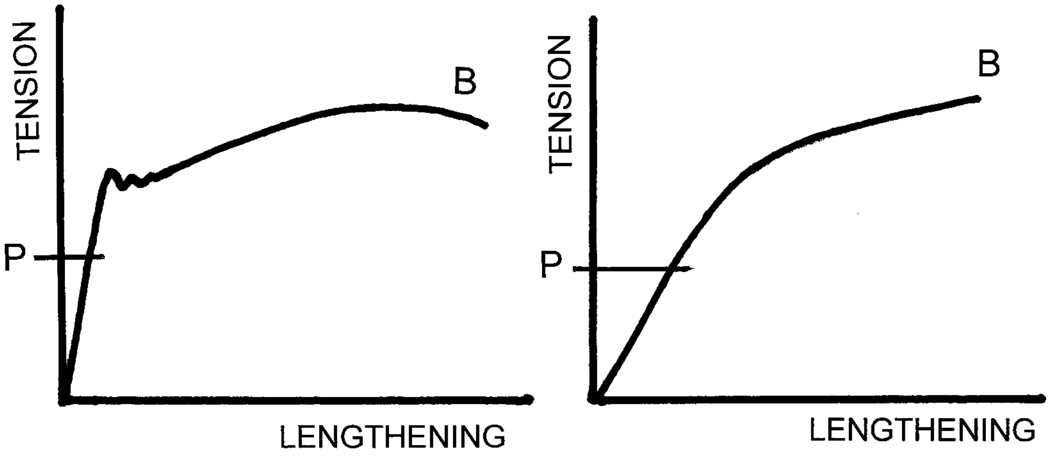

Figure 4.5 a) Typical tension-lengthening (load-strain) diagram and b) diagram of plastic lengthening.

Strings can not be stretched to any limit. First the string tension is increased, thereafter it is stretched and finally it breaks. The breaking limit **B** sets is the maximum frequency the string can be tuned up to but the elastic limit **P** should be the maximum useful limit. The breaking limit for some materials are given in table 4.3.

Table 4.3. Fracture limit for typical string material (107 Pa = 1 "kg/mm^2")

Steel	2 100-2 600	x 106	Pa
Gut	320 - 460	"	"
Silk	350 - 550	"	"
Nylon	600 - 750	"	"

i.e. a steel string can be stretched 3 to 4 times that of the gut, silk and nylon materials.

The string should not be stretched to the breaking limit. Suitable limits are given for loading are given in table 4.4. The table says that strings are loaded to 50 % of the breaking limit. Thus the steel string can still be tensioned 3 to 4 times more than the gut, silk and nylon strings.

Table 4.4. Suitable tension in percent of the fracture limit.

Steel	40 - 75 %
Gut and fibres	35 - 70 %

ON INHARMONIC PARTIALS

(This paragraph is accidentally too mathematical-technical. Do not read it if difficult)

Table 4.5. Elasticity modulus E (typical) and inharmonicity factor I for different string tensions

	E x109 Pa	0.5 Imax	0.75 Imax	1.0 Imax
Steel	220	56	38	28
Gut	5.5-6.5	9.4	6.3	4.7
Silk	5.0-6.0	7.5	5.0	3.8
Nylon	4.5-5.5	4.5	3.0	2.3

Steel has a high inharmonicity factor and must therefore be given a high tension (the piano string has so high a tension that the string can be broken by a hard touch).

Example 4.12. What is the tension of the E-string of the violin if made by steel with the diameter 0.25 mm?

A violin string has a free length of close to 325 mm = 0.325 m, diameter 0.25 mm = 0.00025 m and the density of steel is 7 800 kg/m3, which results in a weight of 7 800 x 3.14 x $(diameter/2)^2$ = 0.000383 kg/m. The frequency is 660 Hz and a little calculation gives the string tension 70 N ("7.1 kg").

Example 4.13 If the A, D and G strings are made of steel and all should have the same tension, how thick should they be made?

For the same tension and the same material it turns out that the diameter should be a constant divided by frequency. The violin is tuned in fifths which means that the string diameters should be increased successively by a factor 3/2 and the A string (3/2) x 0.25 = 0.38 mm, the D string (3/2) x (3/2) x 0.25 = 0.56 mm, and the G string (3/2) x (3/2) x (3/2) x 0.25= 0.84 mm

Example 4.14: What kind of inharmonicity should we expect from the first and the third strings of a guitar if they are made of nylon and with diameter 0.71 and 1.03 mm and with tension 77 and 52 N.

A little calculation shows that the strings are tensioned to 30 and 10 % of the fracture limit and the inharmonicity factor is approximately 8 and 16 respectively. If we multiply with $(d/L)^2$ (1.19 and 2.5 x 1 000 000) and use the formula we find that the third string gives an inharmonicity 7 times that of the first string (note that the calculation example gives a good demonstration but can not be expected to be accurate).

The examples show that one should have a high tension to obtain a low inharmonicity, or to make a heavier string without increasing the stiffness, i.e. by additional winding.

4.3. WOUND STRINGS (mass and material for winding)

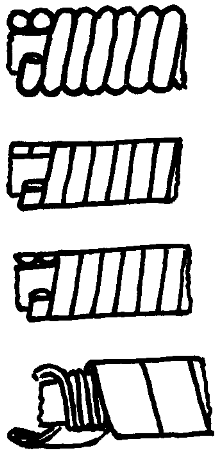

Figure 4.6. Different types of string windings

To obtain strings with strong tone, i.e., heavy strings without making them very stiff, the strings are made in several parts.The core is fairly thin and has little stiffness. Thereafter this core is covered with one or several layers of windings and a wound string is obtained. The winding can be made in several ways, see Fig. 4.6. The winding is made in such a way that the string stiffness is little increased. By means of suitable winding materials, the string is made to the desired weight, see table 4.6

Example 4.15: The G-string of the violin has a diameter of 0.8 mm, of which 0.2 mm is a silver winding. What is the density in kg/m3 and what is the string tension?
a) a little calculation shows that the string density is 4 700 kg/m3
b) the frequency of the open string is 196 Hz which together with some calculation gives the string tension as 38 N ("3.9 kg").

Table 4.6.Density for typical winding materials.

Aluminium	2 700 kg/m^3
Steel	7 700 "
Copper	8 900 "
Silver	11 000 "
Gold	19 000 "
Wolfram	19 000 "

Example 4.16: What will the density of the string become if wound with gold instead of silver?

Table 4.6 shows that the density for aluminium is 2 x Gut, for steel it is 6 x, for silver 8 x and for gold and wolfram (tungsten) it is 15 x. Thus the string mass could be greatly increased by winding with little increase of thickness", i.e., the string weight has been doubled but the diameter has not been changed.

