HARMONY AND TONALITY

Papers given at a seminar organized by the Music Acoustics Committee of the Royal Swedish Academy of Music

Editor: J. Sundberg

Publications issued by the Royal Swedish Academy of Music No. 54 1987.

ISBN 91-85428-49-3

ADEBE Reklam & Tryckservice AB, Stockholm, Sweden

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FOREWORD

In recent years some basic aspects of music listening have attracted psychologists' and engineerers' attention more than in the past. As a result, our understanding of the acoustic fundamentals of harmony and tonality has improved considerably. This new understanding is useful not only to composers of computer music, but also in a more general sense, as it explains some basic phenomena in traditional harmony.

On April 5, 1986, the Music Acoustics Committee of the Royal Swedish Academy of Music arranged a public seminar on HARMONY AND TONALITY. As with the preceding ten public seminars arranged by this Committee it was held at the Royal Institute of Technology (KTH), Stockholm, and it attracted some hundred participants from various places in Sweden and its Nordic neighbor countries. The aim was to present recent advances in this area. Three foreign speakers were invited: Carol Krumhansl, Max Mathews, and William Thompson. In the area of harmony and tonality Max Mathews is renowned for his pioneering research in constructing and exploring the possibilities offered by a new scale, a project that he has carried out together with John Pierce and Linda Roberts. Carol Krumhansl, with various coauthors, has published a series of articles on her excellent experiments on perception and cognition of musical pitch and tonal function. William Thompson just finished his doctoral dissertation on the sensing of tonality in music listening; we were fortunate to receive him as a guest researcher at the Department of Speech Communication and Music Acoustics at the time of the seminar. In addition, I contributed a primer on intervals and harmonic spectra and a short note on a possibility to extend Krumhansl's work to music performance.

Here these authors publish, in a popular form, what they presented at the seminar. As with the other ten volumes in this series of books on music acoustics published by the Committee in the series of publications issued by the Academy of Music the sound examples belonging to the articles are published on a phonogram record, the production of which was handled by Lennart Fahlén. The articles have been typeset by the authors themselves, by means of word processor machinery.

KTH, december 1986

Alf Gabrielson Johan Sundberg
Members of the Music Acoustics Committe

HARMONY AND HARMONIC SPECTRA

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The articles in the present volume center around certain acoustic phenomena related to harmony, scales and the perception of tonality. Some readers might not be fully familiar with the basic concepts of tonality and harmony and this would hamper the reading of most of the subsequent articles or at least slow down the reading tempo beyond the limits of comfort in some of the following articles. The aim of the present article was to review, for all readers who feel they need it, the basic acoustic concepts that are relevant to tonality and harmony.

A Harmonic Spectrum

All traditional instruments except some percussion instruments generate tones with spectra that are called harmonic. This implies that the frequencies of the spectrum partials constitute a harmonic series. A harmonic series is the same thing as a multiplication table; for example:

100, 200, 300, 400, 500, etc, or
150, 300, 450, 600, 750, etc.

If one uses a linear frequency scale in the spectrum, such partials are equidistant on the frequency axis, as shown in Figure 1.

Actually, there are more harmonic things than the harmonic series of frequencies that characterize harmonic spectra. Harmonic partials constitute harmonic musical intervals, such as pure octave, fifth, and fourth; major and minor thirds and seconds. This is illustrated in Figure 1. We can see that the interval between the first and second partial in a harmonic spectrum is a pure octave, there is a fifth between the second and third, and a fourth between the third and fourth. We can see that the musical intervals between adjacent partials grow smaller and smaller, the higher up we go in the harmonic series, in spite of the fact that the frequency difference between these adjacent partials remains the same.

We have seen that the frequencies of the partials form a harmonic series and that the partials form harmonic intervals. In other words, whatever the fundamental frequency F_1 is, there will always be a fifth between the second and third partials, and these partials have the frequencies of $2*F_1$ and $3*F_1$. We are now in a good position to draw an important conclusion: a given musical interval always corresponds to one and the same frequency ratio. For instance, the frequency ratio of an octave is 2:1, that of a fifth is 2:3, that of a fourth is 4:3 and so on.

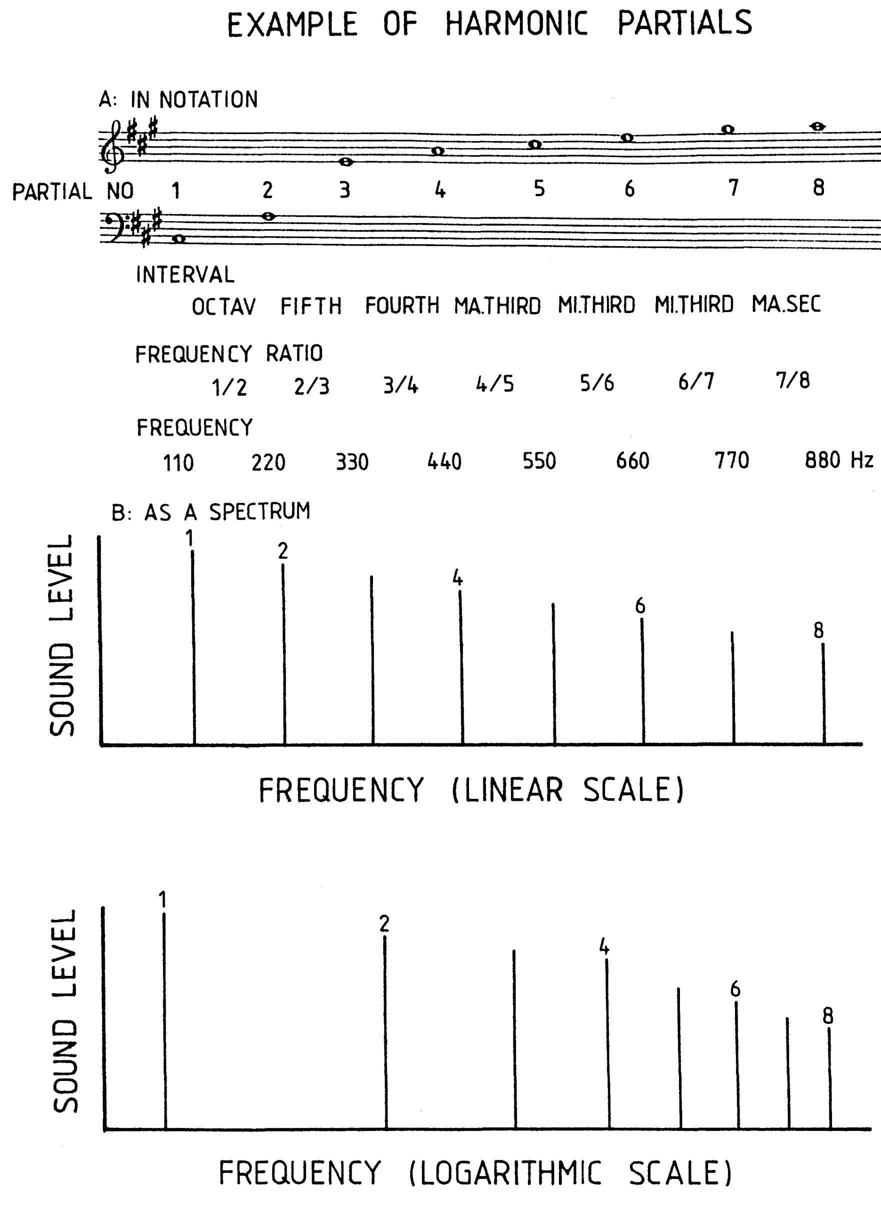

Fig. 1. The series of harmonic partials notated in different ways: on the note staff, on a linear frequency scale, as a series of frequencies, and along a logarithmic frequency scale. Note that a given interval, such as the octave, corresponds to a fixed distance along the logarithmic scale, just as on a piano keyboard, while a frequency difference such as 110 Hz, corresponds to a fixed distance along the linear frequency scale. Note also that the partials in the harmonic series constitute harmonic intervals that are well known in music.

The frequency ratios of some intervals are given in the Table.

Table. Frequency ratios corresponding to musical intervals.

Musical interval	Frequency ratio
Octave	1:2
fifth	2:3
fourth	3:4
major third	4:5
minor third	5:6
major second	8:9

It is important to note that these are the harmonic versions of these intervals; there are also other versions, such as the equally-tempered versions and the Pythagorean versions. We will return to this in a moment.

Summarizing, it is enough to know the frequency ratio of an interval in order to compute the frequency of a tone that forms this interval with another tone of known frequency. For instance, if we want to compute the frequency of the pitch D4 lying a pure fourth below the pitch A4, we should multiply the frequency of pitch A4, or 440 Hz, with the frequency ratio of the fourth, or 3/4:

$$F_{E4} = 440*3/4 = 330 \text{ Hz}$$

We realize that by inverting the frequency ratio 3/4 the direction of the interval is reversed, because obviously, we will then obtain a frequency higher than the reference frequency. Hence,

$$F_{D5} = 440*4/3 = 586.67 \text{ Hz}$$

The same applies to other intervals. Summarizing once again, by recalling the musical intervals between the partials in a harmonic spectrum, one can compute what the frequency of a pitch is that forms a given harmonic interval with a reference tone of known frequency.

On a linear frequency scale, a given frequency difference in Hz corresponds to a given distance along the scale. Therefore, the partials of a harmonic spectrum appear equidistantly spaced along a linear frequency scale, as we saw in Figure 1. If the frequency scale is logarithmic, on the other hand, a constant frequency ratio corresponds to a certain distance. Therefore, on a logarithmic frequency scale a musical interval corresponds to a certain distance. From this we conclude that the frequency scale represented by a piano keyboard is logarithmic, as a given interval always corresponds to a given distance (What a problem to play if this were not the case!). For instance, an octave is always 17,3 cm wide, regardless of whether it corresponds to a frequency difference of 55 Hz, as between pitches A1 and A2, or 880 Hz, as between pitches A5 and A6.

Two Harmonic Spectra

Let us now imagine what happens when two tones with harmonic spectra sound which form a harmonic musical interval such as a fifth. If the lower tone has the fundamental frequency of 200 Hz, the higher one will have the fundamental frequency of 200*3/2=300 Hz. The frequencies of the partials that will be sounding if these notes are sounding together are the following:

```
200 400 600 800 1000 1200 1400 1600 1800
  300   600   900    1200     1500  1800
```

We observe that some partials are common to these two spectra; every third partial in the lower tone's spectrum coincides with every second partial in the higher tone's spectrum. This can be put in a more general form also: if the fundamental have the frequency ratio m:n, m<n, every n:th partial of the lower tone coincides with every m:th partial of the higher tone. This is illustrated in Figure 2.

Roughness arises from simultaneously sounding tones that are close along a critical band frequency scale. (When a tone reaches the ear, it causes maximum vibration at a certain place along the basilar membrane, and when two tones sound, two places are vibrating. The critical band is a unit that roughly reflects this spacing of frequencies on the basilar membrane.) If two complex tones sound simultaneously, a lot of partials close to each other on the critical band scale will sound and make the sound rough. However, if these tones have harmonic spectra and if their fundamental frequencies

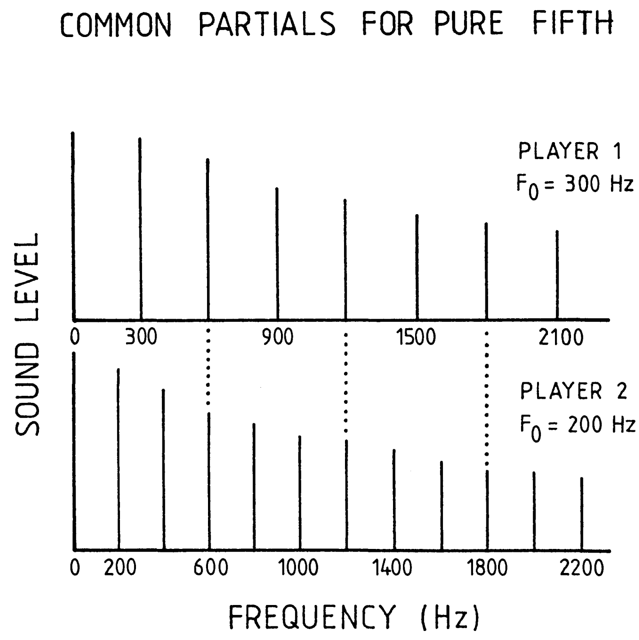

Fig. 2. Illustration of the phenomenon of coinciding partials occurring when two tones sound simultaneously which have with harmonic spectra and which have fundamental frequencies forming a ratio that can be expressed by small integers. In the case illustrated here, the fundamental frequency ratio can be expressed as 3:2; as the spectra are harmonic, every second partial in the spectrum of the upper tone coincides with every third partial in the spectrum of the low tone.

constitute a simple ratio between small integers, several partials will coincide and this will reduce the roughness. If, on the other hand, the frequencies do not correspond to a ratio between small integers, these coinciding partials will not coincide so they will cause roughness. Therefore, we realize that a minimum in roughness will be reached as soon as the fundamental frequencies correspond to a ratio between small integers. Such minima in roughness correspond to consonant intervals. We imagine that the more numerous the coinciding partials, i. e. the lower the integers in the frequency ratio, the less roughness will be generated. We also imagine that roughness will increase as soon as a consonant interval is mistuned, because then the otherwise coinciding partials will start generating roughness.

The Circle of Fifths

The circle of fifths appears in many unexpected contexts, generously offering some kind of "explanation", or at least a relationship between various phenomena. These phenomena appear in melody and harmony in particular. From where the circle of fifths derives these almost magic properties is still an unanswered question. However, it may be relevant that two tones having harmonic spectra and

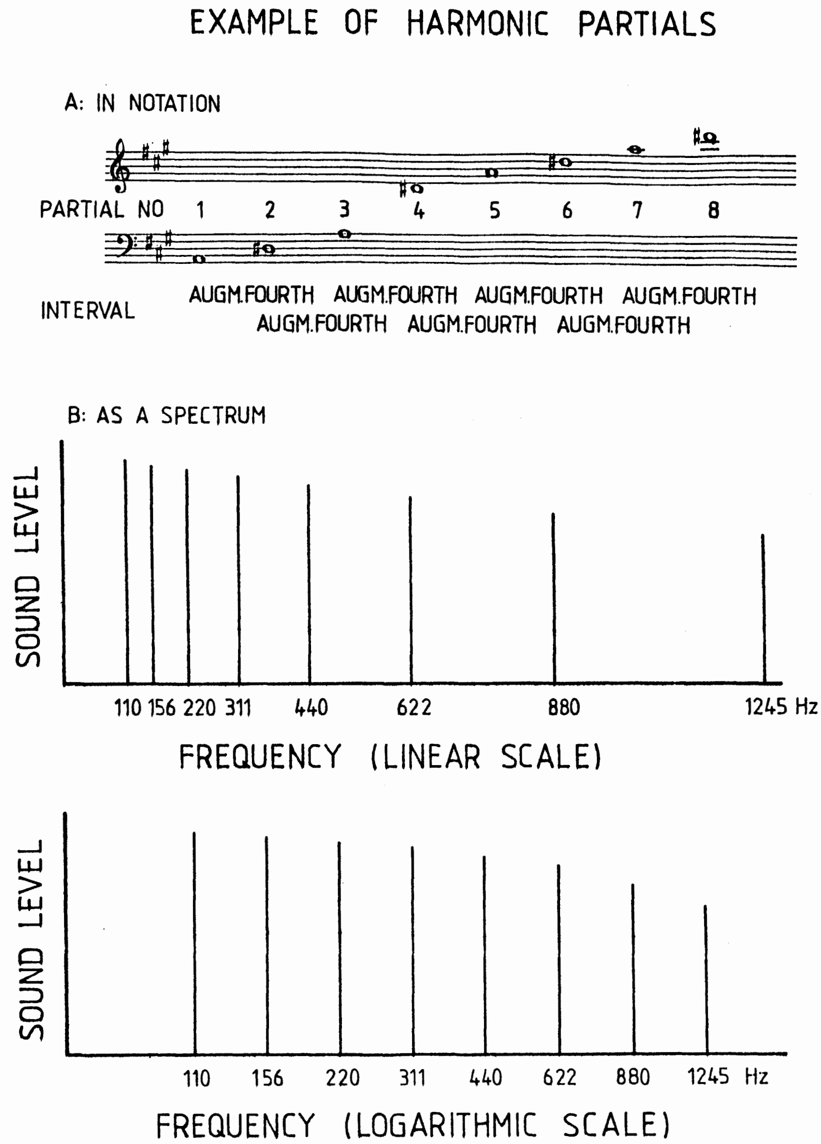

Fig. 3. Example of an inharmonic spectrum, in which the neighboring partials form tritone intervals. The partials are then equidistantly spaced along the logarithmic frequency scale but not so on a linear frequency scale.

constituting a fifth share a maximum number of partials, if we except the octave. In other words, the fifth is the most consonant interval after the octave. Also, no other intervals, except the dissonant minor second, in our diatonic scale has the circular property of returning to the starting point after having visited all twelve tones that we use in our tone system. This is certainly not much of an explanation why the circle of fifths seems so fundamental in both music theory and music perception, but a future explanation is likely to start from or at least include these facts.

Temperament

One problem with the Western tone system is that the twelfth fifth does not produce a pure octave of the starting tone, but rather a tone that is slightly higher in frequency. This discrepancy has annoyed music theorists and, perhaps to some extent, also performers. In the case of instruments with fixed fundamental frequencies, such as keyboard instruments, there is a need for a solution to this problem. Several solutions called temperaments have been proposed over the last centuries. The simplest and probably also the most important one in Western music culture is called the equally tempered scale. This solution is highly beautiful, as seen from the point of view of mathematics: it splits the octave interval in twelve equal parts. Bearing in mind that a musical interval corresponds to a frequency ratio, and that the frequency ratio for the octave is 2:1, it can be realized that the frequencies of the tones in the equally tempered scale are computed by a repetitive multiplication by a factor equal to a twelfth of an octave, i. e. $(2)^{1/2} : 1$.

Inharmonic spectra

In most traditional instruments harmonic spectra are generated. However, in today's computer music studio there is no need to confine oneself to this type of acoustic signals. Inharmonic spectra can also be explored. It is possible to construct spectra in which, for instance, all neighbor partials constitute a fixed interval, such as a third or, as in Figure 3, a tritone. In such cases, the traditional rules for predicting the degree of consonance of intervals must be revised.

SOUND EXAMPLE 1 demonstrates this point. The dyad sequence tritone > minor sixth is played twice. First it is played with tones having normal harmonic spectra. In this case, the first dyad sounds dissonant while the second dyad sounds consonant. The second time the same intervals are played with tones having spectra in which each neighbor pair of partials forms a tritone interval. In this case, the first dyad is much more consonant than the second one. Thus, by changing the spectra we made a consonant interval sound dissonant and vice versa! Inspecting once more Figure 3 we realize the reason: in the tritone, all partials of the higher tone are coinciding with the partials of the lower tone, just as in the case of an octave played with harmonic spectra: the high number of coinciding partials creates a consonant dyad. In the case of the sixth, on the other hand, there will be few coinciding partials, and a good number of roughness-generating partials will appear.

From the above we conclude that the selection of scale tone frequencies and the selection of the main harmonies might need revision when playing with tones having inharmonic rather than harmonic partials. Indeed, an entirely new harmonic - or perhaps inharmonic - world opens up with the possibilities offered by the computer music studio. It will certainly be easier to make an efficient use of all these new possibilities if one is aware of the recent advances in the fields of auditory perception and music psychology.

TONAL AND HARMONIC HIERARCHIES

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Listening to music, we hear the sounded elements not as disconnected units, but in relation to one another. The listener's experience goes beyond the simple registration by the sensory system of the acoustic events in isolation. Each event is heard in its temporal and pitch context, and is understood as it functions within that broader context. To understand the listener's response to music, it is necessary to specify which of the potentially available features of the music are realized by the listener, how they are processed and remembered, and how they contribute to the listener's appreciation of the overall plan of the piece of music.

The listener's experience of a particular musical sequence is driven and constrained by the sounds registered by auditory mechanisms and processed by available mental resources. But, the construction of the music itself must take into account the listener's capacities for appreciating and remembering structured auditory information. In other words, the way musical materials are formed must take into consideration the nature of the listener's perceptual and cognitive systems. This suggests, then, that there are two separate but strongly interdependent components of music psychology: the structured sound material of the music itself, and the listener's capacity for apprehending and remembering relations that obtain among the temporal and pitch events.

The experimental work I will summarize here was directed at the problem of characterizing the listener's knowledge of pitch structure, focusing on traditional Western tonal-harmonic music. The outline of the program of research is shown in the Figure 1. Three different sets of musical elements are considered: tones, chords, and keys. The set of tones consists of the twelve tones of the chromatic scale, and in all cases equal-tempered tuning was used. The chords used in most experiments are triadic harmonies built on the diatonic scale tones. The set of keys consists of the twenty-four major and minor keys.

These sets are restricted in various ways, but the small number of elements in each set makes it possible to exhaustively investigate the perceived relations between the elements.

In the experiments I will describe, listeners made judgments about how closely related different elements are to one another. I have argued that relations between musical events are central to our experience. Different tones are heard as more or less related, depending on the context, as are different

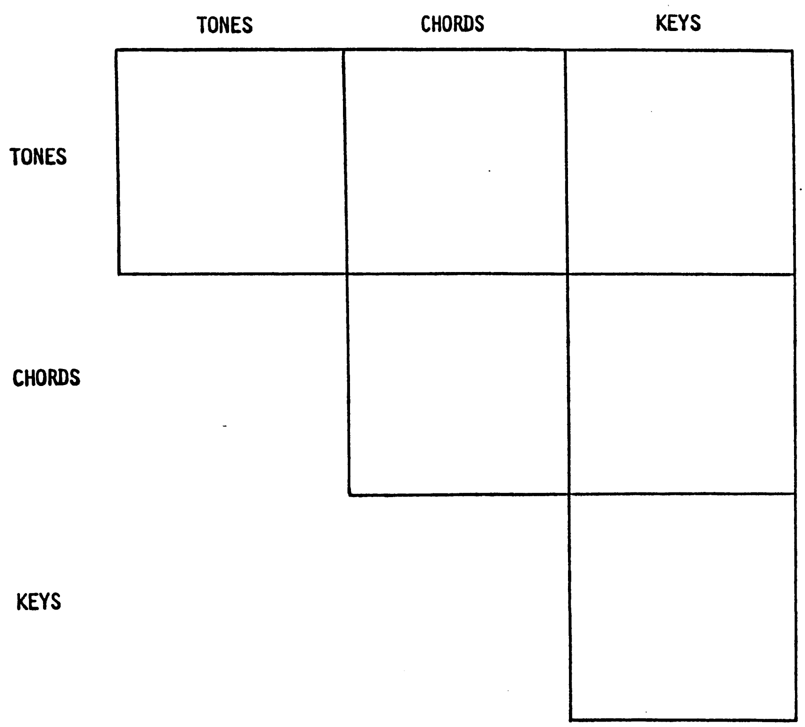

Figure 1. The experiments summarized here investigate the perceived degree of relatedness between elements in three sets: tones (twelve chromatic scale tones), chords (diatonic triads), and keys (twenty-four major and minor keys).

harmonies. In addition, a musical key establishes a kind of hierarchy on the sets of tones and chords; that is, the tones and chords are heard as more or less closely related to the prevailing tonality.

The research is directed at quantifying these perceptual relations.

Tonal hierarchy

For ease of presentation, I will describe first those results concerning the relations between tones, between tones and keys, and between different musical keys. The first experiment concerns the hierarchy that a musical key establishes on the set of twelve chromatic scale tones.

Krumhansl and Shepard (1979) first measured this hierarchy using a method we called the probe tone technique, which was used in a later replication and extension by Krumhansl and Kessler (1982); I will describe the results of the later study here.

Each trial of the experiment began with a musical unit, such as a scale, tonic triad chord, or a chord cadence, that unambiguously establishes a major or minor key. The key- defining context was followed by a single tone, called the probe tone, which was

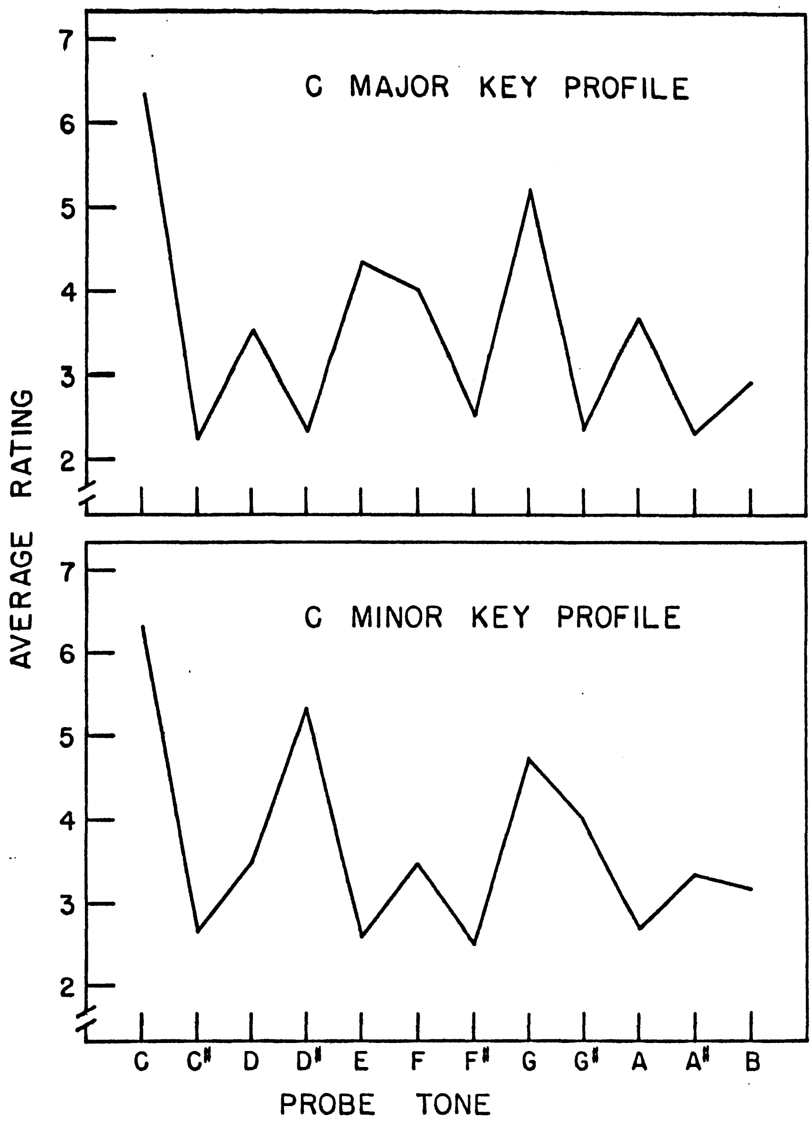

Figure 2. The probe tone ratings from Krumhansl and Kessler (1982) exhibit a hierarchy of stability or structural significance that major and minor keys establish on the set of chromatic scale tones.

one of the twelve tones of the chromatic scale. The listener's task was to rate on a seven point scale how well the final probe tone fit with with the key-defining context at the beginning of the trial. Sound Example 1 plays a number of trials in which the context is a C major scale; these are followed by a number of trials in which the context is a IV V I cadence in C minor.