4.4: SUMMARY: FUNDAMENTALS OF STRINGS

In this part, fundamental string theory has been summarised, i.e., the relations between acoustical properties such as resonance frequencies and vibration sensitivity, and mechanical properties such as the mass (weight), length, and tension of the string have been given. The influence of the string diameter on the string inharmonicity has been demonstrated. Finally the mass increase by winding of a string without making it stiffer has been shown.

4.5: KEY WORDS:

Resonant frequency, string length, string tension, string mass (weight), modes of vibration, antinodes, nodes, vibration sensitivity, string diameter, wound strings, fracture limit, and elasticity modulus.

Chapter 4.
Second part: PLAYED STRINGS AND SCALES

INTRODUCTION

In the second part of this chapter we are approaching the real world of played strings. First properties of real strings are presented. Thereafter the fundamentals of the plucked and the bowed string is introduced, effects of bow gestures and played scales. In the appendix the relations between tone frequency (in Hz) and tone position (name and deviation in cents from nominal value).

4.6 REAL STRINGS

As pointed out in the theory part it is sufficient to weigh a string to be able to calculate its most important properties. The weights of the strings and not their diameters should be given on the string envelopes. For very thin strings extremely sensitive scales are needed and it may be easier to measure the fundamental frequency of a string when tensioned with a well defined force, see Fig. 4.7. Measured frequencies can be recalculated into string weights. Measured string weights are given in table 4.7 for violin strings (Pickering found also that some strings had rather high inharmonicity and large bandwidth) and these are given in table 4.8 for guitar strings. Furthermore an example on inharmonicity for violin strings is given in table 4.9.

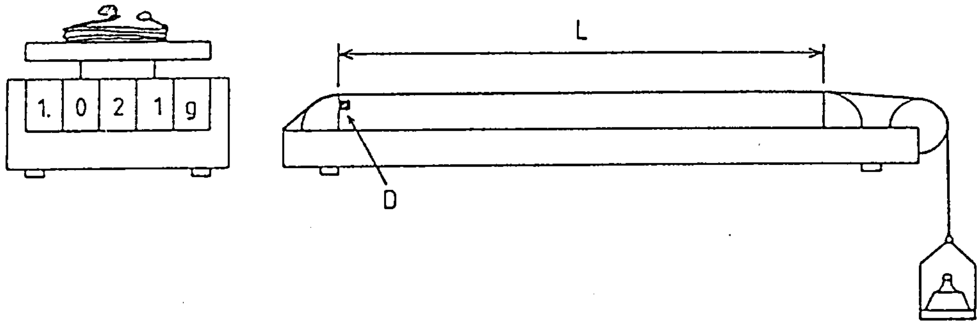

Figure 4.7. Two ways to determine the main properties of a string are a) by measuring the mass (weight) with a precision scale, or b) by measuring the frequency with a well defined tension (L is the string length and D a vibration sensitive detector).

Table 4.7. Measured properties of violin strings (after Pickering)

	E-string	A-string	D-string	G-string
Fundamental frequency (Hz)	660	440	294	196
Diameter (mm)				
min-max	0.249-0.264	0.452-0.701	0.671-0.914	0.790-0.833
Tension (N approx "kg/10")				
min- max	72.25-84.01	48.89-63.51	34.76-61.73	35.43-49.92
Weight (mass) (g/m)				
min-max	0.381-0.443	0.579-0.752	0.924-1.641	2.115-2.799
Spec vibration sensitivity				
(m/Ns) min-max	5.18-6.02	4.57-5.94	3.14-5.58	2.59-3.65

Table 4.8. Measured properties of guitar strings

String	E	H	G	D	A	E
Fundamental frequency (Hz)	330	247	196	147	110	82.4
Diameter (mm)	0.70	0.83	1.03	0.75	0.93	1.07
Tension (N approx "kg/10")	76.7	61.1	57.9	74.5	74.5	61.2
Weight (mass) (g/m)	0.417	0.593	0.892	2.04	3.45	5.33
Spec vibration sensitivity (m/Ns)	5.59	5.25	4.33	2.56	1.97	1.97

Table 4.9. Inharmonicity (cent*) and bandwidth (BD) measured for four violin strings

STRÄNG (Hz)	F1 / BD (cent / Hz)	F2 / BD2 (cent / Hz)	F3 / BD3 (cent / Hz)	F6 / BD6 (cent / Hz)	F10 / BD10 (cent / Hz)
E 656.5	0 0.27	0 0.62	0 0.50	1 0.75	4 1.5
A 440.2	0 0.32	1 0.41	2 0.56	0 1.3	1 2.3
D 296.0	0 0.29	-2 0.79	0 1.1	5 3.5	17 13.1
G 196.6	0 0.16	-1 0.35	1 0.72	9 1.9	19 8.3

*Cent is a measure of deviation from correct nominal value in hundreds of a semi-tone step (see section 4.8).

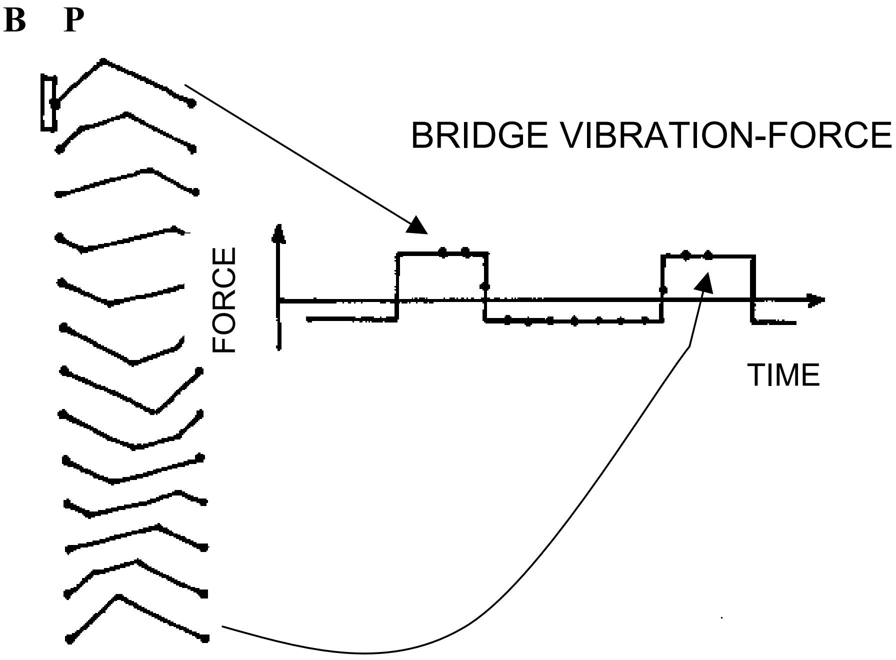

Fig 4.8 Time history (in principle) for the plucked string, bridge B and plucking position P (after Hall) and bridge forces. The different vibration shapes and corresponding bridge forces are indicated by points in the bridge force curve.

4.7 PLAYED STRING IN PRINCIPLE (time history, shape of deflection, spectra, the plucked and the bowed string)

The plucked string has in principle the following time history. The string is pulled out of equilibrium and makes a triangle as showed in the upper left corner of Fig. 4.8. When the string is released it will develop a complicated vibration as sketched in the left column. It can be seen that the string is divided into three straight parts. The string angle in relation to the bridge determines the excitation force by which the string will shake the bridge. If one looks closely at the shape of the string one can see that the string only has two different angles at the bridge, which results in half the maximum force upwards in the first case and the maximum force downwards in the second case. The vibration force will show a time history as shown in right part of Fig. 4.8. It should be noted that the time history shows a downward pulse for one third of the time, i.e., the same relation as the one to three division of the string at the plucking position.

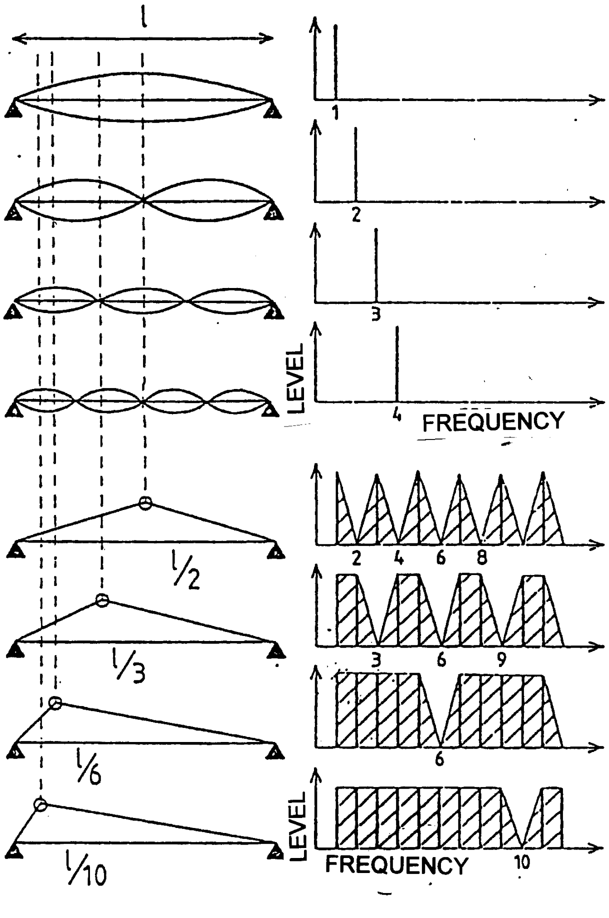

Figure 4.9 Position of plucking and excitation strength of resonance (in principle). Left upper part the lowest four resonances, right upper the corresponding partials, down left the initial shape of the string at plucking for different plucking positions and down right the resulting spectra for the different plucking positions.

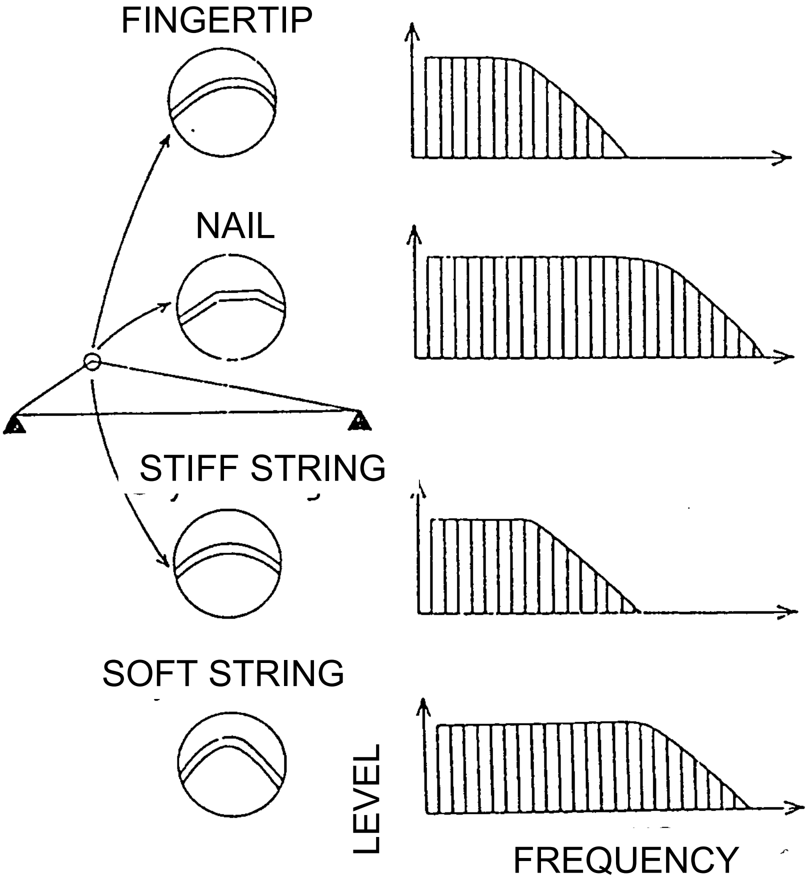

Figure 4.10 Influence of different pluckings and of different strings

The relation between position of plucking and the level of partials have been sketched in fig 4.9 When plucked in the middle, the string is initially displaced in form of a triangle with two sides alike. For a string resonance to be excited it must not be plucked at one of its nodal points. Thus we can understand that for a plucking in the middle the first partial, the fundamental is set into vibration, but not the second, the third is set into vibration, but not the fourth etc.. Thereby we obtain a spectrum like in the uppermost part of the lower right frame. If we choose to pluck at a third of the string length the third, the sixth etc. partials will be missing. If we pluck at a tenth of the string length the tenth,the twentieth partial, etc. will not be excited.

The player has also other possibilities to influence the properties of the played tones, see Fig. 4.10. A soft "plectrum" like the fingertip will give a smoother bend of the string and thus weaker high partials. A stiff string can not be bend sharply and gives therefore a tone with weaker high partials.