The listeners had on average about ten years experience playing music, but little formal training in music theory. Listeners in other experiments had similar music backgrounds. The results are shown in Figure 2. The ratings are plotted as though the context key were C major or C minor. In fact, a number of different keys were used and the results were similar when shifted to a common tonic tone. The results were also similar whether the context key was established by a scale, tonic triad, or chord cadence.

For both major and minor keys, the highest rating was given to the tonic tone, followed by the third and fifth degrees of the scale which, together with the tonic, form the tonic triad chord. Next highest ratings were given to the other tones of the diatonic scale, and lowest ratings were given to tones not in the scale. The pattern fits well with music-theoretic descriptions of the relative degree of structural significance or stability that different tones have in tonal contexts. And, there is good agreement with the "tonic charge" values of Sundberg, Frydén, and Askenfelt (1983).

Moreover, Kessler and I were able to derive from these data a very regular and interpretable map of musical keys. If two keys are closely related, we argued, they should impose similar hierarchies on the set of musical tones. In support of this, the hierarchies for C major and A minor were similar and, in contrast, the hierarchies for C major and F# major were dissimilar. The similarity was quantified as the correlation of the probe tone ratings for all possible pairs of keys. These correlations were then analyzed using multidimensional scaling. This produces a spatial configuration such that keys having similar tonal hierarchies are located close to one another in the space, and keys having dissimilar hierarchies are far apart.

This analysis produced a four-dimensional solution in which the points for the twenty-four major and minor keys fell on the surface of a torus. A torus can be depicted as a rectangle in two dimensions, where it is understood that opposite edges are identified. That is, the top and bottom edges are to be considered the same, as are the left and right edges. Presented in this way in Figure 3, the positions of the twenty-four keys make sense musically. Each major key is flanked by its neighbors on the circle of fifths and its relative and parallel minor keys. Similarly, each minor key is flanked by its neighbors on the circle of fifths and its relative and parallel major keys. This analysis demonstrates that the information contained in the probe tone ratings is sufficiently rich to generate structure at the level of musical keys, and I will describe later how this spatial map of keys can be used to represent how the sense of keys develops and changes over times.

The tonal hierarchy also affects the degree to which different tones are heard as related to one another. The experimental method (Krumhansl, 1979) is a variant of the one just described; sample trials can be heard in Sound Example 2. Again, each trial began with a key-defining context (ascending or descending scale or tonic triad), which was followed by two tones now rather than one. The listeners were asked to rate how closely related the first of these tones is to the second in the key context. This produces a matrix of ratings for each possible pair of tones that can be analyzed by multidimensional scaling to produce a spatial representation.

The results for a C major context are illustrated in a slightly idealized form at the bottom of Figure 4. The twelve tones of the chromatic scale are located on the surface of a cone. They are positioned so that going around the cone, the tones are ordered by pitch proximity as shown in the top of the figure. The vertical axis corresponds to the previously measured tonal hierarchy. The tonic itself is at the vertex of the cone, the other closely related tones, G and E are relatively near the vertex, followed by the other tones of the diatonic scale, and finally the nondiatonic tones which are judged as distant both from each other and the more structurally significant tones near the vertex. In addition, this study showed another effect of the tonal hierarchy, which cannot be represented in a geometric configuration such as that in Figure 4. There was a preference, in general,

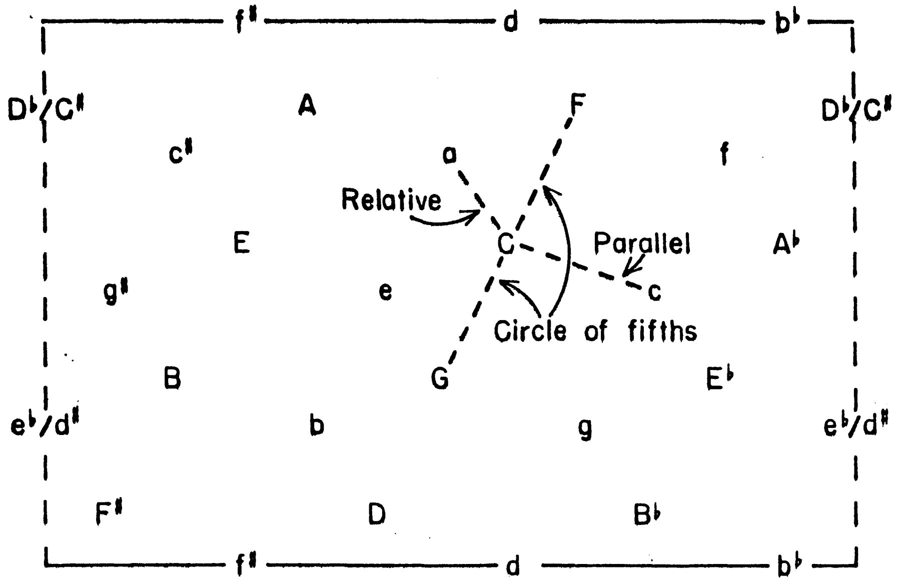

Figure 3. The map of key distances derived from the Krumhansl and Kessler (1982) probe tone data placed the twenty-four keys on the surface of a torus, depicted here as a rectangle with opposite edges identified.

for two-tone sequences ending on more stable tones over sequences ending on less stable tones. In other words, there were differences depending on the order in which the two tones were sounded.

To summarize the results for tones and keys, the perceived relations between different tones can be represented by a conical surface generated by pitch proximity and the tonal hierarchy. The relations between tones and keys are characterized by the tonal hierarchy measured in the probe tone study. And, key distances are summarized by the toroidal configuration containing the circle of fifths and the relative and parallel major-minor key relations; this map was generated from the experimentally quantified tonal hierarchy.

In addition to the rating studies I have described, a number of studies using memory performance have been conducted (Krumhansl, 1979). These studies serve to confirm the basic results. The tonal hierarchy affects listeners' abilities to recognize tones in tonal contexts. Tones that are relatively stable in the system are better recognized than those that are unstable. And, listeners tend to confuse unstable tones with more stable tones rather than the reverse. So, there are a whole variety of psychological effects reflecting tonal structure.

The final point I would like to emphasize is that the patterns depend strongly on the context in which the tones are embedded. According to the Gestalt psychologist, Wertheimer (1924): "The flesh and blood of a tone depends from the start upon its role in the melody: a b as leading tone to c is something radically different from the b as tonic. It belongs to the flesh and blood of things given in experience, how, in what role, in what function they are in the whole."

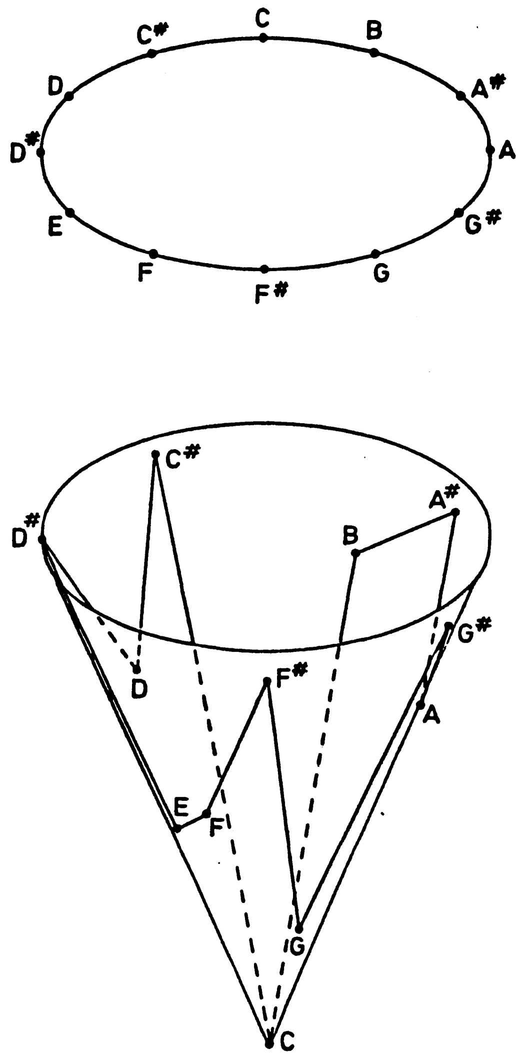

Figure 4. Multidimensional scaling of relatedness judgments of two-tone sequences in a C major context (Krumhansl, 1979) produced a circular dimension corresponding to pitch proximity and a vertical dimension corresponding to the tonal hierarchy.

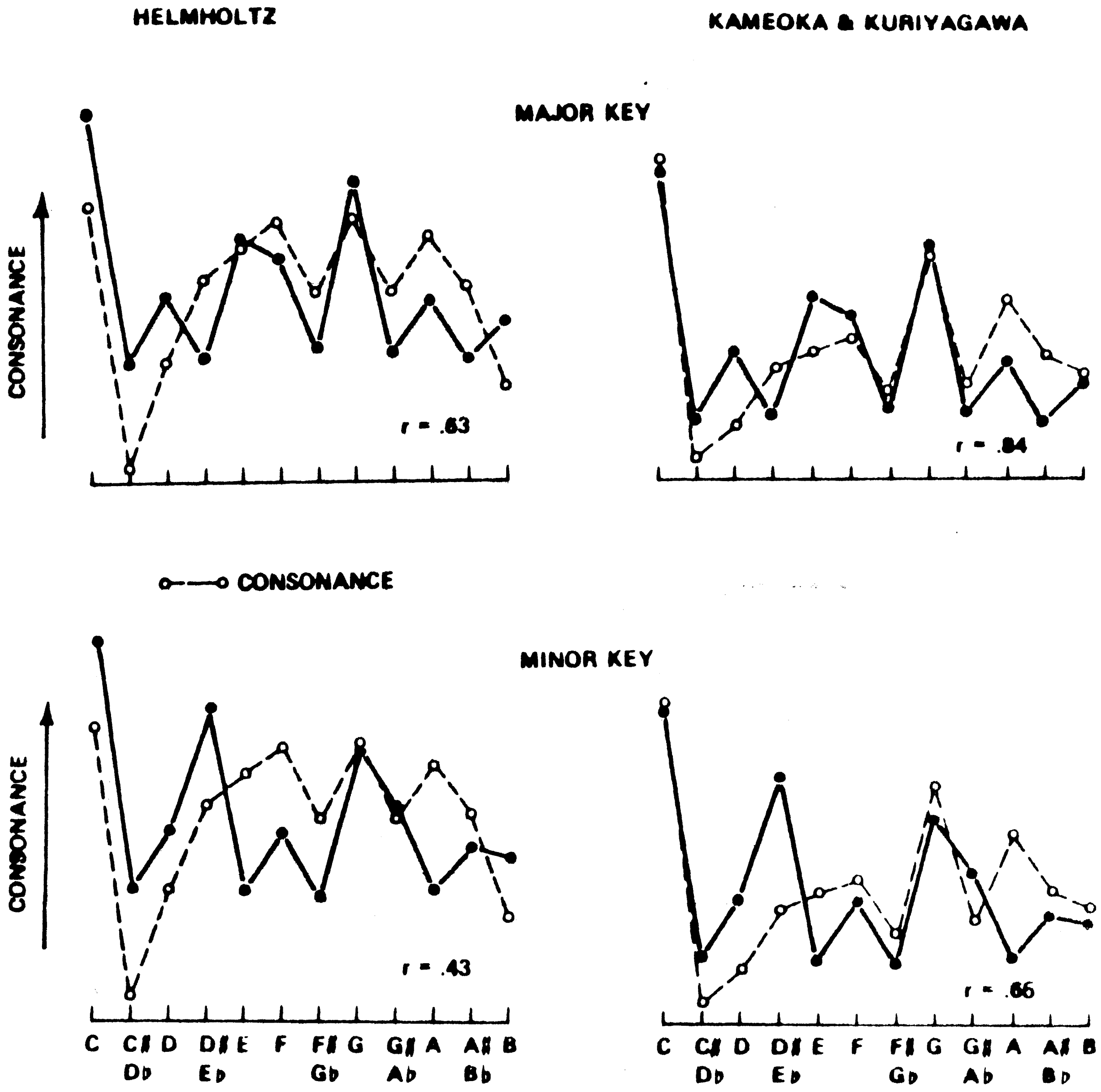

Figure 5. Theoretical values of consonance for intervals formed by each tone of the chromatic scale and a fixed reference tone (C) from Helmholtz (1863/1954) on left and Kameoka and Kuriyagawa (1969) on right; these are superimposed on the tonal hierarchies for C major (left) and C minor (right).

A similar point is made by Meyer (1956, p. 34): "Thus it is pointless to ask what the intrinsic meaning of a single tone or series of tones is. Purely as physical existences they are meaningless. They become meaningful only in so far as they point to, indicate, or imply something beyond themselves... Though the perception of a relationship can only arise as the result of some individual's mental behavior, the relationship itself is not located in the mind of the perceiver. The meanings observed are not subjective. Thus the relationships existing between the tones themselves,... though a product of cultural experience, they are real connections existing objectively in culture." What, then, are the objective musical correlates of the experimental results I have been describing?

Let me approach this question by first considering the possibility that the psychological effects of tonality are in some sense derived from the acoustic structure of tones. More specifically, is it the case that the tonal hierarchy corresponds to the dimension of psychoacoustic consonance, which is turn is presumed to be related to the overtone structure of complex tones?

To explore this possibility, Figure 5 shows on the left the values of consonance taken from Helmholtz's (1863/1954) treatise on music. The value plotted for each tone is that for the tone sounded simultaneously with a constant reference tone which is taken to be the tonic of the key. Superimposed are the tonal hierarchies for major (top) and for minor (bottom). The correlation for major, .63, shows some correspondence between the two sets of values, that is, consonant intervals are rated highly in the probe tone study. The correlation for minor, .43, is lower and not significant. On the right of Figure 5 is the same comparison with a more recent study (Kameoka & Kuriyagawa, 1969). Of the various treatments of consonance considered, this provided the best fit to the tonal hierarchy. Here, the correlations are reasonably high for both major and minor, .84 and .65 respectively, although some fairly large discrepancies can still be seen. It would seem, then, that psychoacoustic consonance may have some role in determining the perceived tonal hierarchy, but other factors are operating in addition.

One possibility is that the tonal hierarchy reflects the way tones are used in music. Hughes (1977) tabulated the total duration of each tone of the chromatic scale in a short piano piece by Schubert, *Moments Musicaux* (Op.94, No.1). The total duration of each tone is plotted in Figure 6, together with the probe tone ratings for the predominant key of the piece, G major. As can be seen, the correspondence is almost perfect. Tones that receive high ratings in a G major context in our experiment are precisely those that are sounded for the longest total duration in this piece.

The next question is: to what extent is this statistical distribution of tones typical of Western tonal- harmonic music? Studies by Youngblood (1958) and Knopoff and Hutchinson (1983) tabled the frequency of occurrence of tones in a variety of vocal melodies, as shown in Table 1. What I did was to correlate these distributions with the probe tone ratings for the appropriate major or minor key. The correlations were uniformly high, and all statistically significant. Thus, the distribution of tones matches closely the ratings from the probe tone experiment.

What about the listeners' ratings of pairs of tones? Are these consistently related to the frequency of two-tone combinations in music? Youngblood (1958) tabled the frequency of all possible pairs of successive tones in twenty songs by Schubert, Mendelssohn, and Schumann. This sample included a total of 1,972 pairs of tones. The frequencies correlated significantly, .67, with listeners' judgments of tone pairs in tonal contexts. So, again, we see a match between the experimental results and the statistics describing the use of tones in tonal-harmonic music. It seems likely that through experience listeners have abstracted and internalized the patterns of tonal distributions, and these are reflected in our empirical studies.

Finally, what are the objective correlates of the relations expressed in the toroidal representation of keys? Here, no statistical summaries were available, but it can be noted that Schoenberg's (1954/1969) maps of musical keys are contained in local regions of the toroidal configuration. Moreover, Daniel Werts (1983) recently developed a toroidal description of key distances very similar to that derived

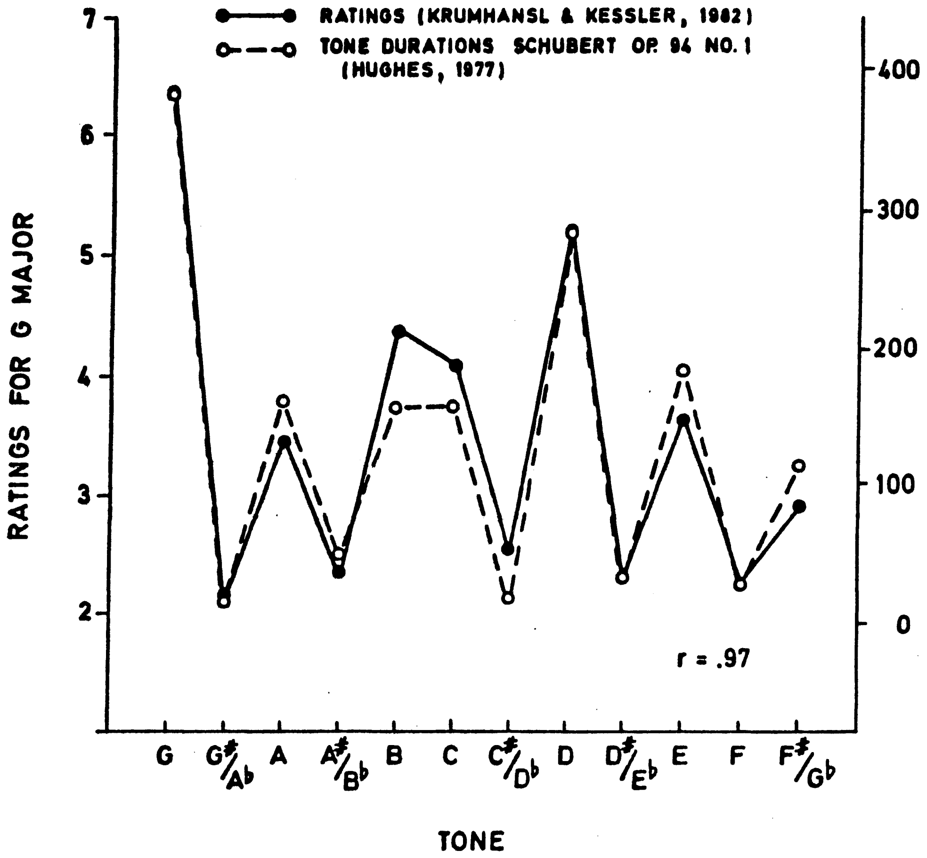

Figure 6. The total durations of each chromatic scale tone in Schubert's *Moments Musicaux*, Op. 94, No. 1, correlate strongly with the tonal hierarchy of the predominant key of the piece, G major.

Table 1. Total frequency of occurrence of chromatic scale tones in vocal melodies (N = 24,852)

Piece	Correlation with Tonal Hierarchy
Schubert Songs (Major)	.88 **
Schubert Songs (Minor)	.86 **
Mozart Arias	.84 **
Hasse Cantatas	.88 **
Strauss Lieder	.93 **
Mendelssohn Arias	.90 **
Schumann Songs	.91 **

** $p < .01$

from the probe tone ratings. His description is based in part on his theory of scale reference, but is also supported by the analysis of a large number of compositions. Changes between keys, both between and within movements, were precisely those that appear close together in the spatial configuration. These correspondences suggest that the psychological measure of key distance has an objective correlate in compositional practice.

To summarize these correlates of the experimental results, the perceived relations between tones are correlated with first-order statistics in a sampling of tonal music, that is, the relative probabilities of successive two-tone combinations. The tonal hierarchy is related to the zeroth-order statistics, that is, the frequency of occurrence of the chromatic scale tones. And, finally, the measure of key distance derived from the tonal hierarchy agrees well with key relations expressed in musical compositions. These correspondences suggest that knowledge of tonal practice evident in listeners' judgments is abstracted from their musical experience, and that through this experience they have internalized the regularities of the musical style.

Harmonic hierarchy

This section summarizes experimental results on perceived relations between chords and considers how the representation of harmonic structure fits with conceptions of interkey distance. Just as a key imposes a hierarchy on the set of tones, it also imposes a hierarchy on the set of harmonies. This hierarchy has been explored experimentally, using methods very similar to those described earlier. In one study (Krumhansl, Bharucha, & Kessler, 1982) pairs of triadic harmonies were presented, and listeners were asked to rate how perceptually related they are to one another. Some sample trials from this experiment can be heard in Sound Example 3.

The results of this study are shown in Figure 7. Here, the chords are labeled by Roman numerals to

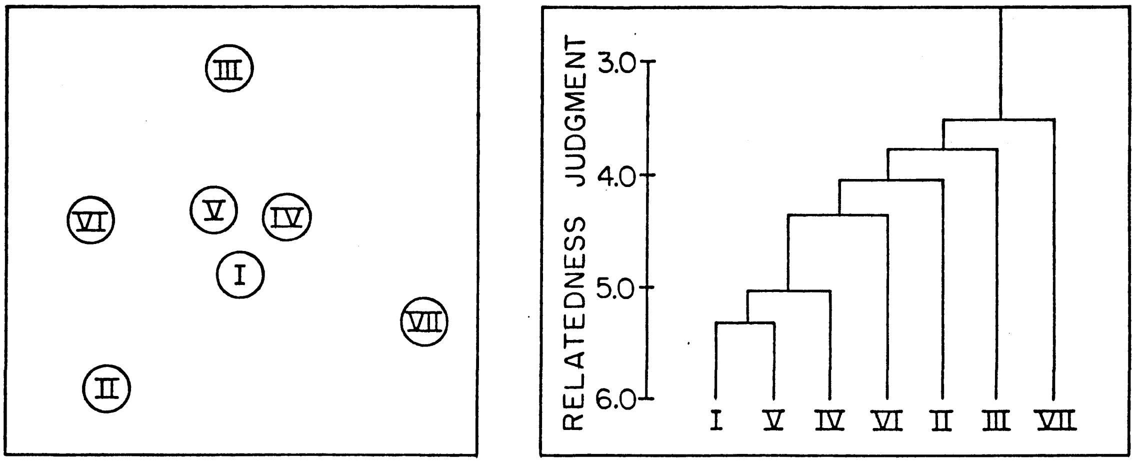

Figure 7. The judgments of chord pairs from Krumhansl, Bharucha, and Kessler (1982) results in a central cluster of harmonically significant chords in the multidimensional scaling analysis (left) and a harmonic hierarchy in the hierarchical clustering analysis (right).

indicate the position of their roots in the diatonic scale. Note that in the multidimensional scaling analysis of the data shown at the left there is a central cluster of the most harmonically significant chords, I, V, and IV. Around these are located chords with weaker harmonic functions. On the right, in a hierarchical clustering analysis of the same data, we see a hierarchy emerging, with the I joined first by the V chord, followed by the IV, VI, II, III, and VII chords.

This hierarchy has, in a number of ways, been shown to be closely tied to key distance. Chords that play significant roles in closely related keys are found to be perceptually related (Krumhansl, Bharucha, & Kessler, 1982) and chords in distantly related keys are perceptually unrelated (Bharucha & Krumhansl, 1983; Krumhansl, Bharucha, & Castellano, 1982). Moreover, the different keys in which chords play harmonic functions are close together in the key map (Krumhansl & Kessler, 1982). And, it is possible to derive from the harmonic hierarchy a measure of key distance that is virtually identical to that derived from the tonal hierarchy (Krumhansl, in preparation). Thus, it appears that in tonal-harmonic music, the two hierarchies converge on precisely the same description of key relationships.

Briefly, let me mention that these descriptions of harmonic structure are supported by a variety of other empirical studies using memory performance (Bharucha & Krumhansl, 1983; Krumhansl, Bharucha, & Castellano, 1982; Krumhansl & Castellano, 1983). Sequences of chords that conform to harmonic conventions are better remembered than those that do not. Chords consistent with the tonal context are easier to recognize and are quite readily confused with one another, but rarely confused with a chord that is inconsistent with the tonal context. So, again, these studies converge on the structural descriptions obtained from the rating studies.

Are there objective correlates of these experimental measures? Again, I will consider acoustic consonance first. Hutchinson and Knopoff (1979) tabled consonance values of all possible three-tone combinations in an octave range based on a model of consonance proposed by Plomp and Levelt (1965). This is Helmholtz's model modified to take critical bandwidth into account. Is it the case that the triads most commonly used are relatively consonant in the set of sixty- six possible triads? Table 2 shows the ranks for major, minor, diminished, and augmented chords in the set of sixty- six. For the first three kinds of chords, there are three consonance ranks depending on how the chord is voiced. As can be seen, these common triads are relatively consonant, with major and minor approximately equal and considerably more consonant than the diminished chord, as would be expected. However, the augmented chord is relatively consonant according to this model. These values do not correspond well to the experimentally measured harmonic hierarchy and deviate in certain ways from musical intuitions, suggesting acoustic consonance may be only one factor influencing the construction of tonal harmony.

Table 2. Consonance of triads. (Hutchinson & Knopoff, 1979)

Triad type	Rank (out of 66)	Average Rank
Major	5, 9, 17	10.33
Minor	7, 11, 14	10.67
Diminished	16, 21, 25	20.67
Augmented	12	12.00

The harmonic hierarchy corresponds much more closely to the frequency of occurrence of chords in tonal-harmonic music. Budge (1943) analyzed representative 18th and 19th century compositions and tabled the frequency with which the diatonic triads occurred. All together there were nearly 66,000 chords considered in this extensive analysis. For both major and minor keys, the psychological harmonic hierarchy correlated significantly with the frequency count; the correlation was .83 for both major and minor. This suggests the hierarchy is established in part through internalizing these statistical distributions which, incidentally, are quite stable over the period of music Budge analyzed.