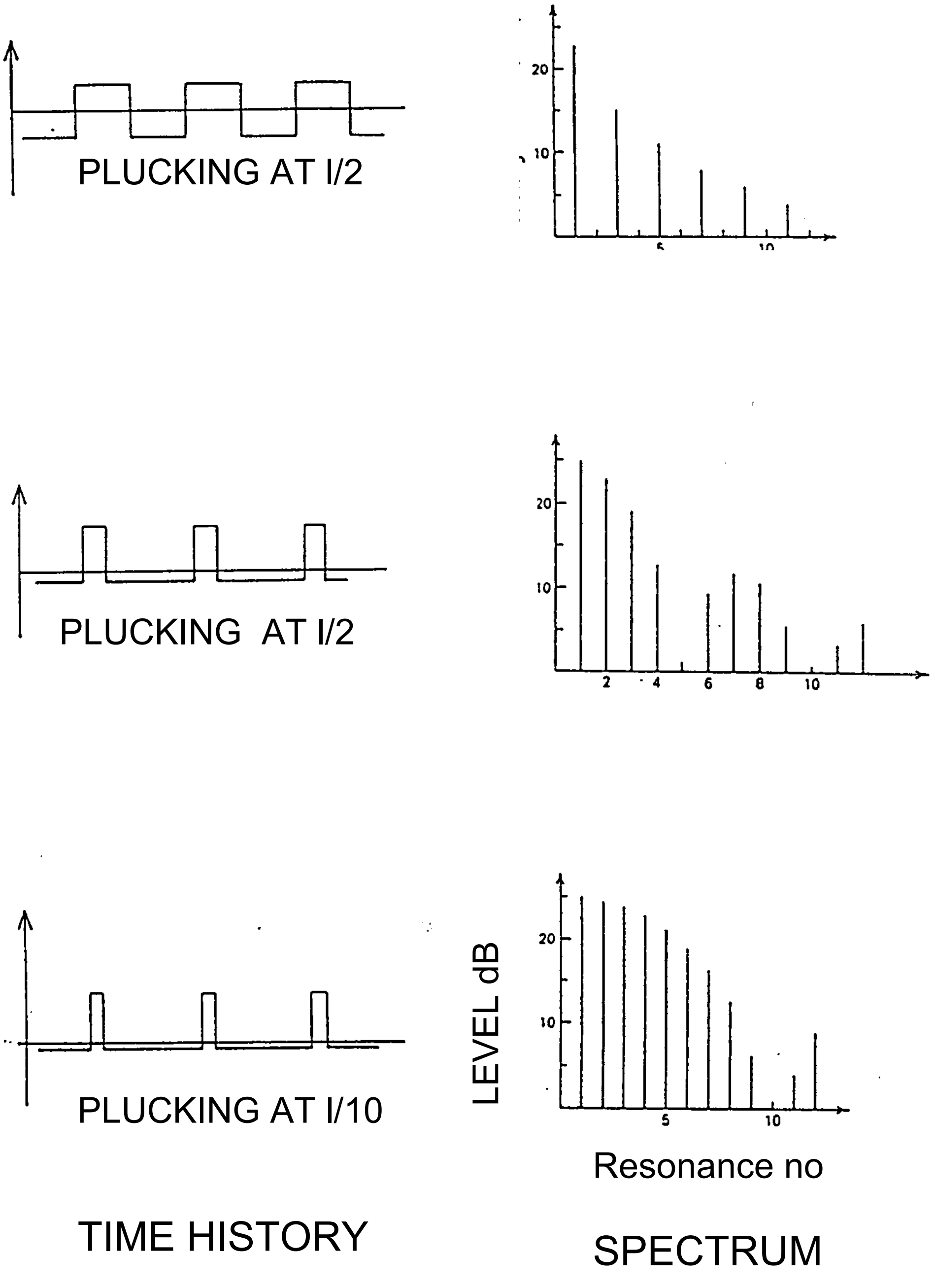

Figure 4.11 Time history and spectrum of string force on bridge for different plucking distances from the bridge.

Different plucking positions give different time histories for the force from the string to the bridge, and thus different vibrations of the bridge. A qualitative explanation was given in Fig. 4.9. As a matter of fact also a varying level for the different partials will be obtained, in principle as in Fig. 4.11.

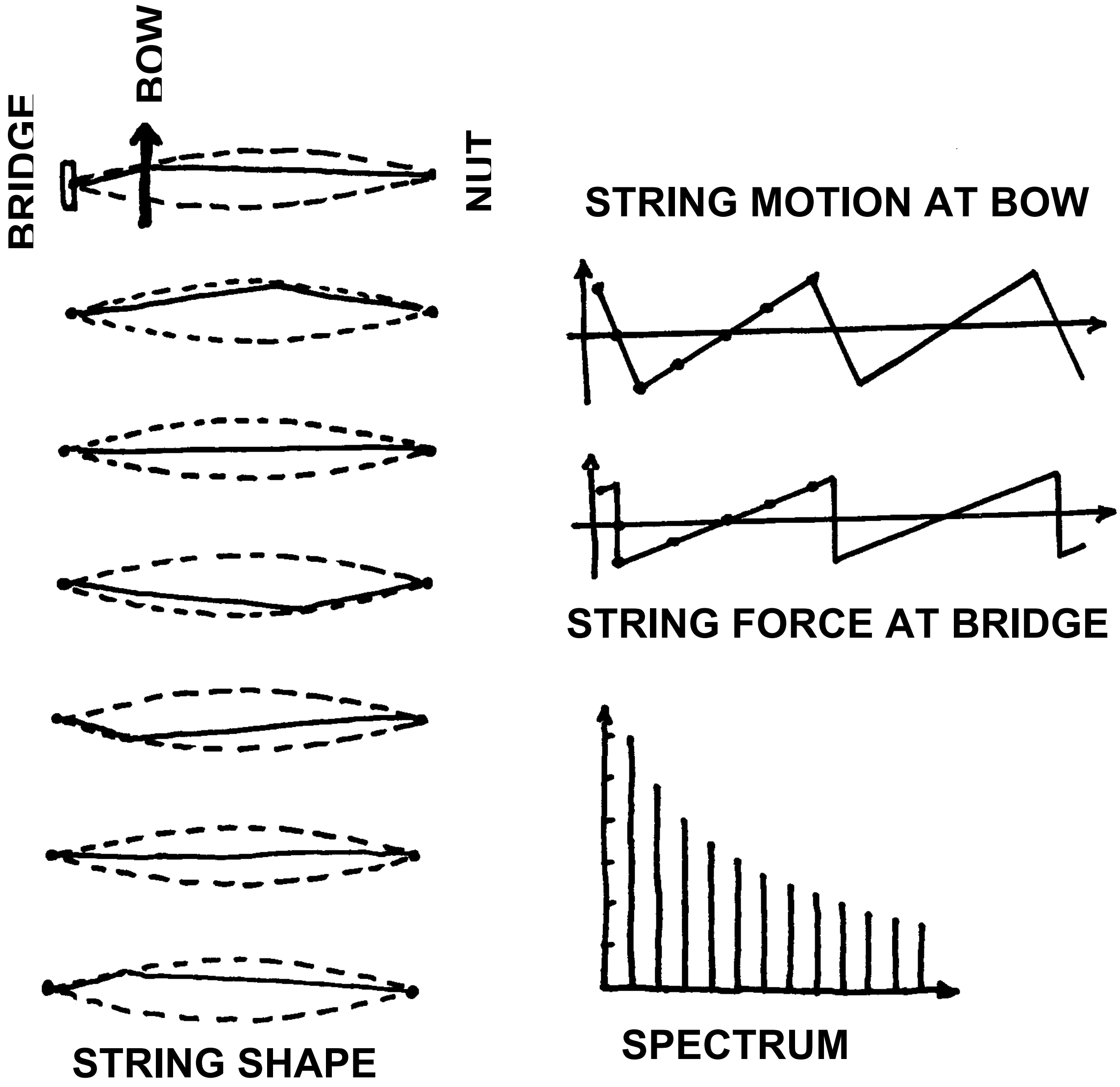

Figure 4.12 Time history for a bowed string etc. (after Hall)

Similar relations are also valid for the bowed string, see Fig. 4.12. When the bow is pulled over the string, the string is displaced as in plucking. The string is pulled aside but tries to regain its straight line. After a while the force to regain equilibrium becomes so large that the string pulls itself loose from the bow hair and moves towards a straight line. After some time the string motion is stopped and the string fastens to the bow hair again, is pulled aside once more, pulls loose, etc.. The excitation here corresponds to a plucking once each fundamental period of the string. Thereby a saw tooth shaped time history is obtained. The nature makes this repeated plucking happen once every period as long as the violin player handles his bow in a correct manner. The ratio of string-lengths, bow to bridge and bow to nut, is the same as the duration ratio of the two straight parts of the saw tooth time history. It should be mentioned that twisting motions of the string (torsion motions) are important and that temperature effects may also play a role.

4.8 GESTURE OF THE BOW AND SCALES

GESTURES OF THE BOW

At our department Dr. Askenfelt has investigated the bow gestures and articulation. With a special experimental apparatus he can simultaneously measure 1) the fundamental frequency of the played tone (pitch), 2) the bow position at the string (contact point), and 3) the bow pressure (the bow force). The three parameters are registered simultaneously as the violin is played. The bow force was found to be between 0.15 and 1.5 N (corresponds to a weight of 15 to 150 g). The velocity of the bow is between 0.04 m/s and 3 m/s (3 m/s is approximately 8 mph).

Examples of different ways of bowing are shown in Fig. 4.13. In the upper frame we can see examples of legato playing, and from top to bottom, music, bow force, bow position, and time respectively. It can be seen that the violin player decreases the bow force just before changing to a new note and keeps the low bow force at the beginning of the following note.

In martellato and the sforzando playing it is found that the violin player adjusts the bowing force and velocity. In the martellato case the the velocity is constant (the bow position is a horizontal line) and the bow force is released just before the bow reaches the tip. In the sforzando case the bow is pressed against the string with a high force in the beginning, the bow is pulled quickly over the string, the bow force is released and the bow velocity has diminished to a third. Furthermore it was found that the normal and the reversed way of playing crescendo-diminuendo gives large differences in the time history of bow force.

From research in psychology it is known that different feelings are reflected in differences in gestures, articulation. An example is shown in Fig. 4.14 for violin playing. Four bars from the Beethoven violin concerto were played softly and aggressively as shown in Fig. 4.14. The figure shows that in the aggressive version the player chose to vary the bow force rather abruptly.