Psychological judgments of two chord sequences also appear to be rooted in compositional practice. Piston's (1978, p. 21) table summarizes the usual chord progressions. He lists, for each chord, the chords that typically follow it with three levels of frequency: often, sometimes, and less often. These can be roughly quantified as three, two, and one, respectively, with zero assigned to a chord pair not appearing in the table. This can be treated as a rough measure of the frequency of two-chord sequences because, according to Meyer (1956, p. 54), "this is actually nothing more than a statement of the system of probability which we know as tonal harmony". The values resulting from quantifying the table in this way correlated significantly (.53) with listeners' judgments of the relatedness between successively sounded chords.

To summarize these objective correlates of the experimental results, the ratings of pairs of chords correlated with Piston's table of usual root progressions (which reflects the probability of two-chord sequences). The harmonic hierarchy correlated with the zeroth-order statistics (that is, the frequency of occurrence of the chords). So, again, the psychological judgments, here about harmonies, appear to find strong correlates in the way these elements are used in tonal-harmonic music.

The final cell of the experimental program concerns the relations between tones and chords. Here the question is whether the harmonic hierarchy can be accounted for in terms of the tonal hierarchy of its component tones. That is, does the structural significance of chords depend on the tones contained within it? To test this, I correlated the psychological harmonic hierarchy with the sum of the values of its tones in the tonal hierarchy. Although some relationship is apparent, for neither major nor minor keys was this correlation significant. Also, I correlated the relative frequency of chords and the frequency of the component tones in the various statistical counts I described earlier. Again, some relationship is apparent, but neither correlation is significant. Thus, it appears that the harmonic and tonal hierarchies operate somewhat independently, both subjectively and objectively. Recall, however, that both converge on the same description of key structure. This suggests that whatever correspondence is found between the two hierarchies it may be mediated through key structure.

Tracing the developing and changing sense of key

The final experiment I will describe concerns how the sense of key develops and changes over time. Kessler and I (Krumhansl & Kessler, 1982) used the probe tone technique to investigate this problem. We constructed ten different nine-chord sequences, some of which contained modulations between keys. For each sequence, the listener first heard just the very first chord, followed by all possible probe tones. This generated a profile for the sequence after the first chord. Then, the listener heard the first two chords, followed by all possible probe tones, generating a profile for the sequence after the first two chords. This process was continued until the sequence was complete. Sound Example 4 illustrates this technique with contexts consisting of one, two, and three chords.

This method gives a way of evaluating the strength of each possible key after each chord of the sequence. More specifically, we correlated the probe tone ratings for each point through the sequence with the probe tone ratings for all twenty-four major and minor keys collected in the experiment described earlier (Krumhansl & Kessler, 1982). This gives a numerical value indicating how strongly each possible key interpretation is felt at each point in time. These values can then be used to generate a point on the key map, using a technique called multidimensional unfolding. The use of the spatial representation reflects a commitment to the notion

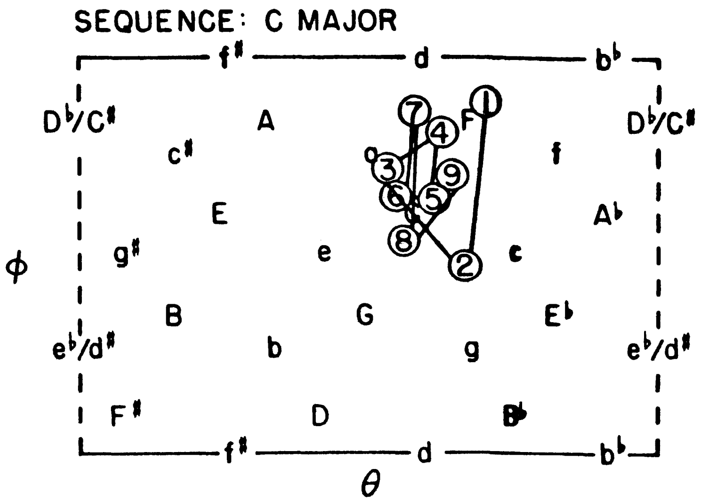

Figure 8. Points representing the relative key strengths after each successive chord for the sequence in C major (Krumhansl & Kessler, 1982)

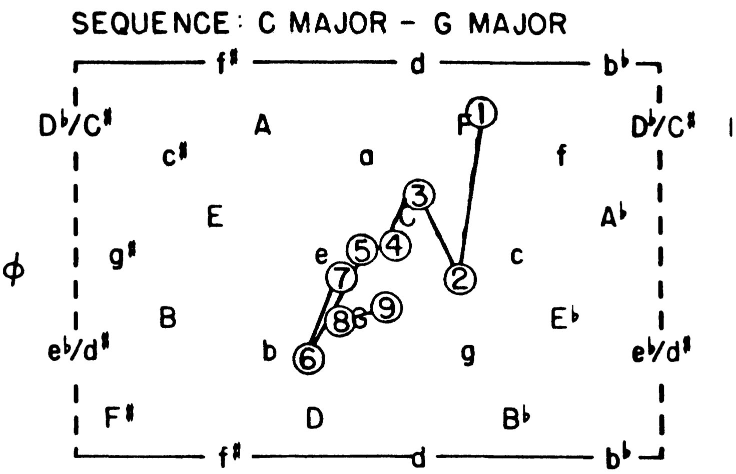

Figure 9. Points obtained as in Figure 8 for the sequence modulating from C major to G major.

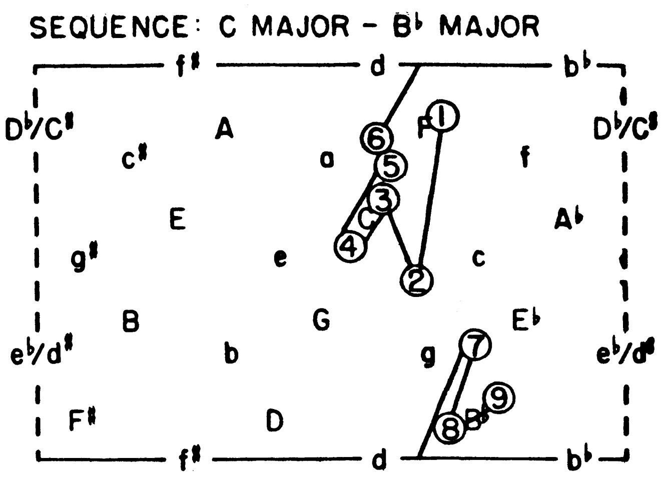

Figure 10. Points obtained as in Figure 8 for the sequence modulating from C major to Bb major.

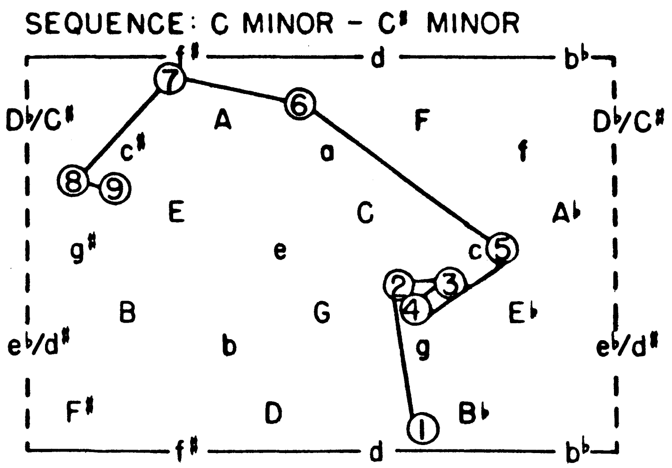

Figure 11. Points obtained as in Figure 8 for the sequence modulating from C minor to C# minor.

that both musically and psychologically tonality is best described with reference to key regions rather than in terms of a single major or minor key.

I will show the results for just four sequences which can also be heard in Sound Example 4. The first sequence was written in C major. However, the tonic triad was not sounded until the fifth position. As can be seen in Figure 8, the points for this sequence moved quickly to the region around C major and remained there throughout. The second sequence was written to modulate from C major to G major. Here, as in all modulating sequences, a pivot chord was contained in position five and a chord unique to the new key was contained in position six. Listeners easily assimilated this key shift, moving readily into the region around G major as shown in Figure 9. The next sequence contained a somewhat more distant modulation, between C major and Bb major. Figure 10 shows that the listeners exhibited a tendency to remain longer in the region of the original key and then shift rather suddenly to the region of the second key. The final sequence contains a very distant modulation, between C minor and C# minor. As can be seen in Figure 11, the trajectory reflects the large distance between keys and, in fact, travels the long way around the torus from one key to the other.

Finally, I will describe a computer algorithm written by Mark Schmuckler and myself that can be applied to musical pieces to identify the initial key and trace modulations throughout. The algorithm is based on the idea that listeners may match the sounded tones within segments of a musical piece to their internal tonal hierarchies, with the best-matching major or minor key being the most likely key interpretation. Specifically, we took each segment and totalled the durations with which each tone of the chromatic scale is sounded using the notated durations. This gives a twelve-dimensional input vector of durations. This vector was then correlated with the probe tone rating profile for each of the twenty-four major and minor keys, producing a twenty-four dimensional output vector, showing the degree of match between the input duration vector and each of the key profiles. The value for any key will be high to the extent that the distribution of tones in the musical segment matches the tonal hierarchy of the key. The results can then be used to find a point on the key map so as to reflect the relative strengths of the different keys.

In the application I will describe here we used the entire Prelude in C minor from Book II of J. S. Bach's Well- tempered Clavier. We selected this prelude because it contains an interesting pattern of shifting tonal centers. To have a basis for evaluating the success of the algorithm, we asked two music theorists to independently give us a quantitative estimate of the relative key strengths in each measure of the piece. These values were then used to find a point on the torus map of keys that best represents the relative weights of the different keys.

Figures 12 - 17 show the match between the experts' judgments (dashed lines) and those of our algorithm (solid line); the music can be heard in Sound Example 5. The first phrase of the prelude, measures one through four, corresponds to points in the region of C minor (Figure 12). The second phrase, measures five through nine, shows a shift to the Eb major region (Figure 13), where the points remain until the end of the next phrase in measure twelve (Figure 14). The next phrase, in measures thirteen through eighteen, contains a pattern of shifting tonal orientations moving from Eb major to the region around F minor (Figure 15). The following phrase, measures nineteen through twenty-two, remains in the F minor region (Figure 16), followed by a shift back to the original key of C minor in the final phrase in measures twenty-three through twenty- eight (Figure 17).

In general, the key judgments of the experts agreed more with one another than with the algorithm, but the algorithm came quite close and indicated the same pattern of shifting tonal centers. Clearly, their key estimates are based on much more detailed and sophisticated criteria than our algorithm, which simply matches the durations of the tones to the tonal hierarchies. But it is interesting to see that a simple algorithm of this sort is as efficient and accurate as it is. Its success suggests that listeners could develop a sense of key by matching the sounded tones against their internalized tonal hierarchies and use this matching process to rapidly orient to the correct tonal center initially and track modulations as they occur. This provides a framework for encoding and remembering the sounded events, which allows the listener to generate

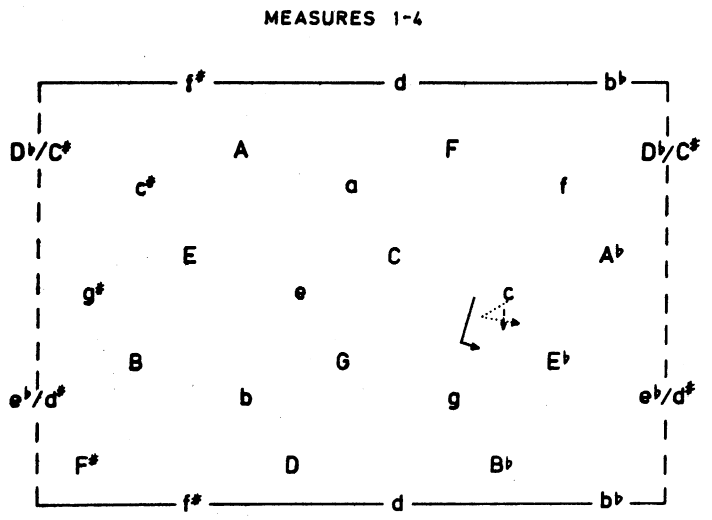

Figure 12. The points obtained from the experts' judgments of key strengths (dashed lines) and the model's analysis of tone durations (solid line) for measures one through four of the C minor prelude, Book II.

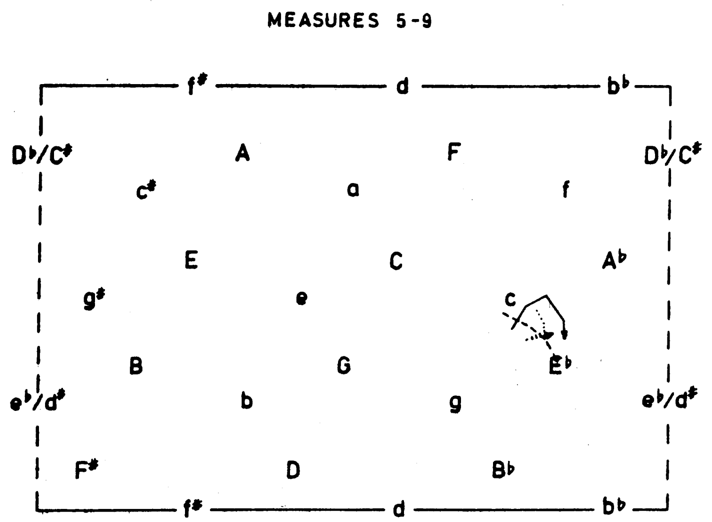

Figure 13. The points obtained as in Figure 12 for measures five through nine.

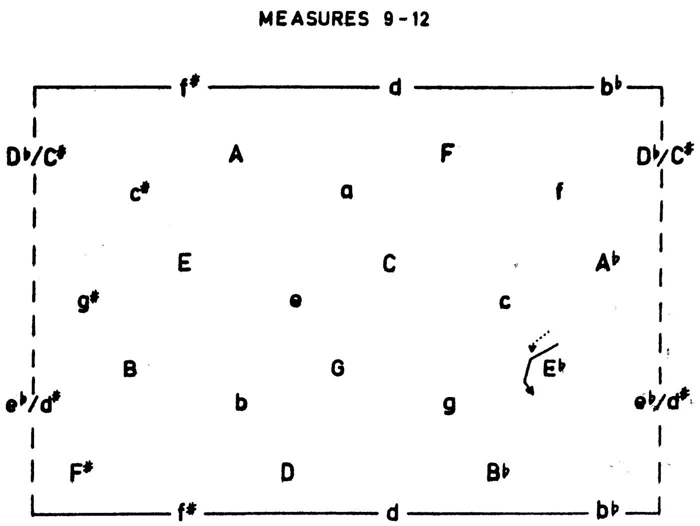

Figure 14. The points obtained as in Figure 12 for measures nine through twelve.

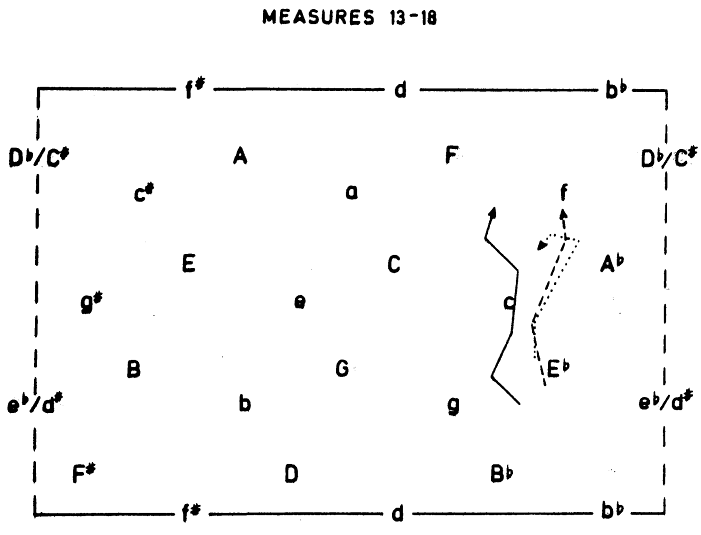

Figure 15. The points obtained as in Figure 12 for measures thirteen through eighteen.

MEASURES 19 - 22

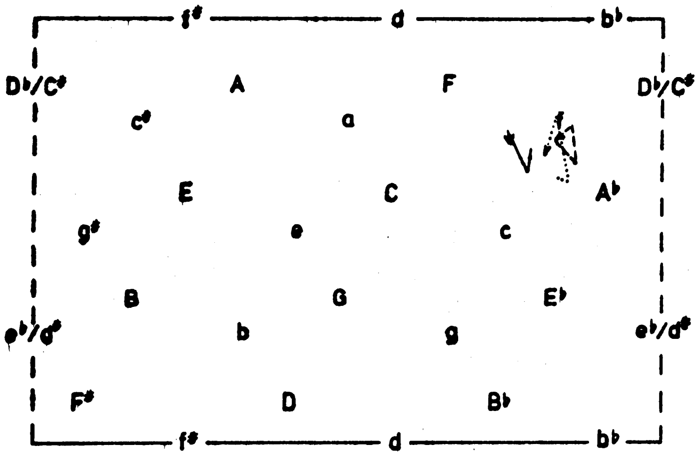

Figure 16. The points obtained as in Figure 12 for measures nineteen through twenty-two.

MEASURES 23 - 28

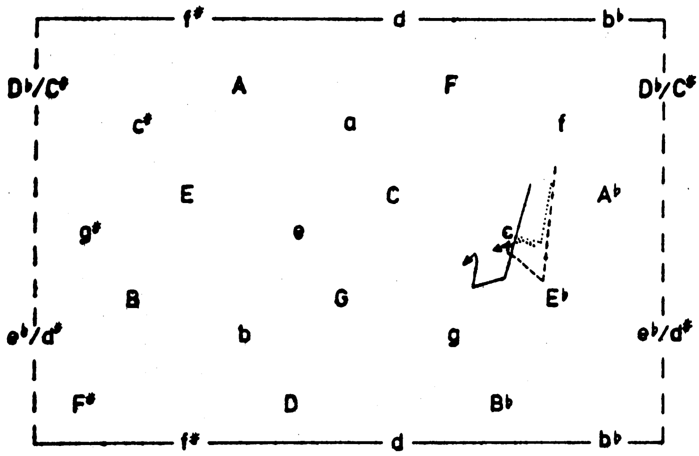

Figure 17. The points obtained as in Figure 12 for measures twenty-three through twenty-eight.

expectations for subsequent events and appreciate tensions and contrast of varying degrees.

Conclusions

This paper provides an overview of a program of research directed at characterizing the internal representation of musical pitch. The approach focuses on relations between elements, because these are held to be central to our musical experience. Three different kinds of elements are identified: tones, chords, and keys. The experiments quantify the perceived degree of relatedness both between elements of the same type and elements of different types. The results show a highly organized and regular pattern of perceptual relations. A tonal context imposes a hierarchy on the sets of tones and chords, which are heard as more or less related depending on their functions within the predominant tonal framework. These regularities are intimately connected to interkey distances, with closely related keys having similar tonal and harmonic hierarchies.

These experimental results find direct correlates in compositional practice. Statistical summaries of the use of tones and chords in tonal-harmonic music correspond closely to the listeners' judgments in the empirical studies. This suggests that the internal representation of musical pitch structure revealed by the experiments has been abstracted from the listeners' experience with the musical style. This correspondence between objective features of music and subjective relations between musical elements was the basis for a computer algorithm which was found to be quite successful in tracing the shifting pattern of tonalities in one application. The dynamic properties of musical experience were investigated in another experiment, the results showing that listeners easily assimilate shifts to closely related keys but resist more distant modulations. Thus, beginning with a set of basic results on perceived relations between musical elements, we have the means with which to begin to characterize more subtle aspects of the experience of music.

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GENERAL PROPERTIES OF MUSICAL PITCH SYSTEMS: Some Psychological Considerations

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In this paper, I would like to consider structural properties of musical pitch systems in somewhat more general terms than in the accompanying paper in this volume. The experiments summarized there all used stimulus materials based on traditional Western tonal-harmonic music. For example, they used diatonic scales, triadic harmonies, traditional chord progressions, and major and minor key contexts. These investigations demonstrated that listeners have a great deal of knowledge about conventional pitch structures. These may reflect psychological constraints from sensory and cognitive processes, as well as knowledge that results from applying various learning strategies to music in the tonal style.

One of the most intriguing questions, however, is how novel pitch systems might be devised. This question is interesting from both a musical and a psychological point of view. Musically, it is a question of expanding available compositional resources. Psychologically, it is a question of exploring the range of sound attributes that can be effectively encoded, organized, and remembered. The issue of how novel pitch systems might be constructed is particularly pertinent because the recent revolution in computer music technology has made possible the manipulation of every conceivably relevant dimension of musical sound. The basic premise is that, for a musical style or individual piece to work successfully as communication, the way the auditory materials are structured must fit or be compatible with the way listeners tend to process and store auditory information. And, the music should be suited to the kinds of learning strategies or principles listeners bring to bear when presented with unfamiliar perceptual information.

My approach will be to begin with traditional Western music and describe in more general terms the structural properties exhibited there. These generalizations may then serve to suggest, by analogy, other ways of constructing scales and chords. In each case, wherever possible, I will indicate the psychological considerations or experimental results that seem relevant. The rationale for this approach is simply to use tonal-harmonic structures as a way of suggesting other possibilities -- but ones that seem consistent with what we have learned about music perception, and the kinds of descriptions that have proved useful in music theory for characterizing traditional pitch systems.

Table 1. Properties of scale structure.

Small number of pitches with well-defined intervals
Intervals selected with respect to overtone structure
Tonal framework established by:
Focal pitches
Asymmetric pattern of large and small intervals
Distinctive intervals
Pattern repeated at interval throughout frequency range
Generates relations between different scales

Pitch materials

I begin with the observation that most listeners do not have absolute pitch. That is, memory for the absolute value of a pitch in isolation is quite poor. This means that relative pitch, the relations between pitches, is what is important for perception and memory. Moreover, the number of items along any continuous dimension that can be reliably remembered and labeled is limited to about seven items or so. These considerations suggest the first property in Table 1 -- that musical materials should be constructed from a relatively small number of discrete pitches with well- defined intervallic distances. A system with this property would seem best matched to the psychological system. Dowling (1978) has noted that, indeed, the scales used in most cultures seem to conform to this constraint.

How might the selection of pitches be determined? In Western music, a number of intervals including the octave, fifth, and major third are prominent, and the octave and the fifth, in particular, appear in many other musical cultures. As is well known, these intervals are formed by tones whose fundamental frequencies have simple numerical ratios. Therefore, they appear relatively early in the overtone series, and when two complex tones forming these intervals are played simultaneously, the result is a smooth, consonant sound. This suggests that one may wish to choose scale tones with respect to the overtone structure. If a number of consonant intervals are included this would allow contrasts with more dissonant intervals.

It should be noted that the exact nature of the overtones can be manipulated. Most string and wind instruments have overtones that are integer multiples of the fundamental, but other relationships are possible, especially with computer generated tones. Mathews and Pierce (1980; see accompanying paper by Mathews) generated tones using a stretched octave and the partials of each tone were similarly stretched. That is, the partials were such that their ratios with the fundamental were slightly larger than the normal ratios of 2, 3, 4, and so on. A short passage was written, and their listeners were quite accurate in judging the tonality of the passage. They did, however, judge that the final cadences produced with the stretched overtones sounded less final than analogous cadences using tones with normal harmonic overtones.

A third property that may be desirable to incorporate into novel systems is establishing a tonal framework through various means. One way is to include one or more focal tones. Elsewhere in this volume, I described the probe tone method used to quantify the tonal hierarchy as applied to major and minor key contexts. The probe tone ratings were strongly correlated with the frequency of occurrence or durations of tones in a number of tonal compositions, suggesting that a piece of music may establish a central reference pitch or pitches through their explicit emphasis in the music.

Castellano, Bharucha, and Krumhansl (1984) obtained evidence that listeners use this kind of information to abstract the appropriate tonal hierarchy with unfamiliar musical styles. That study used themes from North Indian rags and listeners who were either familiar or unfamiliar with the style. Both groups of listeners gave probe tone ratings consistent with music-theoretic descriptions of North Indian music. Moreover, the ratings correlated significantly with the relative durations of the tones in the musical contexts used in the experiment. So, it appears that even naive listeners can appreciate the style- appropriate tonal hierarchy, and use the explicit emphasis given the tones in the music to do so.

Of course, frequency of occurrence and relatively long durations are only two ways to establish focal pitches; these factors are emphasized here simply because they can be easily quantified and compared to empirical measures. Focal pitches may also be established by their position in the phrase, for example if they appear at the beginning and end of rhythmic units, if they fall on strong beats, if they appear as the goal of an ascending or descending contour line, or if they are sounded in a prominent or distinctive timbre. In any case, if certain pitches are emphasized throughout a segment of music, they can serve as reference points in relation to which other pitches can be encoded and remembered.

A second way in which a pitch reference system may be established is through the use of an asymmetric pattern of relatively large and small steps between adjacent scale tones. For example, the major diatonic scale consists of intervals in the following relative step sizes: large, large, small, large, large, large, small. In Indonesian music, the pelog scale has just the complementary pattern: small, small, large, small, small, small, large. These patterns may serve the purpose of helping the listener maintain a constant framework for encoding the sounded pitches or to help in "position finding", to use Richmond Browne's (1981) phrase.

Other scales lack this property of having an asymmetric pattern of relatively large and small scale steps. For example, the slendro scale of Indonesia consists of five approximately equally spaced tones in an octave. That may, in part, account for the fact that in the cross-cultural study done by Kessler, Hansen, and Shepard (1984) both Western and Balinese listeners produced relatively flat and inconsistent probe tone ratings following the slendro context in their experiment.

In a recent study, Krumhansl and Schmuckler (1986) used the octatonic scale as context in a probe tone study. This scale consists of the symmetric pattern: whole tone, half tone, whole tone, half tone, whole tone, half tone, whole tone, half tone. Although there is an alternating pattern of large and small intervals, the pattern repeats four times per octave rather than one. This is why Messiaen (1944) called this a mode of limited transposition. Owing to this property, the pattern would seem less well-suited to defining a pitch reference system than one that repeats only once per octave. Indeed, the listeners in the experiment found it difficult to make consistent probe tone ratings following the octatonic scale context. The rating profiles obtained were relatively flat. Incidentally, the octatonic scale appeared to function perceptually somewhat like a dominant seventh chord; the probe tone ratings resembled the key whose dominant was the tone at the beginning and end of the octatonic scale context.