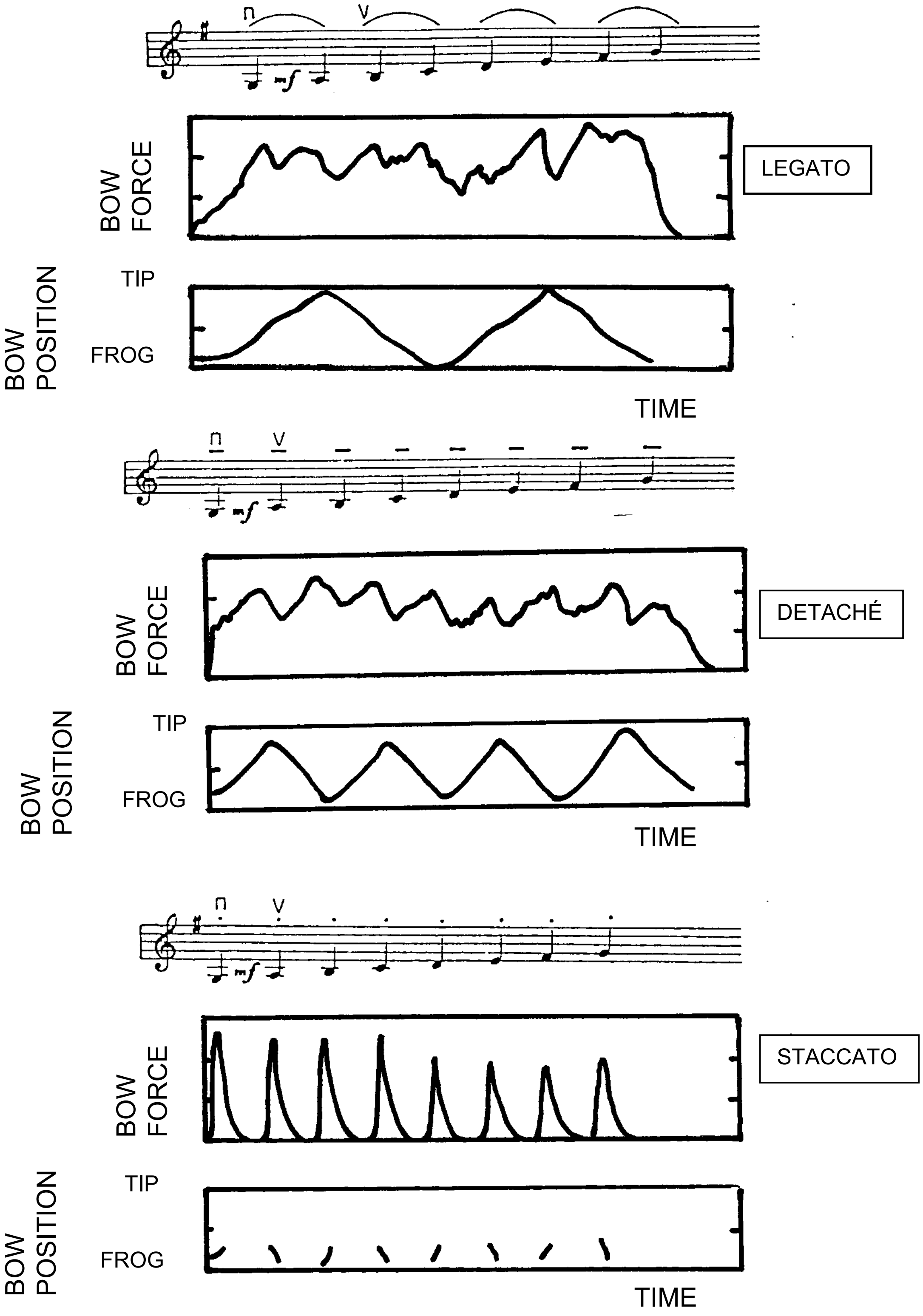

Figure 4.13 "Bow pressure" (bow force) and contact point (bow position) for different types of bowing (from Askenfelt).

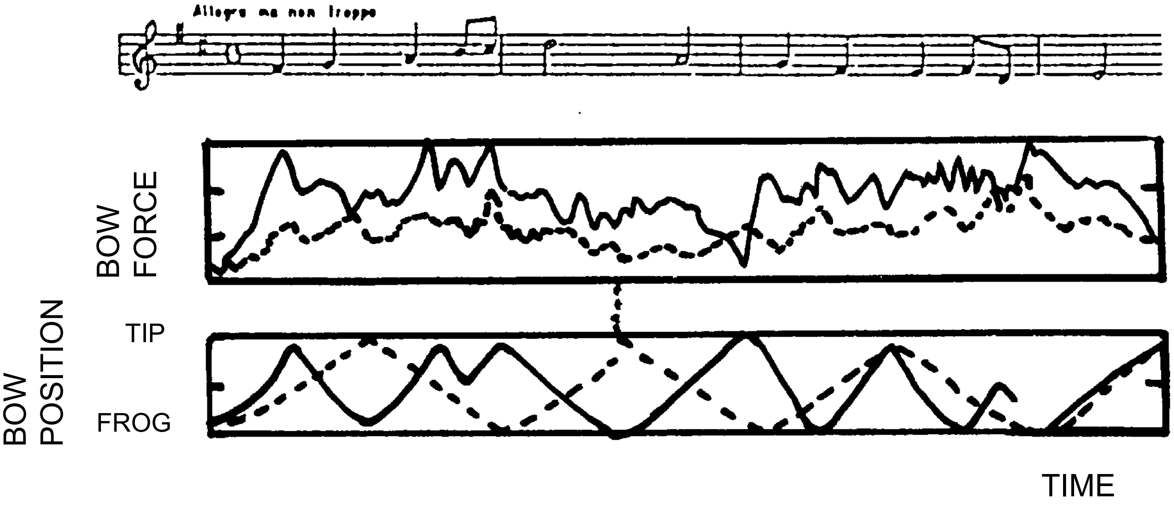

Figure 4.14 "Bow pressure" and contact point for two different types of bowing of four bars of the Beethoven violin concerto (aggressively and smoothly full lines and broken lines respectively, from Askenfelt)

THE EQUALLY TEMPERED SCALE AND PLAYED SCALES (definition of the scale, frequency and cent)

The scale generally used for tuning keyboard instruments and in playing is the equally tempered scale. This scale uses a division of the octave into 12 equal steps called semitones. Each step corresponds to an h times higher frequency and

$$\overbrace{h \times h \times h \times h \times h \ \ldots\ldots\ldots\ldots\ldots \ \times h \times h}^{12 \text{ steps}} = 2$$

i.e. $h^{12} = 2$ and $h = \sqrt[12]{2} = 1.059$

A semitone step thus corresponds to slightly short of a 6 % increase in frequency. Often the frequency of a played tone should be given with higher accuracy than a semitone step. For this purpose every semitone step is divided in 100 equal steps c. .

100 cent steps = 1 semitone step i.e. $c \times c \times c \times c \times \ \ldots\ldots \ \times c \times c = \sqrt[12]{2}$

i.e. $c^{100} = \sqrt[12]{2}$ and $c = \sqrt[1200]{2} = 1.00058$.

The step c is called a cent and corresponds to 6/100 %. Furthermore, for practical use a numbering system which gives the lowest useful octave the number 0 has been devised. This octave goes from C0 to C1 and covers the frequency range from 16 to 32 Hz and notes in the octave are labelled with "zeroes". The next octave goes from C1 to C2 and notes are labelled with a "one" etc. The tuning A = 440 Hz is labelled A4 (to avoid misunderstandings one can write A4 (440 Hz), as the introduced labelling does not agree with any of the ten most common notations used in musical scale systems. Further the introduced system only uses sharp signs, i.e. C#, D#, F# etc.

For tuning of free plates this division in single cent steps is too detailed. Therefore a cent-frequency table has been calculated in 20 cent steps, i.e., steps 1.012 times the lower frequency. Twenty cent (20 cent) is thus 12/10 % i.e., slightly above 1 %. This table can be found in the appendix 4.11 RELATIONS BETWEEN FREQUENCY AND TONE, table 4.10,starting at the 0th octave C= (16.4 Hz) ending at the 7th octave C8 (4186 Hz).

Every octave, from the 0th to the 7th., has its own table.There are thus 8 tables starting with the 0th octave and ending with the 7th octave.

The top line of every table gives the tone names and the first line with numbers the cent values 0 and corresponding frequencies in Hz. The second line gives the tones + 20 cent and corresponding frequency in Hz. The table continues with +40, +60 and +100 cent. The bottom line gives the tone names again but now at +100 cent, i.e. a semintone step higher. The column to the right give the cent values starting from the bottom line, i.e. –0 to –100 cent.

Example 4.16: 440 Hz = tuning A = A4.

Example 4.17. The note C2 has the frequency 65.4 Hz.

Example 4.18. The frequency 119 Hz corresponds to the note A# in the second octave +40 cent (upper line and left column) or if preferred the note B in the second octave -60 cent (lower line and right column).

Let me also present some measurements on scales and played frequencies. In Fig. 4.15 the relations are shown between three theoretically defined scales. As a rule of thumb one hears differences outside ±5 cent, which have been marked with two horizontal dotted lines.

Along the horizontal scale the note numbers in the octave have been marked, where 0 is the lowest tone of the octave and 12 is the highest. A look shows that the differences as a rule are small between the three scales, but in some cases the differences are noticeable ±15 cent. For the construction purpose of a musical instrument the conclusion can be drawn that one should be able to play the equally tempered scale ±15 % variations.

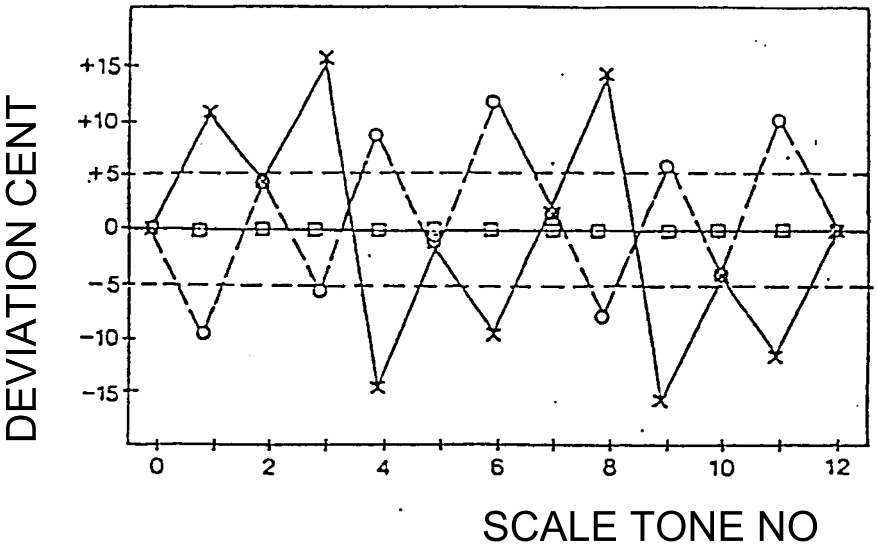

Figure 4.15 Relations between the equally tempered scale (horizontal line), the pure scale of just intonation and the Pythagorean scale.

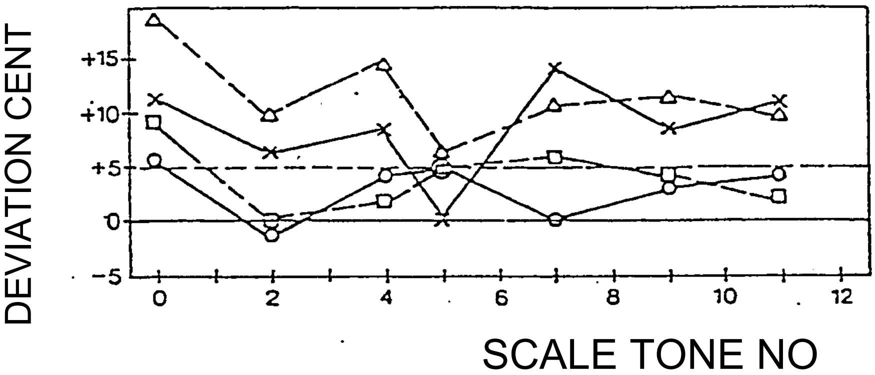

Figure 4.16 Average of intervals used in woodwind quartet playing, students in tutti (triangles), in solo (crosses), teachers in tutti (squares) and in solo (circles) (from Mason).

Let me so show the differences that can be met in actual playing, see Fig. 4.16. The experiments showed that the students as a rule played considerably above the "tuning pitch". The teachers also tend to be high but as a rule less than +5 cent. In solo performance without accompaniment the oboist in Fig. 4.17 played even ±20 cent outside a arbitrarily chosen reference tone.

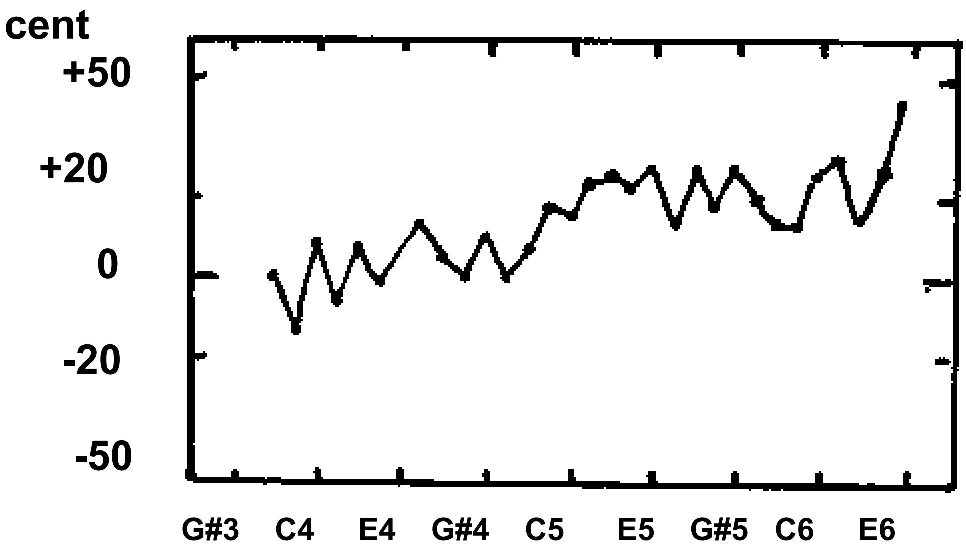

Figure 4.17 Deviations from the equally tempered scale used in unaccompanied solo playing (after Sundberg).

The important conclusion from the presented material is that the played scales do not follow simple mathematical formulas - one plays neither in the Pythagorean scale, nor the pure scale nor the equal tempered scale. It seems rather that the played tone frequency is a part of the musical speech and one chooses the frequency approximately as the mathematically defined scales but with minor deviations for the effects one wants to stress. When playing several instruments together the possibilities of selection for the single musician is strictly limited to avoid unwanted sound effects.

4.9 SUMMARY: PLAYED STRINGS AND SCALES

In this part mechanical and acoustical properties have been presented for violin and guitar strings. Further the principles for the vibrations of the strings have been sketched for differences in plucking and bowing of the string. The gestures with the bow makes it possible for the player to "talk" with the violin. Finally the equally tempered scale has been described, and how the scales are used in playing and how frequency can be measured.

4.10 KEY WORDS:

Plucked string, bowed string, time history spectrum, equally tempered scale, cent, bow velocity, bow force (bow pressure), position.