If the scale contains an asymmetric pattern of intervals, then it will tend to contain a number of relatively rare intervals. Browne (1981) noted that in the diatonic scale, for example, there are just two minor seconds, between the third and fourth scale degrees and between the seventh scale degree and the tonic. And, there is just one interval of a tritone, between the fourth and seventh scale degrees. These intervals, according to Browne, provide the listener with strong cues as to the diatonic set. In support of these observations, Butler and Brown (1984) demonstrated that listeners made accurate key judgments when presented with just three tones if they included the unique tritone interval. Thus, these attributes -- focal tones emphasized by frequent sounding, longer durations and other means, asymmetric patterns of relatively large and small intervals, and distinctive or rare intervals -- would seem to promote the listener's maintaining a reference frame for encoding tones and their interrelations.

The fourth property in Table 1 is that, whatever the pattern of scale steps, it repeats throughout the frequency range. Listeners have limits in the number of discrete pitch classes that can be remembered. So, if one is interested in using a wide frequency range, the number of musical pitches can be increased by repeating the scale pattern at various intervals throughout the range. Most musical cultures use the octave. Presumably this is because the octave appears as the first overtone in the harmonic series and, thus, is a natural choice for acoustic reasons. In addition, there are a variety of empirical results showing that tones separated at octave intervals are psychologically similar. These factors presumably facilitate the listener's coding of the octave equivalent tones in different pitch ranges in terms of the same underlying scale framework. The system described in the accompanying paper by Mathews (see also Roberts, Reeves, Mathews, & Pierce, 1986) employs an alternative interval, the tritave, as the basic interval with respect to which the scale pattern is repeated. This tritave appears as the third partial in the normal harmonic series, and is equal to an octave and a fifth.

The fifth and final property concerning scale structure is that it determines in a natural way relations between different scales. Consider for example the major diatonic scale. Raising the fourth scale degree by a half step generates the collection of scale tones of the next key around the circle of fifths. Repeating this process continues to produce scales around the circle until the initial key is returned to again. A variety of empirical data indicate that listeners are aware of the relations between different scales. Bartlett and Dowling (1980), and Cuddy and collaborators (Cuddy, Cohen, & Miller, 1979; Cuddy & Lyons, 1981; Cuddy, Cohen, & Mewhort, 1981) have shown that key distance affects the recognition of transposed sequences. Thompson (1986; see also the paper in this volume summarizing some of his results) demonstrated that musically trained listeners can judge both direction and distance of modulations. Krumhansl & Kessler (1982) found it was possible to derive distances between keys from probe tone ratings, and that listeners assimilated modulations between closely related keys more easily than modulations between distant keys.

There is a similar principle for generating the circle of thats representation of North Indian scales. In our cross-cultural study (Castellano, et al., 1984), we found that the probe tone ratings, at least those for listeners familiar with North Indian music, could be used to recover the circle of thats. That is, scales that are theoretically similar had similar tonal hierarchies. The natural determination of scale distances is also exhibited by the Pierce scale (see Mathews, this volume; Roberts et al., 1986) and the scale of twenty microtones proposed by Balzano (1980; 1986). This and other aspects of the proposed scales will be described in greater detail below.

At a general level, why might a system of related scales be a desirable feature? Again, I would suggest that it may have to do with cognitive limits in pitch coding. If one scale framework is predominant, and listeners know its close relationship to other scales, then this expands the resources available. It provides for the possibility of shifting smoothly to other scales and allows the expression of varying degrees of contrast and tension.

To summarize so far, I have described attributes of musical scales that may make them well-matched to the psychological system for organizing and remembering pitch sequences. These properties are concerned primarily with the selection of scale pitches, not with their combination into melodic, rhythmic, and harmonic patterns. The formation of these patterns would seem to be governed primarily by aesthetic considerations, although some progress is being made on the representation of melodic and metrical structures with a view to psychological processes (see, for example, Sundberg & Lindblom, 1976; Deutsch & Feroe, 1981; Lerdahl & Jackendoff, 1983). In the following section, I will offer a few suggestions based on empirical work concerning basic properties of harmonic systems.

Harmonic materials

Not all musical cultures emphasize vertical, that is, harmonic, relations to the extent that they are emphasized in traditional Western tonal music. However, this musical system has been shown to contain an extremely rich set of compositional possibilities, so it is natural to inquire what kinds of principles of organization are found in this style and why they work perceptually. The first property in Table 2 is that tones sounded simultaneously are chosen with respect to the effects they will have in terms of consonance versus dissonance. This is not to say that only consonant intervals should be employed, but that whatever choice is made there

Table 2. Properties of harmonic structure.

Chord types selected with respect to overtone structure
Small number of chord types consistently mapped to scale degree
Harmonic hierarchy
Chord tones dispersed throughout range of scale
Generates relations between different scales

will be consequences for the listener owing to psychoacoustic consonance, and this derives largely from the harmonic structure of the tones, as discussed before.

The second property is that there is a relatively small number of chord types employed, and a fairly consistent mapping between chord type and the degree of the scale on which the chord is built. In traditional music, one finds predominantly major, minor, and diminished triads and seventh chords as determined by the structure of the diatonic scale. There may be two psychological principles operating here. First, if chord construction is determined in some principled way by scale structure, then this further serves to maintain the tonal framework for encoding pitch information. Second, memory limitations may constrain the system to a relatively small set of chords whose distinctive functions can be appreciated and remembered.

The third property is the establishment of a harmonic hierarchy. Through their explicit emphasis in the music, certain chords become reference points for perception and memory. This will be true especially to the extent that there is some correspondence between tonal and harmonic hierarchies. Chords built on tonally stable pitches, for example the tonic, would be natural candidates for harmonic reference points. Our perceptual studies, however, show some independence of the chord and single pitch hierarchies; the harmonic hierarchy is not well predicted from the tonal hierarchy of its component tones. This suggests that vertical and horizontal organizations may only loosely constrain one another.

The second to last property concerns the way in which chord tones are selected from the set of scale pitches. In the diatonic scale, triads are formed from every other tone, in part because this results in relatively consonant intervals of fifths and thirds. However, it is interesting to note that choosing the chord tones in this way also disperses the tone throughout the scale range so that smooth voice-leading is possible. That is, when successive chords are sounded, it is possible to choose tones so that most voices do not jump more than a third or so. Proximity is a predominant factor in pitch perception, giving a sense of coherence between successive tones. So whatever tones comprise the scale, chord tones should be dispersed throughout the scale range to allow for smooth voice- leading.

Finally, the harmonic structure should give rise to the same interkey relations as the scale structure. In the diatonic system, harmonically significant chords, such as the tonic, dominant, and subdominant are shared by closely related keys, and Krumhansl (in preparation) shows that the harmonic hierarchy yields the same measure of interkey distance as the tonal hierarchy. This is true even though the two hierarchies seem to be at least partially independent. In developing novel systems, then, to the extent that they are intended to emphasize vertical relations, harmonic hierarchies should be constrained by the intended key relations. More particularly, harmonically significant chords should be precisely those that are shared by closely related keys.

Analyses of traditional and novel pitch systems

Psychological investigations of musical pitch structure employing materials from traditional tonal-harmonic music suggest a set of characteristics or features that may, by analogy, be incorporated into novel systems. The lists above are not intended to be comprehensive or obligatory. But these features seem well-grounded in existing musical literature and psychological evidence. Musical explorations and concomitant psychological investigations promise to further our understanding of the range of musical attributes that are matched to the psychological system for encoding, organizing, and remembering structured auditory information.

In what follows, I consider a number of different musical system in terms of the above characteristics. Included are well-established systems extensively employed in compositions, such as diatonic, pentatonic and North Indian scales, and novel systems that have been proposed recently, particularly those by Pierce (see accompanying paper by Mathews; Roberts et al., 1986) and Balzano (1980, 1986). The objective of these analyses is to make explicit the parallels between these systems and to highlight their differences. The potential of the novel systems can be evaluated finally only in terms of the richness of their compositional resources. But, to the extent that the above list of characteristics are grounded in psychological principles, these observations may serve to guide their development.

Diatonic scale

The first set of analyses is of the diatonic scale and traditional triadic harmony. The properties are summarized in Table 3, and are assumed to be generally well-known. They are given here primarily for the purpose of later comparisons and to establish notation general enough to be employed when considering other systems. The diatonic set can be considered as a subset of the chromatic set, labeled in the first column as 0 through 12 (which corresponds to the octave). The second column gives the ratios in equal- tempered tuning of the pitches with the zero tone, and the third column their values in cents. The fourth column gives simple

Table 3. Diatonic scale.

Pitch	Ratio with 0	Cents above 0	Simple-integer ratio	Interval name	Diatonic scale	Interval vector
0	1.0000	0	1:1 =1.0000	Unison	0	
1	1.0595	100	16:15=1.0667	m2		2
2	1.1225	200	9:8 =1.1250	M2	2	5
3	1.1892	300	6:5 =1.2000	m3		4
4	1.2599	400	5:4 =1.2500	M3	4	3
5	1.3348	500	4:3 =1.3333	P4	5	6
6	1.4142	600	36:25=1.4400	Tritone		1
7	1.4983	700	3:2 =1.5000	P5	7	
8	1.5874	800	8:5 =1.6000	m6		
9	1.6818	900	5:3 =1.6667	M6	9	
10	1.7818	1000	16:9 =1.7778	m7		
11	1.8877	1100	15:8 =1.8750	M7	11	
12(0)	2.0000	1200	2:1 =2.0000	Octave		

Cycle of fifths:

	 1	8	3	10	5	0	7	2	9	4	11	6	1 .....
Diatonic					x	x	x	x	x	x	x		
Pentatonic						x	x	x	x	x			

integer ratios approximating the equal-tempered ratios, and the fifth the names of the intervals. The next column indicates the subset that comprises the diatonic set, the seven tones in an octave range.

The pattern of scale degrees is asymmetric; it maps onto itself only at the interval of an octave. The pitches of the scale generally have small integer ratios with the zero pitch, which means that they would form relatively consonant intervals. If one considers the total set of intervals defined by the diatonic set, there are a total of two minor seconds (or major sevenths), five major seconds (or minor sevenths), four minor thirds (or major sixths), three major thirds (or minor sixths), six perfect fourths (or perfect fifths), and one tritone. This can be summarized in vector notation as <2,5,4,3,6,1>, where the first value corresponds to the number of minor seconds (major sevenths), the second to the number of major seconds (minor seconds), and so on. These values are shown in the last column of Table 3.

Note that the most consonant interval, the perfect fourth (or fifth) is the most frequent interval, and the least consonant intervals, the minor second and the tritone, the least frequent. Beyond this, there is only an approximate correspondence between the simplicity of the integer ratio and the number of intervals in the diatonic set. This interval vector has the property that each entry is unique; the number of intervals of any type is different than the number of intervals of any other type. And, there is at least one interval of each type.

A cycle of scales, the cycle of fifths, emerges naturally from the scale structure; it is shown at the bottom of Table 3. For any scale, raising the fourth scale pitch by a half-step (in conventional terminology) or a step in the chromatic set gives the scale of the next key on the cycle whose tonic is an interval of a perfect fifth above the first tonic. Another way of saying this is that the cycle of keys has a generator of seven (corresponding to a perfect fifth) or, equivalently, a generator of five (corresponding to a perfect fourth). The diatonic set consists of tones that are adjacent on this cycle, and the next scale is obtained by dropping the left-most pitch and adding one on the right. Any generator that is mutually prime with the number of chromatic scale tones (twelve) would have the property of generating a complete cycle of this sort. But it is interesting to note that there is just one such number (five or, equivalently, seven) that is mutually prime with twelve, other than the trivial generator of one.

One can also inquire whether there is a connection between the number of scale tones (seven) and the generator of the cycle of fifths. Under the constraints that scale tones must be adjacent on the cycle, and that the shift between adjacent scales is accomplished by changing a single tone one step in the chromatic set, there are just two possible scale sizes: seven and five. Seven, of course, is the diatonic scale and, as will be discussed below, five adjacent pitches on the cycle of fifths form the pentatonic scale. In contrast, suppose the scale consisted of six adjacent tones on the cycle (the hexachord), for example, the scale with tones: 0 2 4 5 7 and 9. Then the next scale would be reached by deleting the tone 5 and substituting the tone 11 -- not a change of a single tone by one step in the chromatic set.

Another concern is how many scale pitches are needed to give a unique interval vector. Suppose just two adjacent tones on the cycle of fifths are taken; this gives the interval vector of <0,0,0,0,1,0>. Then, suppose three adjacent tones on the cycle are taken; this gives the interval vector of <0,1,0,0,2,0>. The interval vectors for different scale sizes are given in Table 4. Note that the first scale size with an interval vector that has all nonzero entries and no repeated values is seven, the diatonic scale. After this there are duplicated values in the vector. Before this, there are missing intervals.

Turning now to harmonic structure, the present discussion will be limited to triads formed by selecting every other diatonic scale degree. For example, the triad built on 0 has the pitches (0,4,7); the triad built on 2 has the pitches (2,5,9); and so on. For the (major) diatonic set, this gives major chords on pitches 0, 5, and 7, minor chords on 2, 4, and 9, and a diminished chord on 11. Thus, there is a small number of chord types consistently mapped to the scale degree on which the triads are built. Constructing chords in this fashion also has the consequence of dispersing the tones throughout the octave range, making possible smooth voice-leading between successive chords.

Table 4. Interval vectors for cycle of fifths and chromatic set.

Scale size	Scale	Interval vector
2		<0,0,0,0,1,0>
3		<0,1,0,0,2,0>
4		<0,2,1,0,3,0>
5	Pentatonic	<0,3,2,1,4,0>
6		<1,4,3,2,5,0>
7	Diatonic	<2,5,4,3,6,1>
8		<4,6,5,4,7,2>
9		<6,7,6,6,8,3>
10		<8,8,8,8,9,4>
11		<10,10,10,10,10,5>

The tones of the major chords can be expressed in terms of the ratios 4:5:6, which would mean that they are relatively consonant. The tones of the minor chords can be expressed in terms of the ratios 10:12:15, and are therefore less consonant. The tones of the diminished chord can be expressed in terms of the ratios 25:30:36, and are therefore less consonant still. This suggests that the major chords might naturally serve as structural harmonies within the system.

The function of the major chords as structural harmonies is also consistent with the cycle of fifths relation between different scales. Consider the diatonic set built on 0, as in Table 3, and its two neighboring scales on the cycle of fifths, built on 5 and 7, respectively. Table 5 shows the triads in the 0 scale, and their functions in the other two scales. Roman numerals indicate the scale degree of the root tone of the chords and whether they are major, minor, or diminished. Given that the I chord is built on the central reference pitch of each scale, it would be expected to dominate in the harmonic hierarchy. Then, because of the interlocking pattern of chord functions in neighboring keys, secondary structural importance would be indicated for the IV and V chords, which is consistent with harmonic practice.

Table 5. Chords of related diatonic scales.

Chord tones	Chord type	Function in 0 scale	Function in 5 scale	Function in 7 scale
0 4 7	Major	I	V	IV
2 5 9	Minor	ii	vi	-
4 7 11	Minor	iii	-	vi
5 9 0	Major	IV	I	-
7 11 2	Major	V	-	I
9 0 4	Minor	vi	iii	ii
11 2 5	Diminished	vii$^{\circ}$	-	-

To conclude this section, the system of diatonic scales and triadic harmonies satisfies each of the properties given in Tables 1 and 2. The diatonic system has a small set of tones and chords with well-defined intervals, a predominance of relatively consonant intervals, an interval vector with unique entries, and a generator for a cycle of scales compatible with scale membership and the functions of chords in closely related scales. The question to be considered in the next analyses is the extent to which these properties are shared by other systems. I consider first a number of scales that can be constructed from the same basic chromatic set: the pentatonic, octatonic, and the ten North Indian scales on the circle of thats.

Pentatonic scale

As indicated above, the pentatonic scale, shown in Table 6, consists of five adjacent pitches on the cycle of fifths. That is, the scale pitches can be obtained using the same generator as the major diatonic scale, and a natural cycle of pentatonic scales emerges. To obtain the next scale on the cycle, the tone 0 is lowered to 11 -- a half-step (in conventional terminology) or a step in the chromatic set. This can be seen with reference to the bottom of Table 3. The interval vector is <0,3,2,1,4,0>. Neither the minor second nor the tritone is present, the least consonant intervals. Excluding these, there are unique entries in the vector for every other interval, the most frequent of which is the perfect fourth (fifth) which is also the most consonant. The pattern is asymmetric, repeating only at the octave interval. Because of the small number of tones (five) the set would seem less well-suited to the construction of chords than the diatonic set, so harmonic properties will not be considered.

To summarize, the pentatonic scale consists of a small number of scale tones forming an asymmetric pattern within the octave. Ignoring the missing intervals, which are also the most dissonant, the interval vector has unique entries, and the most frequent interval is also the most consonant. There emerges a natural cycle of scales with a generator of seven (or, equivalently, five) which gives the cycle of fifths. The next scale on the cycle results from changing one scale tone by a step in the chromatic set. Thus, the pentatonic scale exhibits all the characteristics of the diatonic scale, with the exception that the interval vector shows two missing intervals.

Octatonic scale

The octatonic scale was chosen for consideration

Table 6. Pentatonic and octatonic scales.

Pitch	Pentatonic scale	Interval vector	Octatonic scale	Interval vector
0	0		0	
1		0	1	4
2	2	3		4
3		2	3	8
4	4	1	4	4
5		4		4
6		0	6	4
7	7		7	
8				
9	9		9	
10			10	
11				

because it emerged as a useful theoretical construct in a perceptual study of Stravinsky's music (Krumhansl & Schmuckler, 1986); it is also found in the music of Messiaen, Bartok, and jazz where it is known as the "diminished scale". The octatonic scale exhibits very different structural characteristics than the diatonic and pentatonic scales. It consists of the set of tones shown in Table 6. There are alternating half and whole steps (in conventional terminology) or, equivalently, steps of one and two in the chromatic set. This pattern repeats four times per octave which means that the set maps onto itself with transpositions of three, six, and nine steps in the chromatic set. The interval vector, <4,4,8,4,4,4>, shows a great deal of redundancy, and no predominance of consonant intervals. There is no cycle which would generate a system of related octatonic scales. The only possible complete cycles for the chromatic set do not contain the octatonic set as adjacent tones. Instead, the octatonic set can be represented as the sum of two subcycles: (0,3,6,9) and (1,4,7,10).

The presence of a number of minor and major thirds allows the construction of traditional chord types, including major, minor, diminished, and dominant seventh chords. The choice of these particular tone combinations does not follow naturally from the distribution of intervals, however. That is, there is no simple rule for forming chords that is analogous to the rule for constructing diatonic triads from alternate scale tones. Nor is there a natural hierarchy of chords resulting from a cycle of scales. Thus, the harmonic possibilities are quite independent of the structure of the octatonic scale, in sharp contrast to the triadic harmonies of the diatonic scale.

To sum up, the octatonic scale is seen to exhibit none of the characteristics of the diatonic and pentatonic scales, other than the rather trivial characteristic that the number of scale tones is again relatively small. This suggests that the choice of simultaneities and the articulation of tonal and harmonic hierarchies is determined compositionally, rather than following from inherent structural properties of the scale.

North Indian circle of thats

North Indian classical music employs a vastly expanded set of scales (called thats) compared with Western music. They are, however, drawn from a basic set of twelve focal pitches per octave with tunings approximately equal to the chromatic set. All together, there are a total of thirty- two different seven-tone scales, all of which have the same tonic tone. Each of these contains an interval of a fifth between the tonic (Sa) and the fifth scale tone (Pa); this is an invariant of the musical system. Each of the other five tones can appear in one of two positions: the second, third, sixth, and seventh scale tones can be lowered by a step (in the chromatic set) and the fourth scale tone can be raised by a step. All combinations of these can occur, generating a full set of thirty-two scales. Different scales are considered theoretically to be more or less similar, and geometric representations are used to summarize the relations. A cycle of scales, called the circle of thats, is embedded in the multidimensional representation of scales. The tones in the scales on the circle are indicated in Table 7. The present discussion will be limited to these because of the cyclical relation between scales. More details can be found in Jairazbhoy (1971) on which the description here and in Castellano et al. (1984) is largely based.

The cyclic nature of the scale relations is brought out clearly at the bottom of Table 7, where the scale tones are indicated with respect to the cycle of fifths. As can be seen, neighboring scales differ only in terms of one step in the chromatic set. The first six scales consist of seven adjacent tones on the cycle of fifths; as such, they correspond to different diatonic scales. It should be emphasized, however, that the tonic of all these scales is the pitch 0, unlike the diatonic scales. For these six scales, the numbers in the interval vector are unique: <2,5,4,3,6,1> with a rough correspondence between the number of intervals and consonance.

The remaining four scales consist of the invariant tones 0 and 7 plus five adjacent tones separated from these. These five tones, thus, comprise a pentatonic scale. In these four cases, the interval vectors do not have unique entries. They are: for Al <3,4,4,3,5,2>; for Todi <4,3,3,4,5,2>; for Purvi <4,3,3,4,5,2>; and for Marva <3,4,4,3,5,2>. There is a less consistent relationship between these numbers and consonance. However, if the 0 and 7 tones are omitted (the invariant tones), the resulting

Table 7. North Indian scales on the circle of thats

Kalyan	Bilaval	Khamaj	Kafi	Asavri	Bhairvi	A7	Todi	Purvi	Marva
0	0	0	0	0	0	0	0	0	0
					1	1	1	1	1
2	2	2	2	2					
			3	3	3	3	3		
4	4	4						4	4
	5	5	5	5	5				
6						6	6	6	6
7	7	7	7	7	7	7	7	7	7
				8	8	8	8	8	
9	9	9	9						9
		10	10	10	10	10			
11	11						11	11	11

Cycle of fifths:

	2	9	4	11	6	1	8	3	10	5	0	7	2	9	4	11	6	1.....
Kalyan											x	x	x	x	x	x	x	
Bilaval										x	x	x	x	x	x	x		
Khamaj									x	x	x	x	x	x	x			
Kafi								x	x	x	x	x	x	x				
Asavri							x	x	x	x	x	x	x					
Bhairvi						x	x	x	x	x	x	x						
A7					x	x	x	x	x		x	x						
Todi				x	x	x	x	x			x	x						
Purvi			x	x	x	x	x				x	x						
Marva		x	x	x	x	x					x	x						

vector, <0,3,2,1,4,0>, has unique entries (ignoring the missing intervals), as discussed above for the pentatonic scale.

Each of these scales forms an asymmetric pattern that repeats only once per octave. The relations between scales are based on the same generator as the diatonic and pentatonic scales. It is interesting to note this characteristic because North Indian music does not employ modulations between scales. Another distinctive feature of North Indian music is that it does not employ tones sounded simultaneously in chords, with the exception that the Sa (0) and Pa (7) are sounded constantly in the drone accompanying the melody -- hence the strict adherence to the invariant perfect fifth between these tones. The greater emphasis on melodic than harmonic aspects may be related to the greater variety of scales and the fact that the distribution of intervals shows less correspondence to consonance.

Despite these differences, the scales on the circle of thats share many formal characteristics with diatonic and pentatonic scales: a small number of tones forming asymmetric patterns repeating only at the octave, a unique generator in terms of which scale relations can be expressed, an influence of consonance, and an unequal distribution of values in the interval vector. The last two characteristics are, however, somewhat less clear than for the diatonic scale.

Pierce scale

Impetus for these analyses originally came from the development of the Pierce scale described in the accompanying paper by Mathews and also by Roberts et al. (1986). This system shares many characteristics with traditional diatonic harmony, and I was interested in formalizing these. Apart from clarifying similarities and differences between the systems, it seemed possible that these observations may, in some way, prove useful as the potential of this system is explored compositionally. In the next section, I give a similar analysis to the twenty- fold microtone system proposed by Balzano (1980, 1986).

Table 8 presents the essential characteristics of the Pierce scale as presently formulated; more details can be found elsewhere in this volume. The full set of tones is generated in equal-tempered tuning with ratios of the thirteenth root of three, which has the consequence that the thirteenth tone has a ratio of 3:1 with the first. Hence, it corresponds to the second overtone in the harmonic series and forms an interval (called a tritave) of an octave and a fifth. A number of tones in this set can be expressed in simple ratios of the integers three, five, and seven (and these multiplied by three which corresponds to a transposition by a tritave). As can be seen from looking at the values of the tones in cents, the tones generated in this way do not correspond to those in the chromatic set, despite the fact that simple integer ratios appear in both systems.

The Pierce scale itself consists of the nine tones indicated, forming an asymmetric pattern repeating only at the interval of a tritave. The choice of this subset of scale tones is such that most tones form a simple ratio with the tone 0. This property can also be seen when the interval vector, <5,5,8,6,5,7>, is examined; the three most frequent intervals have simple numerical ratios. With the exception of these intervals, however, the entries are not unique, although all intervals are present. There is a natural cycle of scales that emerges, with a generator of

Table 8. Pierce scale

Pitch	Ratio with 0	Cents above 0	Simple-integer ratio	Pierce scale	Interval vector
0	1.0000	0.0	1:1 =1.0000	0	
1	1.0882	146.2		1	5
2	1.1841	293.3			5
3	1.2886	438.5	9:7 =1.2857	3	8
4	1.4022	584.6	7:5 =1.4000	4	6
5	1.5258	730.8			5
6	1.6604	876.9	5:3 =1.6667	6	7
7	1.8068	1023.1	9:5 =1.8000	7	
8	1.9661	1169.2			
9	2.1395	1315.4	15:7 =2.1429	9	
10	2.3282	1461.5	7:3 =2.3333	10	
11	2.5335	1607.7			
12	2.7569	1753.8		12	
13(0)	3.000	1900.0	3:1 =3.000		

Cycle with generator three:

	.....	8	11	1	4	7	10	0	3	6	9	12	2	5	8	.....
Pierce				x	x	x	x	x	x	x	x	x				

Table 9. Interval vectors for cycle with generator three and Pierce set.

Scale size	Interval vector
2	<0,0,1,0,0,0>
3	<0,0,2,0,0,1>
4	<0,0,3,1,0,2>
5	<1,0,4,2,0,3>
6	<2,1,5,3,0,4>
7	<3,2,6,4,1,5>
8	<4,3,7,5,3,6>
9	<5,5,8,6,5,7>
10	<7,7,9,7,7,8>
11	<9,9,10,9,9,9>
12	<11,11,11,11,11,11>

three as shown at the bottom of Table 8. In this cycle, the Pierce scale tones are adjacent and the next scale can be found by a change of one step, i.e., by changing the tone 1 to the tone 2.

Given this cycle with generator of three, one can inquire whether there are other scales with different numbers of tones that would permit this kind of modulation by changing a single scale tone by a step. The only other possibility would be a scale consisting of four tones, which presumably would be too impoverished for musical purposes. One can also ask whether scales of other sizes have unique entries in the interval vectors and all intervals represented at least once. Table 9 shows the interval vectors for the different scale sizes. The only scale with unique nonzero values has seven tones -- and it is such that the most consonant intervals are the most common. This scale size, however, does not permit modulation to the next scale by a single one-step change.

Given that the Pierce set has thirteen tones, a prime number, a complete cycle can be generated by any number. Table 10 shows the possibilities, excluding the trivial generator of one. Indicated for each generator are the numbers of scale tones that would allow the next scale to be reached by changing a single tone by a step. Following this are indicated the number of scale tones needed to generate a unique interval vector with all nonzero entries and the corresponding interval vector. In all cases, the number of tones needed is seven. The only case giving a unique interval vector and permitting scale changes by a single step is the seven-tone scale with generator two. Its tones are: (0,2,4,6,8,10,12). Between adjacent tones, there is just one interval of one step (between 12 and 0); the others are two steps. The prevalence of two-step intervals may limit its utility. Moreover, the interval vector does not show a prevalence of intervals with simple ratios. Nonetheless, this alternative scale may warrant further consideration. To conclude, the nine-tone scale with generator three -- that is, the Pierce scale as currently formulated -- may be a reasonable compromise between a number of structural characteristics. It lacks only the uniqueness of entries in the interval vector.

From the Pierce scale tones, it is possible to construct two different types of chords with integer ratios. One of these, called major, has ratios of 3:5:7; in it, there are six steps between the root and the middle tone and four steps between the middle tone and the highest tone. The other chord type, called minor, has ratios of 15:21:35; in it, there are four steps between the root and the middle tone and six steps between the middle tone and the highest tone. The Pierce system has the somewhat peculiar

Table 10. Pierce set. Scale sizes for different generators permitting modulation and with unique interval vectors.

Generator	Scale sizes permitting modulation	Scale size with unique interval vector	Interval vector
2	6,7	7	<1,6,2,5,3,4>
3	4,9	7	<3,2,6,4,1,5>
4	3,10	7	<3,1,5,4,6,2>
5	5,8	7	<2,4,5,1,6,3>
6	2,11	7	<5,3,1,2,4,6>

Table 11. Pierce set. Chords of related scales.

Chord tones	Chord type Pierce label	Function in 0 scale	Function in 10 scale	Function in 3 scale
0 6 10	Major	I	III	VIII
0 4 10	Minor	i	iii	viii
3 9 0	Major	III	V	I
3 7 0	Minor	iii	v	i
4 10 1	Major	IV	VI	-
6 12 3	Major	V	-	III
6 10 3	Minor	v	vii	iii
7 0 4	Major	VI	VIII	IV
9 0 6	Minor	vii	ix	v
10 3 7	Major	VIII	I	VI
10 1 7	Minor	viii	i	-
12 3 9	Minor	ix	-	vii

property that both major and minor chords can be built on the first, third, fifth, and eighth scale degrees. Neither type can be built on the second scale degree; only major chords can be constructed on the fourth and sixth scale degrees; and only minor chords can be constructed on the seventh and ninth scale degrees. Thus, the present formulation does not have the property that there is a consistent mapping between chord type and the degree of the scale on which the chord is built. The distribution of chord tones throughout the tritave range would, however, seem to make possible smooth voice-leading.

One can inquire whether the relations between scales suggests a natural chord hierarchy. Table 11 shows the major chords and the minor chords of the 0 scale and their functions in neighboring keys. There, the chords are labeled by Roman numerals to correspond to the scale degree of the root, and major and minor chords are indicated in capital and small letters, respectively. The diagram shows there are many chords in the 0 scale that also function in the neighboring scales. If primary structural significance is given to the I/i chord, then the relationships between scales would indicate the III/iii and VIII/viii to have secondary structural roles. The diagram, however, suggests no natural way to determine the chord type to be assigned to the ambiguous cases, although the simpler numerical ratios for the major chords may weigh in favor of these.

To summarize, the Pierce scale exhibits virtually all the structural characteristics of the diatonic scale.

Table 12. Twenty-fold microtone scale

Pitch	Ratio with 0	Cents above 0	Scale	Interval vector
0	1.0000	0	0	
1	1.0353	60		0
2	1.0718	120	2	7
3	1.1096	180		2
4	1.1487	240		5
5	1.1892	300	5	4
6	1.2311	360		3
7	1.2746	420	7	6
8	1.3195	480		1
9	1.3660	540	9	8
10	1.4142	600		0
11	1.4641	660	11	
12	1.5157	720		
13	1.5692	780		
14	1.6245	840	14	
15	1.6818	900		
16	1.7411	960	16	
17	1.8025	1020		
18	1.8661	1080	18	
19	1.9319	1140		
20(0)	2.0000	1200		

Cycle with generator nine:

...17	6 15	4 13	2 11	0	9 18	7 16	5 14	3 12	1 10 19	8 17....
Scale:			x x	x	x x	x x	x x			

The only exceptions are the redundant values in the interval vector and the somewhat greater number of scale tones. The primary difference from the diatonic case is in the construction of chords. There is no entirely consistent mapping between the scale degree of the root and chord type. There are numerous chords that have multiple functions in related scales which would support modulations, and a natural harmonic hierarchy is suggested by these scale relations. However, the scale cycle cannot be used to disambiguate the chord types assigned to the scale degrees.

Twenty-fold microtone scale

Balzano (1980, 1986) proposed a system with twenty equally-spaced (in log frequency) microtones per octave based on what he calls group-theoretic principles. The essential characteristic is that the set of tones is generated by two mutually prime numbers (four and five) whose product is the number of tones in an octave (twenty). This is by analogy to the chromatic set, which can be generated by two mutually prime numbers (three and four) whose product is the number of tones of the chromatic set (twelve). The properties of the twenty-fold system are presented in Table 12. As can be seen, the set includes one minor third, one tritone, and one major sixth, but otherwise the tones do not coincide with the chromatic set. From this set of twenty, a nine-tone scale is selected that has an asymmetric pattern repeating once per octave.

The system is constructed without regard to consonance; the intervals cannot be expressed as ratios of integers. This factor, Balzano (1980, 1986) argues, is not essential and the emphasis, instead, is on group-theoretic or geometric properties. The scale is given by a cycle with generator of nine, shown at the bottom of Table 12, where the scale steps are indicated. The next scale on the cycle is achieved by changing the tone 2 by a step to the tone 3. The interval vector for this scale, as shown, has two missing intervals but otherwise the entries are unique. The vector is, thus, similar to that for the pentatonic scale.

For this generator of nine, the number of scale tones needed to give a unique interval vector with nonzero entries is eleven, as shown in Table 13. The eleven-tone scale, like the nine-tone scale, also permits modulation with a change of a single tone by a step. In this case, then, there is a scale size that both supports modulations and has a unique interval vector: the scale with eleven tones and generator nine -- not the nine-tone scale proposed by Balzano.

Moreover, the generator of nine is not the only possibility; any value that is mutually prime with twenty will generate a complete cycle. So, the possibilities, other than the trivial generator of one, are three, seven, and nine. Table 14 shows for each of these values, the scale sizes permitting modulation by a one-step change, and the scale sizes with unique interval vectors. The only case in which these coincide is for the scale, discussed above, with eleven tones and a generator of nine. It would seem, then, that given the twenty microtone division of the octave, this is a unique case in which a number of structural properties converge.

Turning now to the harmonic properties of the twenty- fold microtone scale, Balzano (1980) originally proposed that triads be formed using every other tone of the scale. More recently, Balzano (1986) noted these chords leave large gaps in the octave range, and has proposed using tetrads instead. Both proposals will be considered here. Their commonalities will be described first. The chords in both cases would be highly dissonant and, likely, undifferentiated in this respect. However, the schemes proposed produce a consistent mapping between the scale degree of the root of the chord and the chord type, and a distribution of chord types. The triads are of three types. The first has an interval of five steps between the root and the next tone, and then an interval of four steps. There are four of these on scale degrees one, two, five, and six, denoted I, II, V, VI, respectively. The second type of triad has an interval of four steps between the root and the next tone, and then an interval of five steps. There are four of these on scale steps three, four, eight, and nine, denoted iii, iv, viii, ix, respectively. There is one triad with two four-step intervals on the seventh scale degree, denoted vii . Table 15 shows the triads and their functions in the neighboring scales of 11 and 9. The numerous chords having multiple functions in these related scales would support modulations between them. The interlocking pattern suggests that the V and VI would have secondary structural significance to the I chord, and

Table 13. Interval vectors for cycle with generator of nine and twenty-fold microtone scale.

Scale size	Interval vector
2	<0,0,0,0,0,0,0,0,1,0>
3	<0,1,0,0,0,0,0,0,2,0>
4	<0,2,0,0,0,0,1,0,3,0>
5	<0,3,0,1,0,0,2,0,4,0>
6	<0,4,0,2,1,0,3,0,5,0>
7	<0,5,0,3,2,1,4,0,6,0>
8	<0,6,1,4,3,2,5,0,7,0>
9	<0,7,2,5,4,3,6,1,8,0>
10	<1,8,3,6,5,4,7,2,9,0>
11	<2,9,4,7,6,5,8,3,10,1>
12	<4,10,5,8,7,6,9,4,11,2>
13	<6,11,6,9,8,7,10,6,12,3>
14	<8,12,8,10,9,8,11,8,13,4>
15	<10,13,10,11,10,10,12,10,14,5>
16	<12,14,12,12,12,12,13,12,15,6>
17	<14,15,14,14,14,14,14,14,16,7>
18	<16,16,16,16,16,16,16,16,17,8>
19	<18,18,18,18,18,18,18,18,18,9>

Table 14. Twenty-fold microtone set. Scale sizes for different generators pe <18,18,18,18,18,18,18,18,18,9>

Table 14. Twenty-fold microtone set. Scale sizes for different generators permitting modulation and with unique interval vectors.

Generator	Scale sizes permitting modulation	Scale size with unique interval vector	Interval vector
3	7,13	11	<4,5,10,3,6,9,2,7,8,1>
7	3,17	11	<8,5,2,3,6,9,10,7,4,1>
9	9,11	11	<2,9,4,7,6,5,8,3,10,1>

Table 15. Twenty-fold microtone set. Triads of related scales

Chord tones	Function in 0 scale	Function in 11 scale	Function in 9 scale
0 5 9	I	V	VI
2 7 11	II	VI	-
5 9 14	iii	-	viii
7 11 16	iv	viii	ix
9 14 18	V	-	I
11 16 0	VI	I	II
14 18 2	vii°	-	-
16 0 5	viii	iii	iv
18 2 7	ix	iv	-

that the II chord might follow these. However, because the chord types are undifferentiated by consonance, this factor would not reinforce this harmonic hierarchy. As Balzano (1986) noted, these triads do not span the octave range and would, thus, make smooth voice-leading difficult.

Because of this, Balzano (1986) proposed using tetrads instead. The tetrads, consisting of four tones each, are again formed from every other scale tone. All together there are four different types of tetrads. The first has intervals of five, four, and five steps; they appear on scale degrees one, two, and six, denoted I, II, and VI, respectively. The second tetrad type has intervals of four, five, and four steps; they appear on scale degrees three, four, eight, and nine, denoted iii, iv, viii, and ix, respectively. The third has intervals of five, four, and four; this is one on the fifth scale degree, denoted v . The final tetrad type has intervals four, four, and five; there is one on the seventh scale degree, denoted vii. Table 16 shows the tetrads and their functions in the neighboring scales of 11 and 9. Again, an interlocking pattern of chord functions is found, supporting modulations. However, the pattern is somewhat weaker, and it seems problematic that the I of the 9 scale is not in the 0 scale. Given this, the II and VI tetrads would appear to have secondary significance to the I tetrad.

To summarize, the primary feature lacking from the system proposed by Balzano is respect for the acoustic dimension of consonance. The system, as proposed, does exhibit a number of more abstract

Table 16. Twenty-fold microtone set. Tetrads of related scales.

Chord tones	Function in 0 scale	Function in 11 scale	Function in 9 scale
0 5 9 14	I	-	VI
2 7 11 16	II	VI	-
5 9 14 18	iii		

structural properties. It has an asymmetric pattern of scale degrees, and a cycle of scales in which neighbors differ only in terms of a single one-step change. The generator for this cycle, however, is not unique, nor does the interval vector contain all possible intervals. Ignoring these, it does have nonredundant entries. In these respects, it differs from the diatonic case. There is a consistent rule for generating triads and tetrads with a fairly well-balanced distribution of a small number of chord types. And, the formation of chords is generally consistent with the relations between different scales.

Conclusions

These analyses highlight a number of similarities and differences between the pitch systems considered. Using diatonic harmony as a reference point, its characteristics are found to be shared by other systems to varying degrees. Some of these characteristics are primarily psychological in nature, deriving from psychoacoustic and memory processes. Other characteristics are more formal, but have been shown to have consequences for perceptual organization and memory in the context of traditional tonal-harmonic music. The extent to which other systems with analogous properties produce comparable psychological effects is an interesting question that would seem most fruitfully addressed through a combination of approaches. The perceptual effects can be assessed empirically, but such studies should be guided by theoretical formulations and compositional explorations of the resources of the proposed novel pitch systems.

The two novel systems considered here provide interesting contrasts, but also share a number of characteristics. In both the Pierce system (see Mathews, this volume; Roberts et al., 1986) and the twenty-fold microtone system (Balzano, 1980; 1986), there are nine-tone scales forming an asymmetric pattern. A cycle of scales can be found such that scale tones are adjacent and modulation to the next scale is accomplished by shifting one scale tone by a step. Both systems allow for the construction of a variety of chord types, and closely related scales have a number of chords in common. This interlocking pattern would support modulations between scales and suggest a hierarchy of chord functions.

The Pierce system is strongly influenced by considerations of consonance. The determination of scale tones and the formation of chords are such that intervals which can be expressed in terms of integer ratios predominate. The system as presently formulated, however, does not have the property that there is a consistent mapping between chord type and the scale degree of the root of the chord. In contrast, the twenty-fold microtone system has well-defined rules for constructing chords, but consonant intervals are lacking from the scales and chords. Indeed, the whole system is based on the premise that its abstract structural properties can be perceived without intervals that are differentiated in terms of consonance. Empirical support for this is presently lacking.

The primary characteristic that was difficult to assess given the present formulation of these two systems was how a tonal hierarchy might be determined. This property of the diatonic system has been shown to have a variety of consequences for perception and memory, as summarized in the accompanying paper. The tonal hierarchy, moreover, is strongly correlated with statistical distributions of tones in Western tonal-harmonic music. This raises the question as to whether, in these novel systems, the hierarchy can be determined compositionally or whether there are inherent constraints from other structural characteristics. It would seem that scale tones ought to dominate over nonscale tones, and that the hierarchy should be consistent with the cycles of scales. The degree to which these considerations constrain the tonal hierarchies and the consequences for the relationship between tonal and harmonic hierarchies is a matter for further theoretical and empirical investigations.

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MELODIC CHARGE AND MUSIC PERFORMANCE

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Introduction

This article presents part of a rule system that can be used when note signs are automatically converted into the corresponding sound sequences by means of a computer. In a way, then, these rules describe what the skilled musician is supposed to do when performing music in a musically appropriate way.

Music Performance Rules

The rules have been developed during "music lessons" given by an experienced musician and music teacher (author LF). Thus, rather than a normal music student, a computer has served as the pupil: melodies have been written in conventional notation into the computer memory, and these melodies were then performed by the computer system. The instrument played by the computer was a synthesizer. Thus, the strategy has been analysis-by-synthesis; performance rules have been tentatively formulated, implemented one by one in the computer program, and the effect of each rule on the performance of melodies has been assessed by careful listening. Depending on the musical result, the individual rule has been accepted, modified, or rejected. In this way explicit statements can be made regarding performance rules.

Evidently, the musical judgement of the authors cannot be accepted as the final proof for a rule. Therefore formal listening experiments were also carried out (Thompson & al, 1986). The results show that musically competent listeners (professional musicians) prefer performances where these rules have been applied to performances where no rules were applied. In other words, musically trained listeners prefer performances following these rules to performances which are in a closer agreement with the nominal notation.

Here we will present two rules that relate to the measurements described by Carol Krumhansl elsewhere in this volume. It seems that one of these rules puts emphasis on notes that are remarkable in some sense. We have tried to quantisize the remarkableness in terms of something that we have called the note's melodic charge. The other rule generates crescendos and decrescendos reflecting harmonic progressions. Again we use the notion of remarkableness as the key, and we quantify the remarkableness of chords in terms of the harmonic charge. Thus, the harmonic charge for a chord, derived from the chord notes' melodic charges, is used for computing changes in sound level. A more detailed

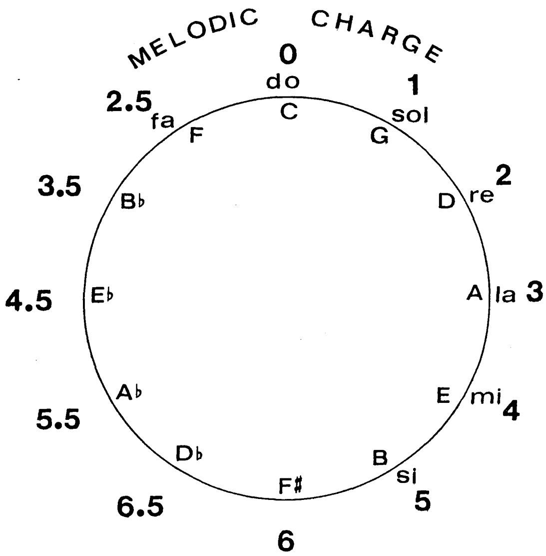

Fig. 1. Definition of melodic charge for notes when appearing over a C major chord. Note that the melodic charge increases by one unit per fifth-step, but all charge values are higher on the subdominant than of the dominant side of the root of the prevailing C major chord.

description of the various steps involved will be given below.

Melodic Charge

The melodic charge of a note reflects its remarkableness, given the harmonic context, as was just mentioned. It is closely related to the "Quintspannung" presented by Ernst Kurth in the beginning of this century (Kurth, 1917). Figure 1 defines, along the circle of fifths, the melodic charge of notes for a C major chord context. Note that the melodic charge increases by one unit for each fifth-step along the circle. Thus, it increases with the distance to the root of the chord as measured in fifths. Note also that the values are higher in the anticlockwise (subdominant) direction than in the clockwise (dominant) direction. In other words, there is no symmetry with respect to the root of the chord.

The melodic charge of a note is computed using the root of the prevailing chord as the zero reference. In the performance program the loudness and duration of a tone is increased in proportion to its melodic charge; the sound level in dB is increased by a factor of 0.19 times the melodic charge. Likewise, the duration is increased by a factor of $[1+0.018*\Delta L]^{1/2}$, where ΔL is the change in sound level due to the melodic charge. For example, the tone one minor second above the root of the prevailing chord has the melodic charge of 6.5. Its sound level is increased by $0.19*6.5 = 1.2$ dB, and its duration is increased by a factor of $[1+0.018*1.2]^{1/2} = 1.0107$, or about 1 %.

Even though the effects are very small in terms

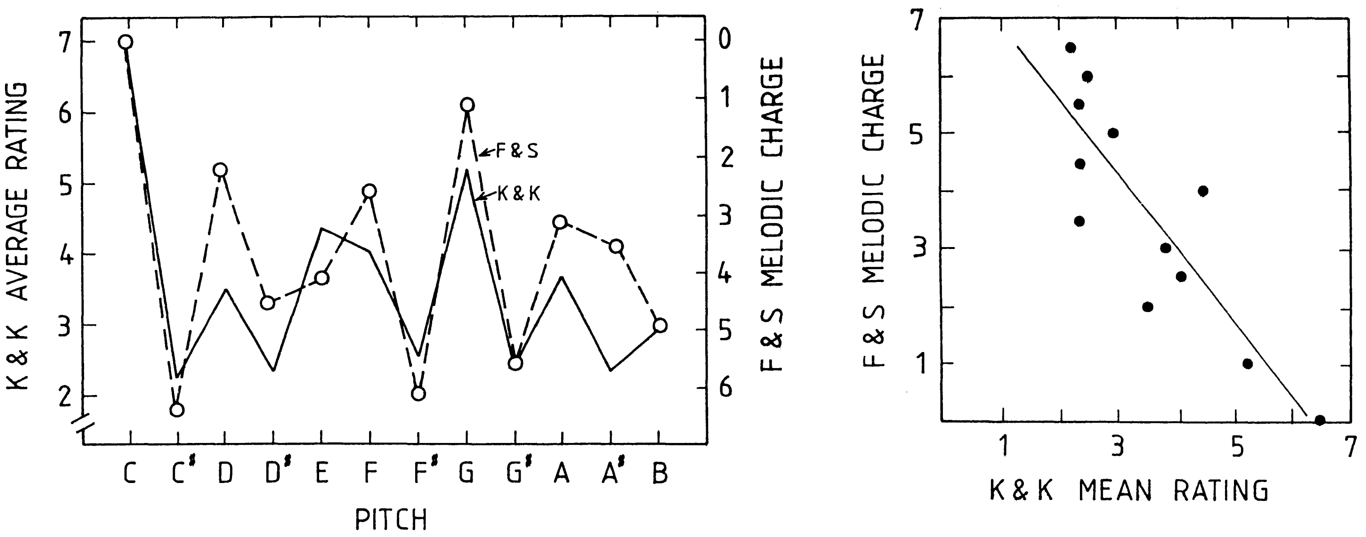

Fig. 2. Correlation between melodic charge and the Krumhansl & Kessler probe tone ratings for a major tonic. To the left the values are plotted for a C major context, to the right as a correlogram. The agreement suggests that our melodic charge and Krumhansl's probe tone ratings might refer to the same thing, although the exact numbers differ slightly.

of dB and msec, this way of marking melodic charge turned out to improve the musical quality of performance (Thompson & al, 1986). Subjectively, it seems that this rule serves the purpose of emphasizing the remarkable notes.

It is interesting that listeners want the musician to announce the melodic charge of the notes, because, evidently, this implies that the listener is able to sense this melodic charge by himself, without assistance of the performer. The situation is not unsimilar to that of listening to a person speaking: it is embarrassing to listen to people reading a text if they do not show, by means of e. g. emphasis, that they understand what is important in the text.

As can be observed in Figure 2 this melodic charge shows a significant correlation with Krumhansl's probe tone ratings. The agreement suggests the possibility that our melodic charge and Krumhansl's probe tone ratings are actually the same thing, although they appear slightly different because of different methods of measurement. It should be realized that the test used for validating melodic charge in our performance program can hardly be very sensitive to small changes of the values. Also, it is possible that the probe tone ratings reflect a combination of more than one factor, and the position on the circle of fifths is merely one such factor.

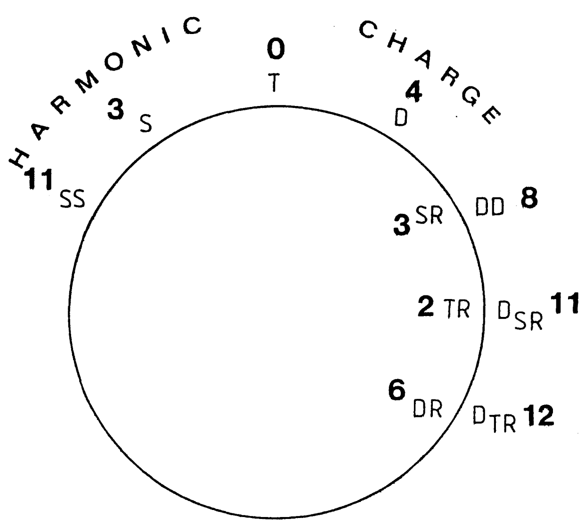

Fig. 3. Harmonic charge values for various chords. In a C major tonality the symbols refer to T=C major, S=F major, SS=Bb major, D=G major, DD=D major, D_{SR}=A major, D_{TR}=E major, D_{DR}=B major, SR=d minor, TR=a minor, and DR=e minor. The harmonic charge is derived as a weighted sum of the chord notes' melodic charge.

Harmonic Charge

The harmonic charge, C_H, is a weighted sum of the chord notes' melodic charges $C_{M,I}$, $C_{M,III}$, and $C_{M,V}$:

$$C_H = 2/3*(3*C_{M,I} + 2*C_{M,III} + C_{M,V} - 9)$$

The harmonic charge of chords in a C major tonality are shown in Figure 3.

The amplitude of the first note after each chord change is increased by ΔL, which is derived from the C_H:

$$\Delta L=1.5*(C_H)^{1/2} \text{ [dB]}$$

Then, the intermediate notes are given intermediate levels, so that crescendos and decrescendos are created. Too slow crescendos are hard to notice, and therefore such crescendos have to be avoided. This is realized by delaying the onset of amplitude increase until 1.9 sec ahead of the chord change. Decrescendos, on the other hand start immediately after the chord change. The duration of each note in a crescendo or decrescendo is lengthened by a factor C_{DR} proportional to the increase in sound level ΔL:

$$C_{DR} = (1+0.018*\Delta L)^{1/2}$$

For example, the level increase from a C-major tonic to the dominant of the relative of the tonic, or E-major, implies an increase in harmonic charge from

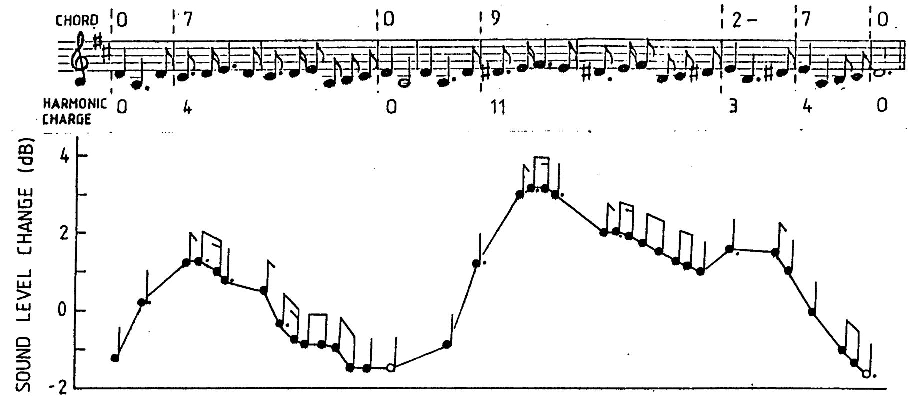

Fig. 4. Illustration of how crescendos and decrescendos are derived from the changes in harmonic charge in the harmonic progression. The chord symbols are shown above the music in a notation specifying the distance in semitones between the root of the chord and the root of the tonic; minus sign denotes minor chord. The harmonic charge values are shown just below the music. The graph illustrates the resulting sound level changes.