4.11 APPENDIX Table 4A.1 RELATIONS FREQUENCY AND TONE POSITION

OCTAVE 0											OCTAVE 0		
TONE	C	C#	D	D#	E	F	F#	G	G#	A	A#	H	TONE
cent	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	Hz	cent
+0	16.4	17.3	18.4	19.4	20.6	21.8	23.1	24.5	26.0	27.5	29.1	30.9	-100
+20	16.5	17.5	18.6	19.7	20.8	22.1	23.4	24.8	26.3	27.8	29.5	31.2	-80
+40	16.7	17.7	18.8	19.9	21.1	22.3	23.7	25.1	26.6	28.1	29.8	31.6	-60
+60	16.9	17.9	19.0	20.1	21.3	22.6	23.9	25.4	26.9	28.5	30.2	32.0	-40
+80	17.1	18.1	19.2	20.4	21.6	22.9	24.2	25.7	27.2	28.8	30.5	32.3	-20
+100	17.3	18.4	19.4	20.6	21.8	23.1	24.5	26.0	27.5	29.1	30.9	32.7	0
TONE	C#	D	D#	E	F	F#	G	G#	A	A#	H	C	TONE

OCTAVE 1											OCTAVE 1		
TONE	C	C#	D	D#	E	F	F#	G	G#	A	A#	H	TONE
+0	32.7	34.6	36.7	37.9	41.2	43.7	46.2	49.0	51.9	55.0	58.3	61.7	-100
+20	33.1	35.1	37.1	39.3	41.7	44.2	46.8	49.6	52.5	55.6	58.9	62.5	-80
+40	33.5	35.5	37.6	39.8	42.2	44.7	47.3	50.1	53.1	56.3	59.6	63.2	-60
+60	33.9	35.9	38.0	40.3	42.7	45.2	47.9	50.7	53.7	56.9	60.3	63.9	-40
+80	34.2	36.3	38.4	40.7	43.2	45.7	48.4	51.3	54.4	57.6	61.0	64.7	-20
+100	34.6	36.7	38.9	41.2	43.7	46.2	49.0	51.9	55.0	58.3	61.7	65.4	0
TONE	C#	D	D#	E	F	F#	G	G#	A	A#	H	C	TONE

OCTAVE 2											OCTAVE 2		
TONE	C	C#	D	D#	E	F	F#	G	G#	A	A#	H	TONE
0	65.4	69.3	73.4	77.8	82.4	87.3	92.5	98.0	104	110	117	123	-100
+20	66.2	70.1	74.3	78.7	83.4	88.3	93.6	99.1	105	111	118	125	-80
+40	66.9	70.9	75.1	79.6	84.3	89.3	94.7	100	106	113	119	126	-60
+60	67.7	71.7	76.0	80.5	85.3	90.4	95.8	101	107	114	121	128	-40
+80	68.5	72.6	76.9	81.5	86.3	91.4	96.9	103	109	115	122	129	-20
+100	69.3	73.4	77.8	82.4	87.3	92.5	98.0	104	110	117	123	131	0
TONE	C#	D	D#	E	F	F#	G	G#	A	A#	H	C	TONE

OCTAVE 3											OCTAVE 3		
TONE	C	C#	D	D#	E	F	F#	G	G#	A	A#	H	TONE
0	131	139	147	156	165	175	185	196	208	220	233	247	-100
+20	132	140	149	157	167	177	187	198	210	223	236	250	-80
+40	134	142	150	159	169	179	189	201	213	225	239	253	-60
+60	135	143	152	161	171	181	192	203	215	228	241	256	-40
+80	137	145	154	163	173	183	194	205	217	230	244	259	-20
+100	139	147	156	165	175	185	196	208	220	233	247	262	0
TONE	C#	D	D#	E	F	F#	G	G#	A	A#	H	C	TONE

OCTAVE 4 OCTAVE 4

TONE	C	C#	D	D#	E	F	F#	G	G#	A	A#	H	TONE
0	262	277	294	311	330	349	370	392	415	440	466	494	-100
+20	265	280	297	315	333	353	374	397	420	445	472	500	-80
+40	268	284	301	318	337	357	379	401	425	450	477	505	-60
+60	271	287	304	322	341	362	383	406	430	456	483	511	-40
+80	274	290	308	326	345	366	387	411	435	461	488	517	-20
+100	277	294	311	330	349	370	392	415	440	466	494	523	0
TONE	C#	D	D#	E	F	F#	G	G#	A	A#	H	C	TONE

OCTAVE 5 OCTAVE 5

TONE	C	C#	D	D#	E	F	F#	G	G#	A	A#	H	TONE
0	523	554	587	622	659	698	740	784	831	880	932	988	-100
+20	529	561	594	629	667	707	749	793	840	890	943	999	-80
+40	535	567	601	637	675	715	757	802	850	901	954	1011	-60
+60	542	574	608	644	683	723	766	812	860	911	965	1023	-40
+80	548	581	615	652	690	731	775	821	870	922	976	1034	-20
+100	554	587	622	659	698	740	784	831	880	932	988	1047	0
TONE	C#	D	D#	E	F	F#	G	G#	A	A#	H	C	TONE

OCTAVE 6 OCTAVE 6

TONE	C	C#	D	D#	E	F	F#	G	G#	A	A#	H	TONE
0	1047	1109	1175	1245	1319	1397	1480	1568	1661	1760	1865	1976	-100
+20	1059	1122	1188	1259	1334	1413	1497	1586	1681	1780	1886	1998	-80
+40	1071	1135	1202	1274	1349	1430	1515	1605	1700	1801	1908	2022	-60
+60	1083	1148	1216	1288	1365	1446	1532	1623	1720	1822	1930	2045	-40
+80	1096	1161	1230	1303	1381	1463	1550	1642	1740	1843	1953	2069	-20
+100	1109	1175	1245	1319	1397	1480	1568	1661	1760	1865	1976	2093	0
TONE	C#	D	D#	E	F	F#	G	G#	A	A#	H	C	TONE

OCTAVE 7 OCTAVE 7

TONE	C	C#	D	D#	E	F	F#	G	G#	A	A#	H	TONE
0	2093	2217	2349	2489	2637	2794	2960	3136	3322	3520	3729	3951	-100
+20	2117	2243	2377	2518	2668	2826	2994	3176	3361	3561	3773	3997	-80
+40	2142	2269	2404	2547	2699	2859	3029	3209	3400	3602	3816	4043	-60
+60	2167	2296	2432	2577	2630	2892	3064	3247	3440	3644	3861	4090	-40
+80	2192	2322	2460	2607	2762	2926	3100	3284	3480	3686	3906	4138	-20
+100	2217	2349	2489	2637	2794	2960	3136	3322	3520	3729	3951	4186	0
TONE	C#	D	D#	E	F	F#	G	G#	A	A#	H	C	TONE