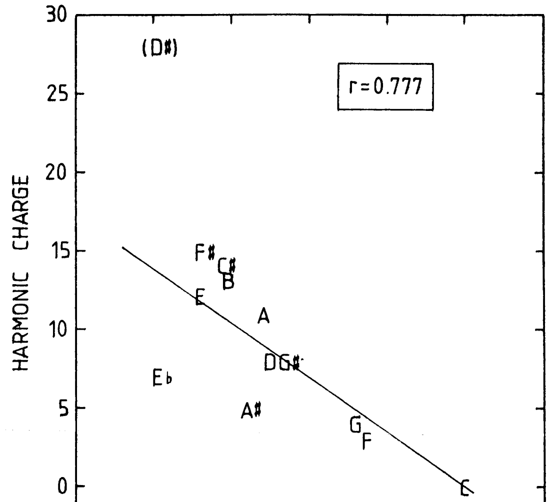

Fig. 5. Correlation between harmonic charge and data gathered by Krumhansl and Kessler mirroring listeners' ratings of major chords in a tonality context. The line is the linear regression line with the correlation coefficient r=0.777. In computing this regression the Eb chord was included while the D# chord was not; the line comes close to the mean of the values for Eb and D# chords.

0 to 12. The level will then increase by a $\Delta L = 1.5*(12)^{1/2} = 5.2$ dB, and the maximum increase of duration, occurring at the onset of the new chord, will amount to a factor of $C_{DR} = (1+0.018*5.2)^{1/2} = 1.046$, or 4.6 %.

Chord changes suggested by each test melody are decided upon by the user. The principles for creating crescendos and decrescendos are illustrated in Figure 4. This rule improved the musical acceptability of the performance of melodies (Thompson & al., 1986). This supports the conclusion that the harmonic charge is an essential property of harmonies in music.

Again, there is a correlation with regard to the major chords with the data gathered by Carol Krumhansl and coworkers from listeners' experiences of various chords in a tonality context, as shown in Figure 5. The correlation coefficient (r=0.777) is reasonably high but would have been considerably higher if the E flat major chord were omitted. The harmonic charge of this chord would be much higher (C_H=28) if regarded as a D sharp major chord. Interestingly, the correlation coefficient would have increased if the average of these two harmonic charge values had been used.

This correlation between listeners' evaluations of scale tones and chords, on the one hand, and rules for the performance of music, on the other, is by no means unexpected. On the contrary, given the fact that music is communication between a sender (the performer) and a receiver (the listener), it seems almost trivial that listeners' preferences are in some sort of accordance with performers' playing; it would merely reflect the self- evident (though rarely appreciated) fact that they have a code in common.

In any event, given the fundamental differences between Krumhansl's and our methods of measurement, it is encouraging to see the correlations in Figures 3 and 5; they point toward the possibility of bridging the traditional gap between music psychology and music performance research, two fields that ideally should mutually support each other.

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HARMONY AND NEW SCALES

M.V. Mathews, J.R. Pierce, and L.A. Roberts

INTRODUCTION

Modern composers have a large appetite for new musical materials. This has lead to big changes in music including the generalization of the rules of traditional harmony in late romantic music, twelve-tone music, and music in which periodicity is deemphasized, such as percussive music, multiphonic music, and music depending heavily on random noises. Although many successful pieces have been written with these new materials, both composers and listeners have had to struggle to provide and appreciate alternatives to the rich harmonic structure which is so important in diatonic music. For the most part, the new materials do not provide alternatives to keys, modulations, and chord progressions.

Is it possible to design new scales which are different from diatonic scales but do have perceptible and rich underlying harmonic structures? This question is very much unanswered. We can certainly say it is difficult to create such new materials. The major difficulty is that it takes a long time to learn to hear and appreciate harmony, and it is difficult to predict what can, and what cannot, be learned. Our appreciation of diatonic music is an overlearned cultural state that has developed over our entire lifetimes. Experiments with long-term learning are almost impossible to do because of the time involved.

Although long-term learning is not one of its notable successes, psychoacoustic science has made great progress in the last decades. Much has been learned about perception, particularly at peripheral levels; computers have provided ways of generating completely general test sounds; paradigms for reliably collecting listener's opinions have been developed; data analysis and multidimensional scaling methods to understand the experimental results have been created. Can we use these scientific techniques to design new musical materials which have prespecified properties? We do not know the answer to this question, but we are optimistic and hopeful that it is yes.

In this paper we will discuss three studies. The first proposes "intonational sensitivity" as a measure of the validity of a chord to form the harmonic basis of a scale. Intonational sensitivity is a measure of the ability of a listener to hear whether or not a chord is in tune. The second study develops a set of new scales from two nontraditional chords that have the same intonational sensitivity pattern as has the traditional major chord from which the diatonic scale can be developed. The third study develops a new scale by "stretching" the intervals in the diatonic scale and also stretching the intervals between the partials of the timbres used to play the scale by a similar amount so as to preserve coincidences of partials in chords in the stretched and unstretched scales. Although some theories of harmony would predict similar harmonic relationships in the stretched and unstretched music, our studies show that differences exist.

INTONATIONAL SENSITIVITY

We here propose a tool to characterize and study chords. We call this tool "intonation sensitivity." The operational test for intonation sensitivity is some form of judgment test in which listeners say, for example, which chord of a pair they prefer. The sensitivity of their preference to changes in intonation is a possible quantitative measure of intonation sensitivity.

In order to explore and validate this test, we have studied perceptions of four different chords. Two of these chords are traditional: the major triad, in which the notes have frequency ratios 4:5:6; and the minor triad, with ratios 10:12:15. In addition, we have used two nontraditional chords. These nontraditional chords are also triads but with frequency ratios 3:5:7 and 5:7:9, respectively.

These particular nontraditional chords were chosen for comparison with the traditional chords for two reasons. First, they involve frequencies which are not part of any traditional scale system (their notes fall in the cracks between the keys of a piano). Thus we can study perceptions of these chords apart from harmonic biases based on musical exposure and training. Second, they have a clear pattern of coincident upper partials and a reasonably high, unambiguous, fundamental bass. Coincident partials are the most important factor in Helmholtz' (1954) theory of harmony. The fundamental bass is the key to Rameau's (1971) theory of harmony and is also important in Terhardt's (1974, 1977) recent extension of Rameau's theory.

For the chords we are studying, the pattern of the coincident partials and the fundamental bass frequency is immediately apparent from the frequency ratios of the tones in the chords. For example, for the major chord with ratios 4:5:6, the 5th harmonic partial of the first note will have the same frequency as the 4th harmonic partial of the second note. For the 3:5:7 chord, the 5th partial of the first note will be coincident with the 3rd partial of the second note, and so on.

The ratio of the fundamental bass to the frequency of the lowest note in the chord is simply the number specifying the frequency ratio of that note. Thus for the major chord (4:5:6), the frequency of the fundamental bass is 1/4 the frequency of the lowest note of the chord. For the 3:5:7 chord, the frequency of the fundamental bass is only 1/3 that of the lowest note.

The minor chord (ratios 10:12:15) differs from the major and nontraditional chords. The frequency of its fundamental bass is at 1/10 the frequency of its lowest note and coincidences occur at higher numbered partials than for the other three chords. Consequently, we might expect results for the minor chord to be unique.

In order to implement a test of intonation sensitivity, several versions of each chord were synthesized in which the frequency of the center note of the chord could be varied from its just value (where "just" means integer frequency ratio). The upper and lower notes were kept at their just frequencies.

Listeners heard two versions of a chord, one following the other, and were asked to make judgments about these chords. The center note of each chord deviated from just intonation by 30 cents below to 30 cents above just intonation (1 cent = 1/100 semitone). This range was selected on the basis of preliminary listening tests. However, it is also in the same order of magnitude that some intervals in the equally-tempered scale deviate from just intervals. For example, the equally-tempered major third is 14 cents larger than the just major third; the equally- tempered minor third is 16 cents smaller than the just minor third.

If listeners' preference curves for intonation are a monotonic function of the absolute deviation of the intonation from some ideal intonation, then a quantitative evaluation of intonation sensitivity should be easily made. Our results show that this view is too simplistic. Intonation preferences are nonmonotonic in interesting ways. The nonmonotonicity complicates the quantitative evaluation of intonation sensitivity, but does not invalidate the concept.

Experiment on intonational sensitivity

In each session of a listening experiment, subjects listened to a series of 80 trials. Each trial consisted of two chords, each lasting 1 second and separated by 1 second of silence. A period of 5 seconds was allowed between trials for the subjects to write an opinion about the chords on an answer sheet. Each session lasted about 15 min.

The stimuli were generated by a computer, using the Music V sound synthesis program (Mathews, 1969). The samples of the sound waveform were read from a disk by an SEL 32 computer at a rate of 20 000 samples per second, converted to an analog signal with a 16 bit digital-to- analog converter, filtered with a Rockland 752A low-pass filter set at 8 kHz, and played over a loudspeaker.

Each chord contained three tones. Each tone consisted of ten partials whose amplitude decreased with frequency relative to the fundamental at 9 dB per octave. Phases of the partials were randomly selected to be 0 or 180 degrees, in order to achieve a tone with a reasonably low-peak factor. The tones commenced with a linear attack lasting 15 ms and ended with a linear decay lasting 18 ms. These parameters were selected to achieve a musical sound-quality. The timbre can be described as a typical bland electronic organ timbre. The sound pressure level in the test booth was not measured, but was set at a comfortable listening level estimated to be about 70 dB at the subjects' ears.

As discussed above, four different kinds of chords were studied: major chords (just frequency

TABLE 1. Frequencies in Hertz of the three tones in the stimulus chords.

	Deviation of f2 from just intonation, cents	f1	f2	f3
Major	-30	262	321.8	393
(4:5:6)	-15	262	324.7	393
	0	262	327.5	393
	+15	262	330.4	393
	+30	262	333.2	393
Minor	-30	262	309.0	393
(10:12:15)	-15	262	311.7	393
	0	262	314.4	393
	+15	262	317.1	393
	+30	262	319.9	393
3:5:7	-30	262	429.2	611.3
	-15	262	432.9	611.3
	0	262	436.7	611.3
	+15	262	440.5	611.3
	+30	262	444.3	611.3
5:7:9	-30	262	360.5	471.6
	-15	262	363.6	471.6
	0	262	366.8	471.6
	+15	262	370.0	471.6
	+30	262	373.2	471.6

ratios 4:5:6); minor chords (just frequency ratios 10:12:15); and two nontraditional chords (just frequency ratios 3:5:7 and 5:7:9). Five versions of each type of ed from their just ratios. The frequency of the lowest note of the chord was synthesized as 262 Hz (middle C) for all chords. Table I gives the actual frequencies used for the chords.

Subjects were presented with all possible pairings of the five different intonations within each of these four chords, thus resulting in 20 different trials. Trials for the different chord types were intermixed.

For each trial, the subjects indicated on their answer sheets which of the two chords sounded more in tune. In a later experiment, which we will not discuss here, they judged two other qualities of these chords: which chord was more smooth, and which was more pleasant. Results for these questions were essentially the same as for the intonation question.

For each chord, we derived a score based on the number of times a particular chord was selected as being more in tune. Each particular chord was paired with eight other chords within the same chord type (e.g., major). If a chord was always selected, it was given a score of 8; if it was never selected, it was given a score of 0, and so on. Thus we were able to compare perceptions of these chords across the four different chord types.

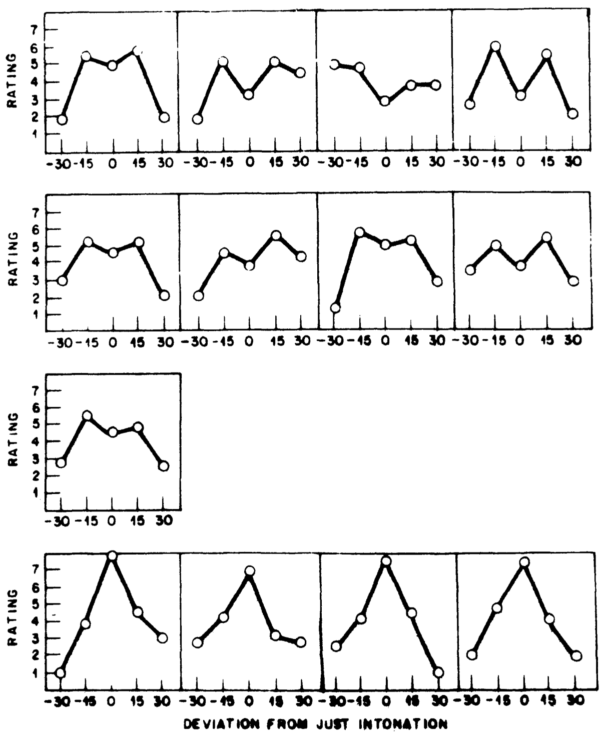


Fig. 1. Ratings, averaged across chord type, for each of the 13 subjects. Rating (based on the derived score) is plotted as a function of the deviation of the center note from just intonation (in cents).

Subjects were 13 volunteers from Bell Laboratories in Murray Hill,N.J. They varied greatly in terms of their musical expertise, ranging from professional violinist to musically untrained.

Figure 1 shows the results for each of the 13 subjects, averaged across chord type. The abscissa gives deviations from just intonation and the ordinate gives the derived scores, described above. It is important to note that all listeners were able to discriminate among the different intonations.

We were very surprised to observe that subjects showed two distinct patterns. Nine subjects (top three rows of Fig.1) had "M" patterns, in which chords deviating from just intonation by +15 and -15 cents are judged to be most in tune and chords with just intonation or deviating from just intonation by +30 and -30 cents are judged to be less in tune. For reasons to be discussed later, we describe these subjects as "rich" listeners. A second distinct pattern is apparent for the four subjects shown on the bottom of Fig. 1. It shows inverted "V" patterns, in which just chords were preferred to all others. We refer to this group as "pure" listeners. The grouping of the listeners does not correspond to any classification according to training that we have been able to discover.

As a result of this apparent grouping, we combined the data for each group. This is shown in Fig. 2, where results are averaged across chord and across subjects for each group. This figure shows very clear "inverted-V" and "M" functions, each of which is very symmetrical.

We were interested in seeing whether these patterns occurred for all four chord types. Figure 3 shows the results for the "rich" listeners. For the major, 5:7:9, and 3:5:7 chords, patterns are very

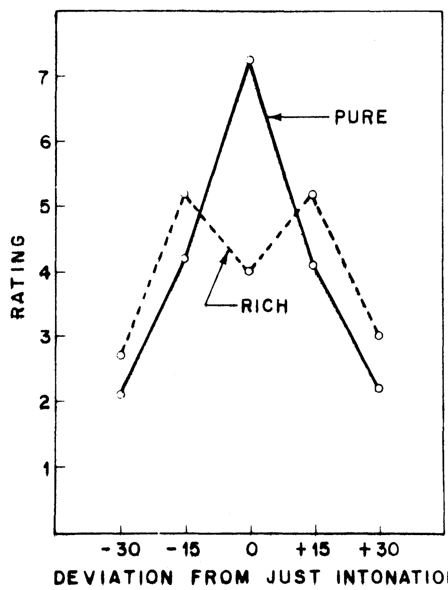


Fig. 2. Ratings for the two groups of listeners, averaged across chord type. Rating (based on the derived score) is plotted as function of deviation of the center note from just intonation (in cents).

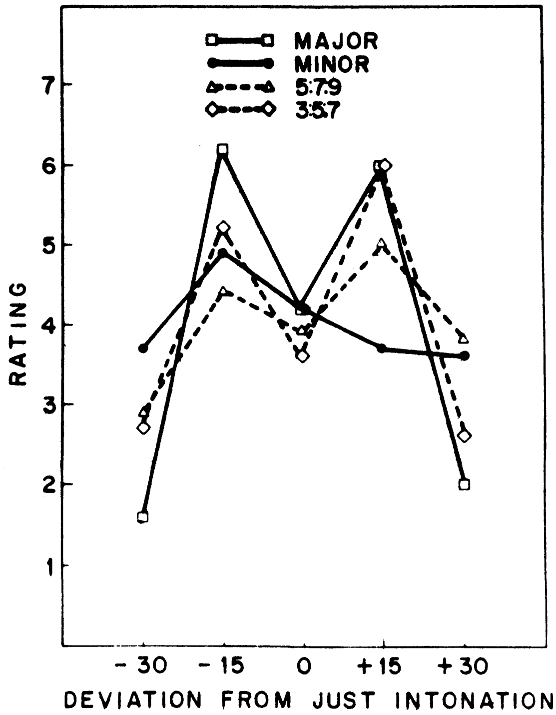


Fig. 3. Rating for the "rich" listeners for each of the four chords.

similar. The minor chord shows a different pattern, in which subjects preferred the chord in which the third of the chord deviated from just intonation by -15 cents.

Figure 4 shows results for the "pure" listeners. The major 5:7:9 and 3:5:7 chords again have very similar patterns, whereas the minor chord appears to behave somewhat differently.

From this experiment we conclude that:
(1) the nontraditional chords have a pattern of intonational sensitivity similar to that of major chords;
(2) that listeners fall into two groups: one of which, the "pure" group, prefers chords with just intonation and the other of which, the "rich" group, prefers chords which deviate enough from just intonation so a pleasant beating can be heard; and
(3) the minor chord is judged differently from the other chords.

Discussion of intonational sensitivity

Intonation sensitivity appears to be useful for studying harmonic structures such as chords. All subjects were able to discriminate chords that had only small intonation differences. Furthermore, their discrimination functions resulted in regular patterns for both traditional and nontraditional chords.

Our subjects divided into two classes according to the shape of their intonation preference functions. The "purists" prefer just intonation. Their preferences decrease monotonically as intonation deviates from just intonation. The "rich" listeners do not prefer just intonation, but rather prefer intonation

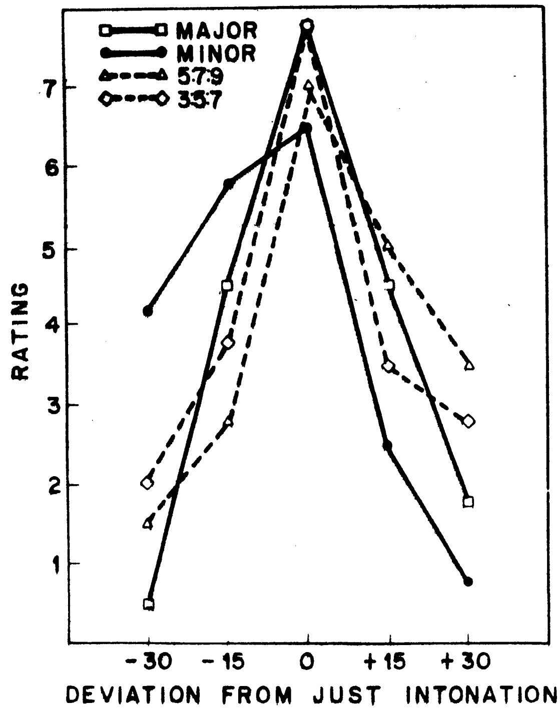


Fig. 4. Rating for the "pure" listeners for each of the four chords.

slightly different from just intonation. In most cases, their preference curves have an M-like shape. In some cases, particularly for the minor chord, the M is asymmetrical with a greater preference for the intonation deviation which is present in equally-tempered chords.

We have two possible explanations for these group differences. The first is that whereas "pure" listeners do not like beats, "rich" listeners do. Judgments for the latter group may be due to the perceptual similarity between slow beats and vibrato rates. The beat rate of the preferred stimuli (deviations from just intonation by +/- 15 cents)is similar to the average vibrato rate of 6.5 pulsations per second reported by Seashore.

Another explanation is that "rich" listeners like chords in equal temperament. For the equally-tempered major chord, the major third frequency is 14 cents higher than the just major third. "Rich" listeners tended to favor chords that deviated from just intonation by + 15 cents. This explanation is further supported by the finding that,for both groups, the minor chord was preferred when the third deviated from just intonation by -15 cents.

The nontraditional chords which we studied have clear and strong patterns of intonation sensitivity. These regularities suggest that these nontraditional chords are appropriate both for experimental studies of harmony and for new compositional systems. Our results strongly suggest that intonation for the two nontraditional chords (just frequency ratios of 3:5:7 and 5:7:9) is perceived in the same manner as that for the major chord (just frequency ratios of 4:5:6).

Finally, it is not too surprising that the minor chord is judged differently from the other chords. It has fewer overtones that can beat with each other. Its fundamental bass is at 1/10 the frequency of its lowest note, which is lower than any of the other chords. Consequently, we might expect results for the minor chord to be unique.

SCALES BASED ON THE NONTRADITIONAL CHORDS

Following the intonational sensitivity study, Mathews designed several scales based on the 3:5:7 and 5:7:9 chords using techniques similar to the way in which the just diatonic scale can be derived from the major (4:5:6) chord (Mathews & al., 1984). Although the harmony of the chords on which the scales were based clearly contrasted with the harmony of other random triads formed from scale notes, many important features in diatonic music could not be represented in these scales and, so far, no music has been composed with them. In particular, different keys and modulation are not included.

The Pierce scale

The Pierce scale was actually first discovered by H. Bohlen (1978), who published an article in Acustica, in which he described the scale almost exactly as we have done here. The work was not known to us at the time we did our research or prepared the draft for this article. However, now that we have read the Bohlen paper, we believe that it is clear that he proposed exactly the same scale including the tempered form of the scale using the 13th root of 3 as the tempered factor and including timbres which have only odd partials such as a square wave for playing this scale. Bohlen derived his scale from a theory of combination tones and, in particular, combination tones involving $3*f1 - f2$. We derived the scale based on experimental measurements of the intonational sensitivity of the 3:5:7 and 5:7:9 chords. The two sources for the scale, although different, are not in conflict, since Bohlen's derivation was primarily theoretical and ours was primarily experimental. We find it heartening that we arrived at the same result from a completely different direction. Obviously there is no question about who derived the scale first, since Bohlen published a decade sooner than we did.

After learning about intonational sensitivity and the 3:5:7 and the 5:7:9 chords, Pierce devised a set of scales based on both of these chords. More specifically he devised a 13 tone equal-tempered chromatic scale and a set of 13 different scales each of which has 9 tones that are subsets of the the chromatic scale. The chromatic Pierce scale is analogous to the normal chromatic scale and the 13 other scales are analogous to the 12 diatonic major scales that are the different keys of normal music.

The Pierce chromatic scale is based on a 3:5:7:9 tetrachord and consists of 13 steps, each having a frequency ratio of the 13th root of 3. The repetition interval in the Pierce scale is 1:3. This is analogous to the octave or 1:2 repetition interval in the diatonic scale. The 1:3 ratio is the ratio of the highest to the lowest tones of the tetrachord. We have chosen to call this 1:3 ratio, the tritave.

Appropriate powers of the 13th root of 3 give excellent approximations to the frequency ratios in the 3:5:7 chord. Table 2 computes the approximation of these powers to the chord in musical cents and for comparison does the same computation for the equal-tempered diatonic scale to its major triad, 4:5:6. The maximum error in the Pierce scale approximation is 6.6 cents which compares to a maximum error of 15.6 cents for the diatonic. The average absolute error in the Pierce scale is 4.4 cents while the average absolute error in the diatonic scale is 10.4 cents. The equal- tempered approximation of the Pierce scale to its chord is somewhat better than is this approximation for the diatonic scale.

The 13th root of 3 seems to be an appropriate choice to approximate a 3:5:7:9 chord. How was it discovered? The factor 3 came from the chord itself. The 13th root was chosen by trial and error from a series of integer roots. This root just happens to approximate the chord well and to produce an appropriate number of tones in the tritave.

The structure of the Pierce scale is diagramed in Figure 5. A circle, representing a 3:1 frequency ratio, is divided into 13 equal steps which are noted

Table 2. Comparison of the equal tempered intervals in the Pierce and the diatonic scales to integer ratios of the chords which form the basis of the scales.

Table 2 (a) Pierce Scale

Scale Step	Interval in cents	Integer Ratio	Integer Ratio in cents	Difference
N	ZP	CR	CRC	ZP - CRC
0	0	1:1	0	0
4	585.2	5:7	582.5	2.7
6	877.8	3:5	884.4	-6.6
10	1463.0	3:7	1466.9	-3.9
13	1902.0	1:3	1902.0	0

Table 2 (b) Diatonic Scale

Scale Step	Interval in cents	Integer Ratio	Integer Ratio in cents	Difference
N	ZD	CR	CRC	ZD - CRC
0	0	1:1	0	0
3	300	5:6	315.6	-15.6
4	400	4:5	386.3	13.7
7	700	4:6	702.0	-2.0
12	1200	1:2	1200	0

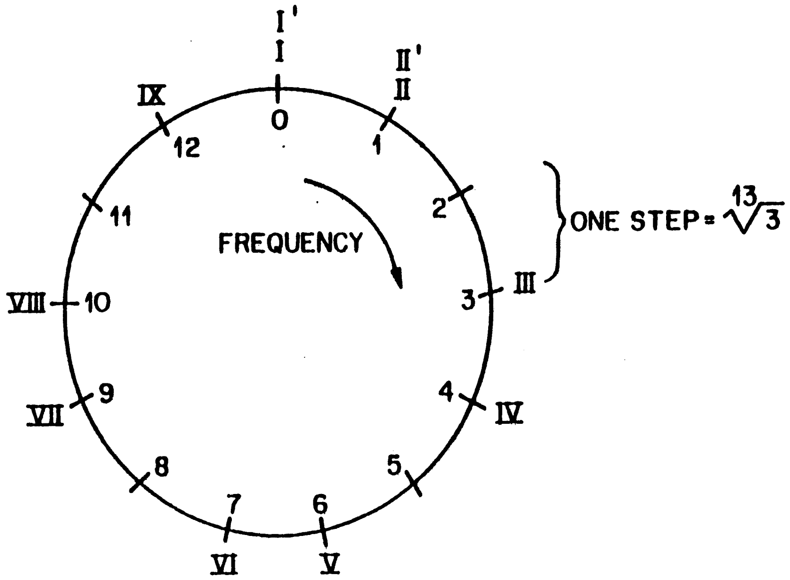
0,1,...,12. Also shown are the nine tones selected from the 13 to be the scale for a particular key. These are designated I,II,...,IX.

The selection of the nine tones is much more arbitrary than the selection of the 13 chromatic steps. However, their choices are important because they determine both the harmonic "structure" and the modulation properties of the scale.

With the particular choices we have made, the intervals in the scale are alternately one-step and two-steps throughout the scale except for the tonic (I) which is surrounded by two one-step intervals. The position of these two single steps identifies the tonic and hence the key. The thirteen possible keys correspond to the thirteen different positions of the tonic.

In the diatonic scale, the 2:1 octave is related to the 2:1 ratio between the frequencies of the fundamental and the first partial of most periodic tones. Many of the perceptible properties of the diatonic scale depend on this 2:1 ratio. The Pierce scale is intended to be played with timbres which have only odd partials--1st, 3rd, 5th, etc. Thus the ratio of the frequency of the first two nonzero partials is 3:1, the same as the 3:1 ratio of the tritave.

What kind of instruments could play the Pierce scale? Although one might design acoustic instruments with appropriate timbres and pitches, such tasks would require a great deal of work. On the other hand, computers and electronic synthesizers can easily be programmed for these purposes, and they are the obvious first choice to play this music.



MAJOR CHORDS				MINOR CHORDS		
I	V	VIII		I	IV	VIII
III	VII	I'		III	VI	I'
IV	VIII	II'		V	VIII	III'
V	IX	III'		VII	I'	V'
VI	I'	IV'		VIII	II'	VI'
VIII	III'	VI'		IX	III	VII'

Fig. 5. Diagram of Pierce scale and the "major" and "minor" chords falling within one key of the scale.

Triads

The Pierce scale has three special triads, the first being a major chord having a lower interval of 6 steps and an upper interval of 4 steps. This triad approximates the 3:5:7 chord. The minor triad reverses the intervals in the major chord and has a lower interval of 4 steps and an upper interval of 6 steps. The third triad, which approximates the 5:7:9 chord, is the first inversion of the 3:5:7 chord.

The positions of the major and minor triads which fall inside a given key are indicated on Figure 5. There are 6 major chords and 6 minor chords in a key. All scale steps are in both a major chord and a minor chord; hence any scale step can be harmonized by either a major or a minor chord, suitably inverted.

Modulation

The tonic can be positioned at any one of the thirteen steps which span the tritave, thus allowing thirteen possible keys. Different keys share various numbers of notes. In particular, each key can modulate to two adjacent keys by changing only one note.

Following the notation on Figure 5, moving the II note up by one step will cause the new tonic to rise to the III note of the original scale (3rd position of the chromatic scale). We will call this third position the dominant and the corresponding modulation a movement into the dominant key. Moving the IX tone down by one step will cause the tonic to fall to the VIII note of the original scale (10th position of the chromatic scale). We will call this VIII position the subdominant and call the corresponding modulation a change into the subdominant key. A major chord begins on both the dominant and subdominant notes of the scale.

Validation of the Pierce scale

After creating a new scale, how can one quickly find out what it is good for? Are there listening tests and laboratory studies that can precede the long slow process of trying to compose significant music with the the new scale? Can the laboratory results aid the composer in coming to terms with the new medium?

Attempting to answer questions like these has led us into new and uncharted waters where much remains to be learned. We will describe the results of two sorts of test here. The first: consonance judgments of all possible triads, seem to us to be musically interesting and encouraging. The second: similarity judgments of chords and their inversions, show some of the difficulties which appear to arise from long-term learning effects. These two kinds of studies are obviously far from sufficient to understand new musical materials. Finding other good paradigms is one of the most interesting questions we presently face.

Consonance judgments of 78 triads

One of the simplest and best defined properties of chords is consonance and dissonance. Consequently, we collected consonance ratings for chords in the Pierce scale from musically trained and untrained listeners. In addition to chords that lie within one key, one can play chromatic chords which have arbitrary combinations of the 13 tones in the tritave. Exactly 78 triads can be formed which span no more than one tritave.

Twelve musicians and twelve untrained listeners participated in the tests. Musicians were graduate and undergraduate music students at the Juilliard School in New York; untrained subjects were selected from the subject pool at AT&T Bell Laboratories in Murray Hill, N.J. All of the musicians had at least 10 years of private instruction on an instrument, and subjects in the untrained group had little or no formal musical training.

Subjects listened to a triad and rated its consonance on a 7-point scale, where 7 was designated as very consonant and 1 was very dissonant. All subjects listened to three repetitions of the 78 triads. There were three blocks of trials. Each block had different random orderings of the 78 triads. A triad was heard for a duration of 1 second and subjects had 5 seconds in which to record their responses on the provided answer sheets.

The triads were generated on a computer, using the CMUSIC sound synthesis program. Each tone of a triad consisted of odd-numbered partials (1,3,5,7,and 9) with amplitudes respectively of 1, -.35, -.19, .125, and -.089. The negative amplitudes corresponding to 180 degree phase shifts were chosen to reduce the peak factor of the waveform. The tones began with a linear attack lasting 15 msec. and ended with a linear decay of 18 msec. The root of each triad was always 175 Hz.

The principal results from the study are as follows:

a) A wide range of perceived consonance and dissonance is observed between the most consonant and the most dissonant chords. Thus, consonance can be a major property of a chord.

b) The strongest factor explaining the dissonance of a chord is the presence of one scale-step interval(s) in the chord.

c) Major and minor chords are relatively consonant compared to the average chords.

d) A "critical band dissonance" model gives a good fit to much of the data.

Table 3 gives the mean consonant ratings for the 8 most consonant and the 8 most dissonant chords for each group of listeners. Each chord is indicated by the position of its tones along the Pierce

Table 3. Mean consonance ratings for the 8 most consonant and the 8 most dissonant chords for the two groups of listeners.

Most Consonant				Most Dissonant			
Musicians		Untrained		Musicians		Untrained	
Chord	Mean	Chord	Mean	Chord	Mean	Chord	Mean
0,11,13	5.31	0,7,10	4.97	0,1,2	1.61	0,1,2	2.47
0,2,5	5.25	0,2,11	4.94	0,11,12	1.67	0,1,3	2.64
0,5,11	5.22	0,7,13	4.89	0,12,13	1.89	0,12,13	2.75
0,6,8	5.17	0,6,13	4.83	0,9,10	1.89	0,2,3	2.81
0,2,8	5.14	0,8,11	4.81	0,10,11	1.94	0,9,10	3.06
0,3,5	5.03	0,4,7	4.78	0,8,9	2.00	0,4,5	3.06
0,3,13	5.11	0,7,9	4.78	0,1,9	2.06	0,8,9	3.11
0,7,11	5.08	0,6,10	4.75	0,1,13	2.22	0,11,12	3.11

chromatic scale. For example, the 0,1,2, chord comprises the first 3 three tones of the scale. For both groups, mean judgments encompassed a fairly wide range, which indicates that listeners were able to differentiate these chords. For the musicians, mean judgments ranged from 1.61 to 5.31 and for the untrained group, judgments ranged from 2.47 to 4.97. It is not surprising that musicians had a wider range of scores; their training may have resulted in more consistent ratings of these chords.

All of the 8 most dissonant chords have intervals of one scale-step. The most dissonant chord, as judged by both groups, has two one-step intervals, and there is considerable agreement between the two groups on what are the most dissonant chords.

For the most consonant chords, agreement between groups is not as good and neither the major (0,6,10) chord and nor the minor (0,4,10) chord is rated as outstandingly consonant. However, as shown in table 4, these two chords and the first inversion of the major chord are ranked as relatively consonant among all possible chords. Although we would have been happy to see greater uniqueness for the major and minor chords, we feel that their consonance is musically useful. Also, we hope that learning effects will enhance their distinctiveness.

Table 4 compares the consonance ratings of the special chords with that of adjacent chords in the chromatic scale. These tables do not show maxima corresponding to the peaks of intonational sensitivity which these chords exhibited (see Fig 4). We feel that this lack may be explained by the different experimental methods used in the two studies. In particular, the paired comparisons made in the intonational sensitivity studies are more sensitive than the consonance judgments of individual chords made here. However, it was necessary to use a faster method to judge 78 chords; paired comparisons of this many stimuli would have been impossibly long.

Model to explain consonance judgments

In an attempt to explain the dissonance judgments more completely, we have proposed a model similar to that of Plomp and Levelt (1965). In this model, the dissonance of two sinusoids is maximum if they are separated by a particular number of cycles per second, usually thought to be about 20, and the dissonance diminishes if the separation is either less than or greater than the maximum. Specifically, our model proposes the dissonance measure as

$$D = F(|f1-f2|) + F(|f2-f3|) + F(|f1-f3|) \qquad (1)$$

where f1, f2, and f3 are the fundamental frequencies of the three tones in the chord and F is the function shown in Figure 6. In Figure 6, q is the separation

Table 4. Mean consonance ratings for both groups of listeners for the "Major" and "Minor" chords and for the chords that are in the region of these chords

Chord	Mean for Musicians	Mean for Untrained
0,5,11	5.22	4.69
0,5,9	2.97	4.03
0,6,9	3.42	4.12
0,5,10	3.44	4.36
0,6,10 (major)	4.39	4.75
0,7,10	3.58	4.94
0,6,11	4.11	4.25
0,7,9	4.22	4.78
0,7,11	5.08	4.53
0,3,8	3.61	4.53
0,3,6	3.81	4.41
0,4,6	4.67	4.08
0,3,7	4.69	3.92
0,4,7 (major inv)	4.58	4.78
0,5,7	4.81	4.56
0,4,8	3.28	4.33
0,5,6	2.83	3.22
0,5,8	3.42	4.47
0,3,11	4.75	4.69
0,3,9	4.00	4.31
0,4,9	2.81	3.92
0,3,10	3.50	4.53
0,4,10 (minor)	4.08	4.67
0,5,10	3.44	4.36
0,4,11	3.56	4.72
0,5,9	2.97	4.03
0,5,11	5.22	4.69

frequency for the maximum dissonance, s is the maximum separation for which components produce dissonance. The model makes the following assumptions: 1) Dissonance of separate pairs of components is additive (Eq. 1); 2) Only interactions between the fundamentals in the tones are significant (since the tones had only odd harmonics, none of the audible overtones are less than a critical bandwidth apart); 3) The function F is piecewise linear as shown.

Figures 7 and 8 show the correlation between the Eq 1 model and the subjects' dissonance ratings for various values of q and s, for trained and untrained subjects respectively. The magnitudes of peak correlations, .89 for the untrained subjects and .75 for the trained subjects, are surprisingly high. The lower correlation for the musicians may be attributed to their expertise with diatonic chords which may confuse their judgments of the Pierce scale chords.

The high correlations give us confidence in the

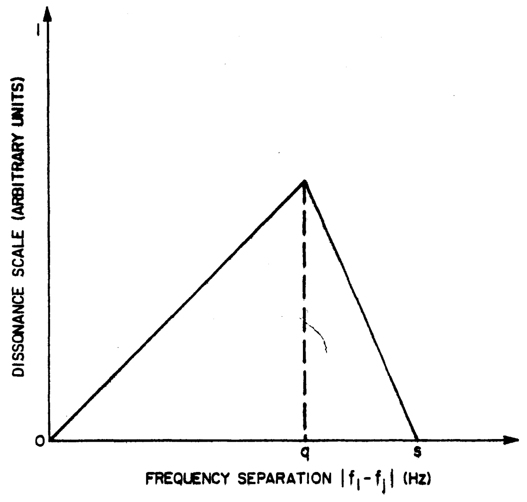


Fig. 6. Hypothesized function with two parameters, q and s, which relates dissonance to the frequency separation of two components, f_i and f_j.

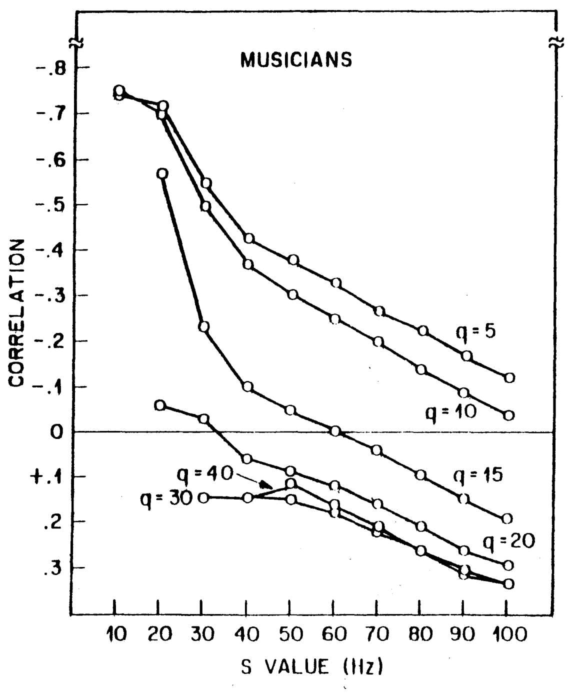


Fig. 7. Correlation between the consonance ratings of 78 chords by musicians and the dissonance model in Eq. (1) as a function of the model parameters q and s.

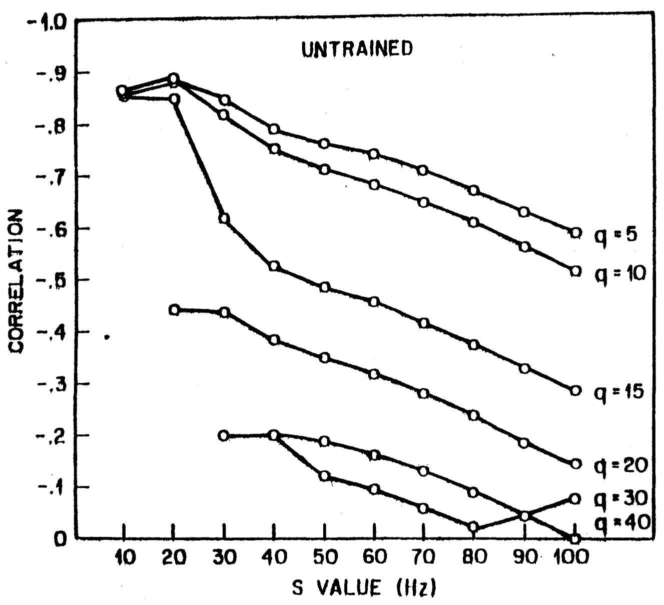


Fig. 8. Correlation between the consonance ratings of 78 chords by untrained subjects and the dissonance model in Eq. (1) as a function of the model parameters q and s.

usefulness of the model for predicting subjects' reactions to the various chords. However, some peculiarities in the results diminish the attractiveness of the model. Maximum correlations occur for q = 10, s = 10 for the trained subjects and for q = 10,s = 20 for the untrained subjects. For these small values of q and s, only a few one scale-step intervals make a nonzero contribution to the sum in Eq 1. It seems to us that intervals other than one scale-step intervals must also be important but we have not yet appropriately modeled their contributions.

Similarity of chords and their inversions

In order to harmonize any note of the Pierce scale with a major or a minor chord, it may be necessary to invert the chord. The use of inverted chords is only justified if listeners perceive a chord and its inversion as similar. There is no guarantee that this similarity holds for Pierce scale chords, which are inverted around the tritave rather than the octave. Consequently, we decided to study these similarity perceptions. For comparison we have collected similar data for traditional major and minor chords. Again, we used both trained and untrained listeners as subjects. Ten musicians and ten untrained listeners listened to pairs of triads and judged their similarity on a 9-point rating scale, where 9 was designated as very dissimilar and 1 was very similar.

All subjects listened to major and minor triads from the Pierce scale as well as to traditional major and minor triads. For both types of stimuli, triads were in root position, first inversion and second inversion. In addition, for both the Pierce and the traditional chords, three different root notes were chosen. Table 5 shows the specific stimuli that were

Table 5. The stimuli used for the similarity experiment.

I. Pierce Chord Set

Chord	Notes of the Chord	Chord	Notes of the Chord
I Maj	0,6,10	I min	0,4,10
I Maj 1	6,10,13	I min 1	4,10,13
I Maj 2	10,13,19	I min 2	10,13,17
VIII Maj	-3,3,7	VIII min	-3,1,7
VIII Maj	3,7,10	VIII min 1	1,7,10
VIII Maj 2	7,10,16	VIII min 2	7,10,14
VIII# Maj	-2,4,8	VIII# min	-2,2,8
VIII# Maj 1	4,8,11	VIII# min 1	2,8,11
VIII# Maj 2	8,11,17	VIII# min 2	8,11,15

Note: 0 = 262 Hz, 1 = first inversion, 2 = second inversion. The roots of the chords are depicted in terms of the roman numerals in the outside of the circle shown in Figure 1; the notes of the chords are indicated by their position in the inner circle shown in Figure 1.

II. Traditional Chord Set

Chord	Notes of the Chord	Chord	Notes of the Chord
C Maj	C,E,G	C min	C,Eb,G
C Maj 1	E,G,C'	C min 1	Eb,G,C'
C Maj 2	G,C',E'	C min 2	G,C',Eb
G Maj	G-1,B-1,D	G min	G-1,Bb-1,D
G Maj 1	B-1,D,G	G min 1	Bb-1,D,G
G Maj 2	D,G,B	G min 2	D,G,Bb
A Maj	A-1,C#,E	A min	A-1,C,E
A Maj 1	C#,E,A	A min 1	C,E,A
A Maj 2	E,A,C#	A min 2	E,A,C'

Note: C = 262 Hz, 1 = first inversion, 2 = second inversion.

used. Given the set of 18 triads for each stimulus type, there were 153 pairs of chords for both the Pierce and the traditional chord set. Stimulus pairs were randomly ordered and they were presented in four blocks of 40 trials per block. Half of the subjects rated the Pierce triads first and half rated the traditional triads first.

Hierarchical clustering analyses (Becker & Chambers, 1984) were carried out on two stimulus sets for both groups of listeners. The clustering

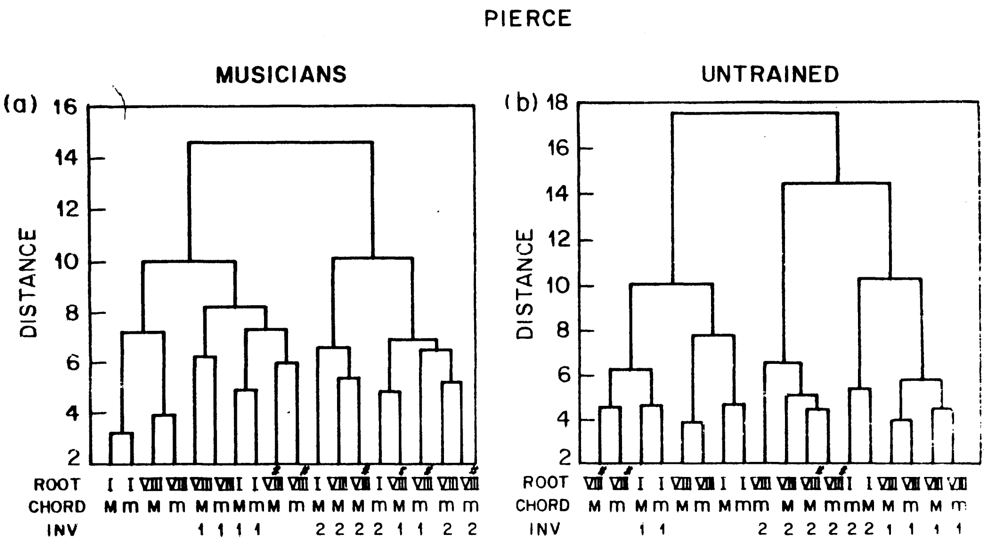


Fig. 9. Similarity judgements of Pierce chords and their inversions by musically trained (9a) and untrained (9b) subjects.

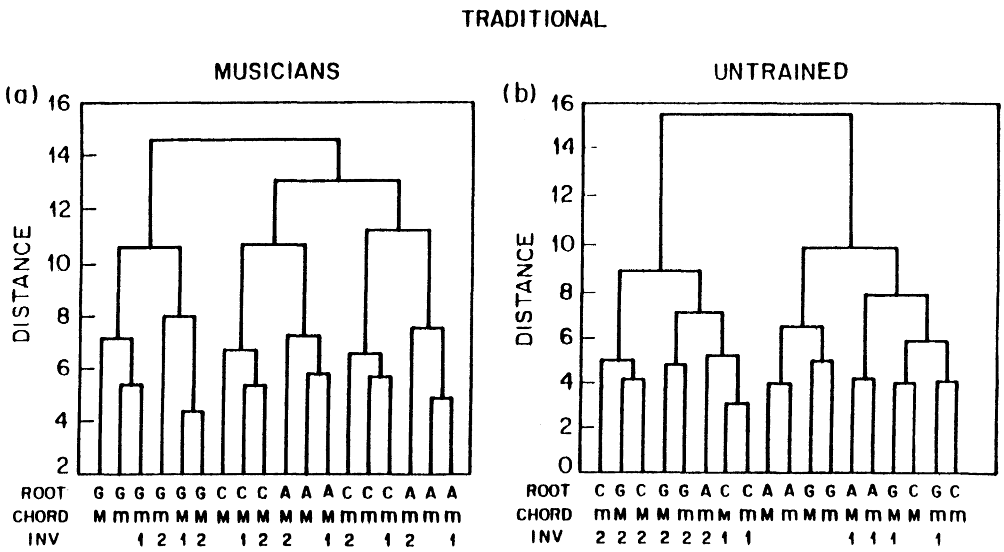


Fig. 10. Similarity judgements of diatonic chords and their inversions by musically trained (10a) and untrained (10b) subjects.

solutions are given in Figures 9 (results for the Pierce chord set) and 10 (results for the traditional chord set).

For both groups of listeners, the clustering solution for the Pierce chord set depends primarily on pitch height or average pitch of the chord. There is little indication that similarity judgments were dependent on chord type, inversion, or key relationships.

Results for the traditional chord set are also dependent on pitch height for the untrained group, as shown in Figure 10b. There is a configuration of the lower pitched chords in the right-hand side and a configuration of higher pitched chords in the left-hand side. Within these larger clusters, there are five pairs of clusters in which the bass notes are identical.

Results differ markedly for the traditional chord set for the musicians. The strongest result is that chords were clustered by inversion. Root positions, first inversions, and second inversions are clustered together for the A major, A minor, C major and C minor chords. In the case of the G chords, the G minor root-position chord is clustered with its first inversion and the first and second inversions of the G major chord are clustered together.

Similar findings for traditional chords were reported in previous research (Roberts & Shaw, 1984). In that study, highly-trained and less-trained listeners judged the similarity of root positions and first inversions of major, minor, diminished, and augmented chords. As was observed in the present solution for the musicians, the clustering solution for the highly-trained group was dependent on chord type and inversion, but results for the less-trained group appeared to be based on pitch height only.

Together, these results suggest that, when listeners are not taught what to listen for, they abstract information mainly about pitch height. But, as a result of a great deal of training and experience with particular scale systems, they are able to abstract information about interval structure, key relationships, the relationship between chords and their inversions, etc. Therefore, it is not surprising that in the present study, highly-trained musicians judged the Pierce chord set in the same way as the untrained group. And we may hope that with sufficient experience and training that people could learn to perceive similarities between root position and inverted Pierce chords, as they do for diatonic chords.

Conclusions from the Pierce study

We have explored a new set of scales based on the frequency ratios 3:5:7:9, the ratio 3:9 = 1:3 being the tritave, which replaces the octave of the diatonic scale. Although one can construct just versions of these scales, we have been mostly concerned with 9 tone tempered scales which are subsets of a 13 tone chromatic scale having steps whose frequency ratios are the 13th root of 3. The tempered scale allows 13 keys, reached successively by moving one note of a scale by one step.

In the chromatic scale, exactly 78 triads can be formed which span no more than one tritave. These were rated by both untrained subjects and by music students at Juilliard. A wide range of perceived consonance and dissonance was observed. The strongest factor explaining dissonance is the presence of one scale-step interval(s). A "critical band dissonance" model fits the data well. Major and minor chords are relatively consonant.

Trained and untrained subjects rated the similarities among chords and their two inversions on three roots, both for the diatonic scale and the new scale. Trained subjects rated the inversions of chords as similar to the root position chord in the diatonic scale but not in the new scale; untrained subjects did not find the inversions similar to the chord in root position in either scale. This seems to indicate that the similarity of inversions might be learned in the new scale.

STRETCHED DIATONIC SCALE

The last study discussed in this paper deals with a stretched diatonic scale in which both the intervals between the notes of the scale and the intervals between the partials of the timbres that are used to play the scale have been stretched or increased by the same factor. In this way the relative sizes of the intervals between partials of the various notes in a chord are the same in the stretched and the unstretched material. In particular, partials that are coincident in frequency in a diatonic chord will also be coincident in the stretched version of that chord. Thus harmonic effects that depend on coincident partials should be preserved in the stretched music.

Originally we had hoped that many harmonic properties of the diatonic scale would be present in the stretched scale and therefore that stretching would provide a way to generate new scales with rich harmonic properties. Our expectations so far have been fulfilled only to a limited extent.

Theories of harmony

In a book first published in 1863, Helmholtz (1954) proposed that dissonance arises from unpleasant beats between partials whose frequencies are too close together; for example, partials separated by 10 - 50 Hz. The octave is the most consonant of intervals because all of the harmonic partials of the upper tone coincide in frequency with partials of the lower tone. The fifth is consonant because the frequencies of the fundamentals (first partials) of the two tones have a simple ratio, 3/2, and because of this the lower partials of the two tones either coincide, or they are considerably separated in frequency and do not beat together objectionably.

Helmholtz's work has been added to by that of others. Particularly, Plomp and Levelt (1965) found in 1965 that two (or more) partials that lie within what is called a critical bandwidth produce an unpleasant sensation (unless they differ very, very little in frequency). For frequencies above, say, 500 Hz, a critical bandwidth is about 1/4 octave (a minor third). However, Plomp and Levelt were careful to call the consonance of tones whose partials are separated by more than a critical band a "tonal consonance," and not to imply that this consonance is all there is to musical harmony.

Rameau (1971) had another view of harmony. He observed that in a major triad all frequencies present are integer multiples of a basse fundamentale or fundamental bass which, in the root position of the chord (C,E,G) lies two octaves below the root of the chord (C). Thus, if the frequency of the fundamental bass is F, the frequency of the root (C) is 4F, the frequency of the third (E) is 5F, and the frequency of the fifth (G) is 6F. Because Rameau regarded the octave as essentially an equality, he could identify the fundamental bass of the chord with its root.

Neither Rameau nor Helmholtz knew of the phenomenon of residue pitch or periodicity pitch, which Schouten (1938) described. When we are presented with harmonic partials in the absence of the fundamental, we perceive the pitch as the least common denominator of the frequencies present, that is, as the frequency of the missing fundamental. Thus, when we listen to a major triad we might well hear Rameau's fundamental bass. Terhardt has discussed this (1974) and has demonstrated the effect (1977).

The experiments we shall describe are relevant to the two views of harmony described above. But, one might hold that musical harmony is merely a matter of brainwashing; that we accept combinations of tones we have been taught are correct, and reject those that we have been taught are incorrect. We have some experimental evidence that bears on this.

Tones used and intent of experiments

In our experiments, we have used tones with stretched partials, as Slaymaker (1970) did. The frequencies f(i,j) of the partials of such a tone are given by

$$f(i,j) = A^{(i/12 + \log2 j)} \qquad (2)$$

Here i is the scale step (i = 12 for an octave), j is the number of the partial and A is the frequency ratio of the pseudo octave. For a true octave, A = 2 and

$$f(i,j) = j\, 2^{i/12} \qquad (3)$$

we see that f(i,j) gives the frequencies of harmonics of the notes of an equally-tempered scale.

Within the accuracy of the equally-tempered scale, major triads made up of notes of the true octave scale satisfy the criterion of Rameau; the frequencies of partials are all integer multiples of a fundamental bass. They also satisfy the criterion of Helmholtz. All lower partials either coincide or are well separated. If all the frequencies present in the triad are stretched according to Eq.2, all the lower partials still coincide or are well separated. So, Helmholtz's criterion will be satisfied in the stretched triad. But, in the stretched triad the partial frequencies are no longer multiples of a fundamental bass frequency. There will no longer be a basis for a periodicity pitch. Rameau's criterion will not be satisfied.

To summarize our conjectures about stretched tones:
1) Harmonic effects which exist in stretched materials are produced by interactions of stretched partials.
2) Harmonic effects which disappear in stretched materials are produced either by periodicity pitch or by brainwashing.
3) It is hard to separate brainwashing effects from other effects.

Besides the tones mentioned above, one experiment was made with tones in which partials are separated by a fixed fraction of an octave, as described by Pierce (1966) and used by him in an eight-tone canon (phonogram).

The experiments

Our experiments with stretched partials involved:
1) The generations of stretched and unstretched materials.
2) Test subjects' ability to identify the key of stretched and unstretched material.
3) Test subjects' perception of "finality" of cadences for stretched and unstretched materials.

Acoustical description of the materials

All the sounds used in the experiments were synthesized on the PDP-10 computer at the Institute for Research and Coordination of Acoustics and Music (IRCAM), Paris, France, by use of the Music V program. Each sound had seven partials whose frequencies were specified by Eq 2. For A = 2, these were in the approximate frequency ratios 1, 2, 3, 4, 5, 6, 8 (note the 7th partial is omitted). The exact partial frequencies as specified by Eq 2 are equal-tempered. That is, the partial frequencies coincide exactly with the fundamental of some note in the equal-tempered scale.

The amplitudes of the partials relative to the fundamental diminished at 9 dB per factor of 2 in frequency ratio between the partial and the fundamental. The value 9 dB per factor of 2 was selected to approximate normal musical instruments which tend to produce a spectrum which decreases faster than 6 dB per octave, but not as fast as 12 dB per octave.

The envelope of the notes was chosen to give a sustained sound with a moderately fast, but not percussive, attack, a slight diminuation (6 dB) over the duration of the note, and a smooth decay. The attack and decay times were each 10% of the duration of the note. Typical note durations ranged from 0.5 - 2s.

The overall timbre can be described as a pleasant, bland, musical sound.

Key-sensing experiments

Judging the tonality of a short passage was chosen as an experimental test. Previous experiments (Corso, 1957) show that keys can be successfully identified but with difficulty.

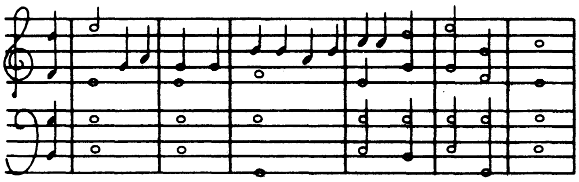
Three short passages designated X, M, and T were synthesized. The scores of X and M are shown in Fig 11. Both X and M are long enough to clearly establish a key. Both end in a clear cadence. T consists of M transposed to a different key. X was synthesized either in the key of M or T.

The key of T differed from that of M by either being a minor third away from M, or being a minor second from M, or simply being the minor of the key of M.

A test was prepared containing 24 test sequences. Each test sequence consists of the sequence XMXT. This sequence was selected over the more traditional MXT test in order to make the test slightly harder. In the MXT order, the cadence at the end of M is very close to the cadence at the end of X because X is a very short passage.

After listening to a test, the subject was asked whether X was in the key of M or T. The test was taken by ten subjects. Five subjects (the nonmusicians) had almost no experience as performers and described themselves as having little interest in music. The other five (the musicians) had extensive experience as amateur or semi- professional musicians.

The results of the experiment are shown in Table 6. We draw the following conclusions:
1) Everyone, musicians and nonmusicians alike, performed at better than chance (50%) for both stretched and unstretched materials. Thus stretching



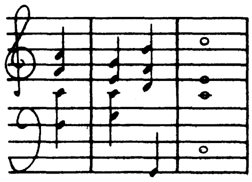


Fig. 11. Scores of passages used for key-sensing tests.

Table 6. Percentage of correct sensing of key in XMXT tests, by use of musicians and nonmusicians as subjects. 50% is chance performance.

	Major-minor	Minor 2nd transposition	Minor 3rd transposition
Musicians			
Normal	100	100	100
Streched	80	100	95
Nonmusicians			
Normal	80	85	95
Streched	60	75	85
Everyone			
Normal	90	92	97
Streched	70	87	90

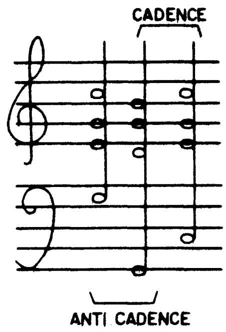


Fig. 12. Cadence (dominant to tonic) and anticadence (tonic to dominant).

did not completely destroy the ability to sense key in this test.
2) In general the larger the key change, the better was the performance.
3) Normal materials were easier to judge than stretched materials.
4) Musicians performed better than nonmusicians.

In general, this experiment supports Helmholtz more than Rameau, because the stretching removed periodicity pitch and the fundamental bass, but left a substantial key sensing ability.

Cadence studies

In the cadence studies we ask subjects to rate the feeling of finality imparted by a two-note chord sequence. Two sequences were used which we call a cadence and an anticadence as shown on Fig 12. The cadence is a normal dominant to tonic cadence. The anticadence starts with the tonic and ends with the dominant. Both stretched (A = 2.4) and unstretched forms of the cadence and the anticadence were synthesized. The use of chord pairs as cadences implies that the subjects already know the tonic chord. The same key was used throughout the tests and the results indicate that subjects clearly knew the tonic.

Thirteen "musicians" and seventeen "non-musicians" rated the finality of the four conditions. There were no apparent patterns of differences between the responses of the musicians and the non-musicians, so the results of these two groups were averaged together.

Results are shown in Fig 13. It is clear that the unstretched cadence gave a strong sense of finality and the stretched cadence did not. The normal anti-cadence and the stretched anticadence were equally nonfinal. Thus, stretching seems to destroy the impression of finality in cadences. This supports Rameau rather than Helmholtz.

Cadence with equally-spaced partials

In an experiment not described here involving a dominant seventh chord we noted that, even in this strongly dissonant chord, relatively few partials lie close together. Perhaps Helmholtz' effects are weak simply because too few partials interact closely. To study this hypothesis we generated a chord pair as shown in Fig 14 using notes with partials equally spaced along a musical scale. The partials are separated by a major fourth in this specific example. As a result of this spacing the first chord of the pair is very dissonant, almost all partials being one half-

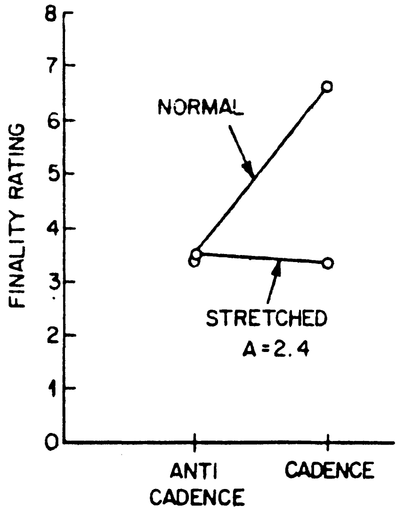


Fig. 13. Finality ratings of stretched and normal cadences and anticadences. With normal unstretched partials, the cadence is judged as having a great deal of finality and the anticadence as having little finality. With stretched partials, the finalities are essentially equal.

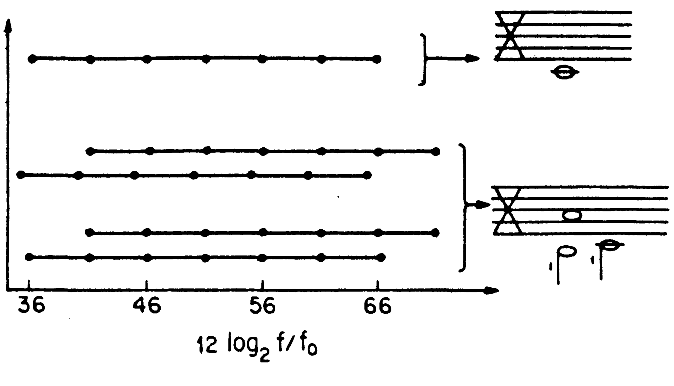


Fig. 14. A pseudo-cadence with uniformly spaced partials, going from a tonally dissonant to a tonally consonant chord. The clef sign X was used to indicate that the staff is not normal musical notation. The plot of the partials is the basic description of the sound.

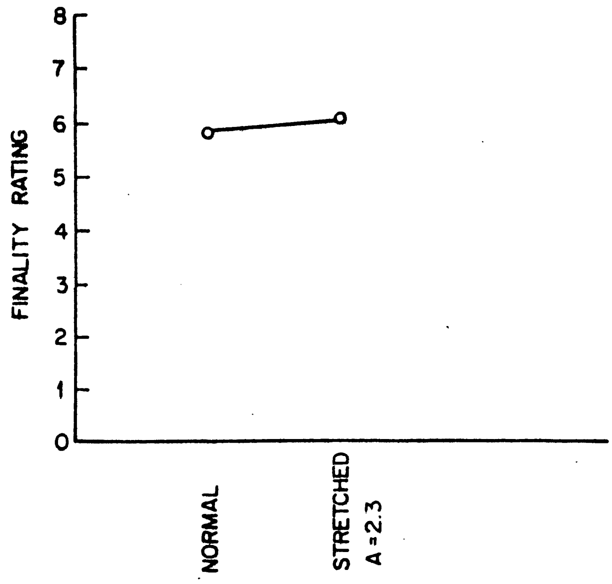


Fig. 15. Judgements of finality of the cadence from Fig. 14. It is judged as final with or without stretching.

step apart and the second chord is very consonant, almost all partials being coincident.

Figure 15 shows that this chord pair is given a relatively high finality rating. Furthermore, this rating, as we would expect, is not diminished by stretching (A = 2.3) the material. Thus, if one makes strong enough interactions between partials an impression of finality can be conveyed by a transition to an acoustically consonant situation. This would argue for Helmholtz' tonal consonance view of harmony.

Supplementary observations

Before concluding our discussion of stretched scales we will mention some informal observations. It is our observation that melodies are easily recognized despite stretching. A stretched (A=2.4) version of the round, "Are You Sleeping, Brother John," was instantly recognized by an audience. The melody of harmonized stretched (A=2.4) versions of "The Coventry Carol" and "Old Hundred" hymns were recognized by an audience, but these had also just been played unstretched.

In the harmonized stretched "Old Hundred" it seemed difficult to distinguish the inner voices. In fact, in single tones stretched (A=2.4), one tended to hear the partials as separated sounds rather than as fused into a tone of a single pitch. We believe that such fusion depends in part on the phenomenon of residue or periodicity pitch (Schouten, 1938). This has been noted by Cohen (1970). She has observed further that the degree of fusion of a stretched tone depends on the envelope of the tone, and is greatest (Cohen, 1980) for an exponentially decreasing amplitude which gives a "struck" quality.

We observe that stretched tones sounded singly tend to fall apart into a group of partials, but when a sequence of such tones is played as a known melody the tones are heard as the individual notes of the melody.

It appears that whether or not a collection of stretched partials is heard as a single tone can depend on both the time evolution of the partial amplitudes and on the context (melodic or otherwise) in which the tones are heard.

Conclusions from the stretched scale experiment

One purpose of the experiments performed was to try to decide among three explanations of harmonic effects: (1) Rameau's fundamental bass, which can be related to Schouten's residue pitch or periodicity pitch; (2) the tonal consonance of Helmholtz and Plomp, which depends on the spacings of the partials that are present; (3) brainwashing, that is, learned expectations and reactions. The experiments did not distinguish clearly among these views.

We found:

1) Subjects can identify keys of both stretched and unstretched materials in an XMXT test.

2) Stretching destroys the perception of finality of cadences.

Results (1) above may be a melodic rather than a harmonic effect, and, as we have noted, melody seems more robust under stretching than harmony does.

Result (2) argues for either Rameau or brainwashing.

Nonetheless, by using tones with equally spaced partials, we can get a sense of finality by going from a tonally dissonant chord, to a tonally consonant chord. This argues for Helmholtz-Plomp.

The acoustic nucleus

Where do the diverse results we have discussed leave us with regard to the question of new music materials? Is it now possible to manufacture new scales and harmonies by systematic techniques or is the diatonic scale unique? We would like to end by proposing an unproved, but interesting hypothesis, that of the acoustic nucleus.

So far our results are compatible with two conditions. First, present psychoacoustic techniques are effective in studying the peripheral hearing process and many important low level perceptions such as the acoustic dissonance of sounds with arbitrary patterns of partials can be understood and taken advantage of by suitable listening tests. Second, many musically important perceptions are higher level mental processes, which are much harder to study in the laboratory. In particular effects which depend on long term learning of new materials are especially intractable. Similarity of Pierce chords and their inversions is an example. With traditional diatonic materials, effects that depend on long term learning seem more powerful than low level acoustic effects.

The acoustic nucleus hypothesis maintains that with new materials, it is necessary to have an acoustic nucleus on which to grow powerful musical connotations via long term learning. The acoustic nucleus consists of sound qualities that are perceivable at a low peripheral level, such as the relative dissonance of various Pierce chords.

We believe that our present results show that we can usefully deal with the acoustic nucleus in the laboratory. What we have not shown is how to predict or control the subsequent learning process that leads from sounds to music. This is an important direction for future research.

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MUSICAL JUDGEMENTS OF BACH CHORALE EXCERPTS

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Abstract

This paper reviews some selected experiments on sensitivity to modulation, or key change, within short musical passages excerpted from Bach chorales. Following presentation of each excerpt, musically trained listeners were asked to identify the distance and the direction of key change with reference to the circle of fifths. Presentations consisted of either the full four voices of the excerpt (harmonic sequences) or one of the individual voices (single-voice sequences).

Overall accuracy of identifying key change was not significantly different for the two types of presentation. However, judgements of key change in harmonic sequences were affected by three musical factors whose influence was not predictable from judgements of key change in individual voices. These factors were: 1. construction of the final chord of sequences; 2. the harmonic progression, and; 3. the direction of modulation around the circle of fifths. The influence of the latter factor in harmonic sequences was supported in a subsequent experiment in which listeners untrained in formal music theory rated the perceived extent of key change. The data are addressed to questions and issues surrounding the hierarchical representation of musical pitch organization.

Introduction

Listening to tonal music includes attending to individual voices (e.g., a melody), a progression of chords, and an overall key structure. It has been suggested, both in theory and in research, that these aspects of music are not independent: a melody is heard and understood in terms of an underlying chord progression, and chords, in turn, are understood in terms of the prevailing key. Thus, the perception of tonal music may involve a hierarchical organization of musical elements, with individual voices, chords, and keys represented at progressively ascending levels of a cognitive organization of musical pitch structure.

A typical music-theoretic account of the hierarchy of levels describes individual tones according to the chord in which they belong; if they belong to no chord, they are considered "ornamental" or "unstable". Each chord is then described with reference to the overall key (Piston, 1978). Schenker's approach to musical analysis relies heavily on the notion that each level of musical abstraction is determined by removing the less stable elements from the previous level (Schenker, 1906/1954, 1935/1979). However, as pointed out by Deutsch and Feroe (1981), there has been considerable disagreement among theorists about the relation between melodic and harmonic structures. In a recent discussion of theoretical problems, Clarke (1986) comments that, "If we consider pitch structures at a number of a different levels, we are forced to recognize that notes, chords, keys and tonal areas are all entities of different *types*, with different properties, and more importantly different principles governing the way they may be structured together" (p. 11). Krumhansl (1983), while acknowledging the unique features that characterize melody, harmony, and key, nevertheless emphasizes the similarity of structural principles at different levels. Each level characteristically displays a hierarchy of tonal stability, and at each level, representation in memory is determined by tonal function. Krumhansl further argues that strong interlevel influences in musical pitch organization are active, and reflect an overall knowledge system consistent with the regularities of music of our culture.

Empirical investigations have verified that the different levels of musical (pitch) elements are structured and inter-related. For example, Cuddy, Cohen and Mewhort (1981) demonstrated that the harmonic organization of a melodic sequence (the implied chord progression) affected its perceived form. Listeners heard short 7-tone melodic sequences and were asked, in different experiments, to rate structural "goodness" of the sequences or to recognize an alteration in a transposition of the sequence. Both ratings of structure and accuracy of recognition decreased as cues implying a simple underlying chord progression (I-V-I) were removed. For another example, findings obtained by Bharucha and Krumhansl (1983) suggest that chords are encoded with respect to an overall key. Listeners were asked to identify a changed chord in a short tonal sequence. If a diatonic chord (i.e., a chord in the key) was changed to a nondiatonic chord (i.e., a chord not in the key), the alteration was easier to detect than if a nondiatonic chord was changed to a diatonic chord. This asymmetry supports the notion that diatonic chords act as anchors in the perception of harmonic sequences. Chords that are consistent with the established key are perceptually anchored. Thus, removing a diatonic chord removes an anchor to which nondiatonic chords are heard in relation, and the removal is highly noticeable.

Although empirical work has identified principles of organization within and between levels, there has been little work addressed to questions concerning the overall hierarchical structure of the levels. In particular, there is the question of how key structure and key movement (modulation) are abstracted. Key relationships between two melodic sequences has been shown to affect the ease with which the melodies are compared (Bartlett & Dowling, 1980; Cuddy et al, 1981). Modulation of key within a melodic sequence may affect perceived structure even when all the notes of the sequence are members of the scale of the initial key (Cuddy & Lyons, 1981). However, these studies did not ask listeners to identify key structure or modulation directly; the effect of key was inferred indirectly from recognition scores or from ratings of overall perceived structure. In addition, the studies did not compare sensitivity to key structure in melodic sequences with sensitivity to similar key structure in harmonic sequences.

In the experimental work reported here, a direct comparison was made between identification of key modulation within melodic (single-voice) sequences and identification of key modulation within harmonic sequences. The basic expectations of a simple hierarchical model of musical organization were examined. In this model, a single voice is perceived with reference to an underlying harmonic structure, or chord progression. Chords, in turn, are understood with reference to the overall key. No direct links are assumed between the individual tones of a single voice and the key. Rather, a set of tones relates to the key through the level of chords. The process of inferring chord progression from melodic lines may be susceptible to ambiguity or misinterpretation. The model predicts, therefore, that the abstraction of key and key change will be more accurate for full harmonic sequences than for any of

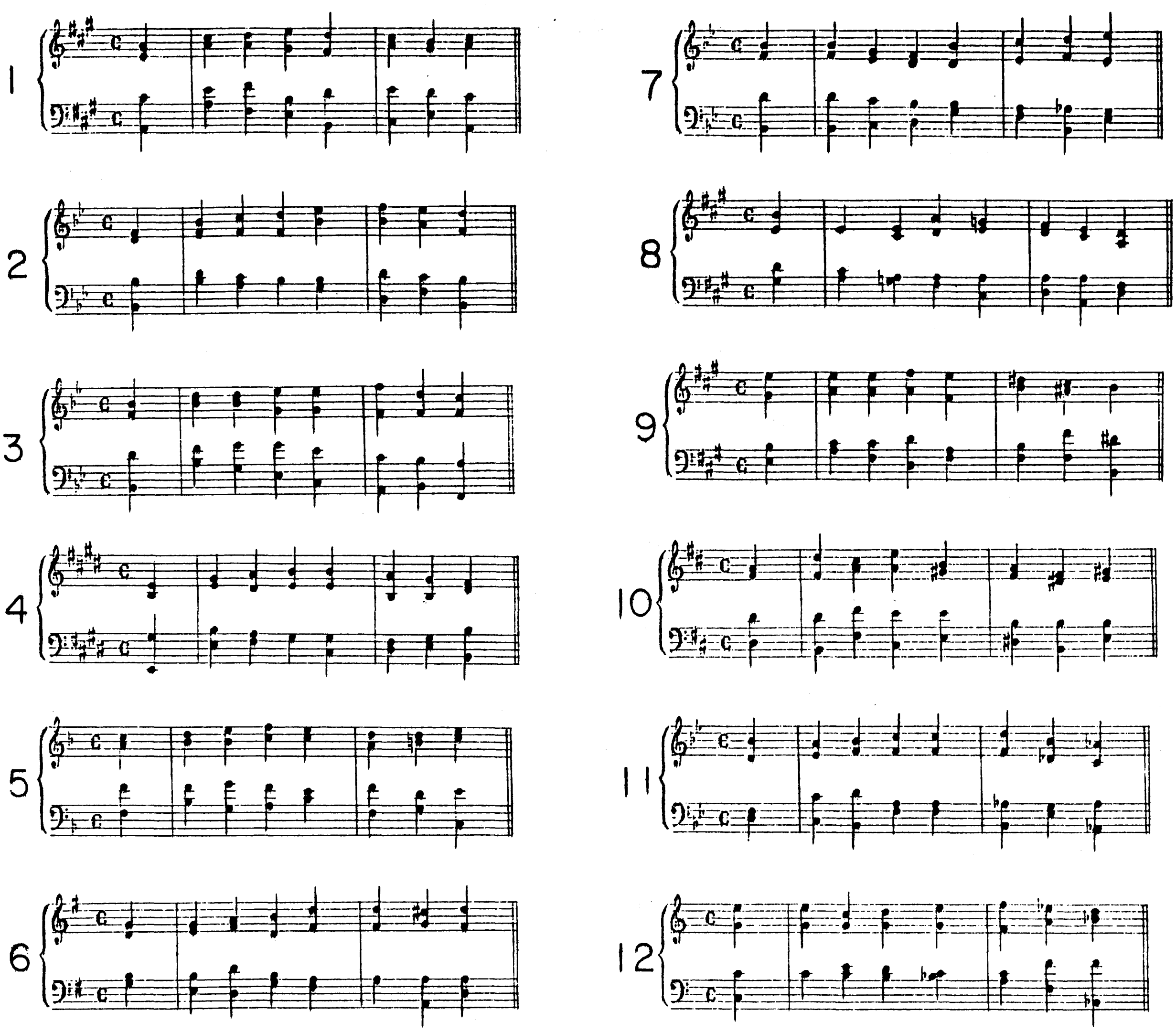

Figure 1. Chorale excerpts used in the presentations.

Table 1. Sequence types used in the presentations. The sequences shown in Figure 1, which correspond to each sequence type, are listed in brackets.

1. Nonmodulating and ending on the tonic chord (sequences 1 & 2).
2. Nonmodulating and ending on the dominant chord (sequences 3 & 4).
3. Modulating to the key of the dominant (sequences 5 & 6).
4. Modulating to the key of the subdominant (sequences 7 & 8).
5. Modulating to the key of the supertonic (sequences 9 & 10).
6. Modulating to the key of the flattened seventh (sequences 11 & 12).

the individual voices that make up the harmony.

Musical materials used

Sequences were phrases excerpted from the complete set of Bach Chorales (Leuchter, 1968). The chorales provide a wide selection of various types of key modulation within short chord progressions. Moreover, the chorales provide melodically continuous single voices with implied harmonic structures comparable to the realized structures of the chord progressions. For each of the following five types of modulation two representative examples were selected: no modulation; modulation to the key of the dominant (V); modulation to the key of the subdominant (IV); modulation to the key of the supertonic (II); modulation to the key of the flattened seventh (VIIb). With respect to the circle of fifths the types of modulation are respectively: no change of key on the circle of fifths; modulation one step clockwise; modulation one step counterclockwise; modulation two steps clockwise; modulation two steps counterclockwise.

The last two chords of these sequences formed a perfect cadence (V-I) in the final key of the sequence. Two additional sequences were selected: these sequences were classified as nonmodulating ending with an imperfect (I-V) cadence. They were included to emphasize to the listener that the task of identifying key change could not be solved by merely attending to the first and last event of the sequence. These sequences, however, are not included in the data analyses discussed below.

Sequences were equated for length and surface complexity; all ornamental and passing notes were removed so that all sequences contained a progression of exactly eight four-note chords. The final selection and classification was verified in consultation with a music theorist.

The sequences are shown in musical notation in Figure 1. The type of modulation for each sequence is listed in Table 1.

The two examples of each type of modulation differ not only in the harmonic progression leading from the first to the final key of the sequence, but also in the construction of the final chord. For sequences ending with a perfect cadence, (the experimental sequences), the fifth of the final triad was omitted in example 1, and retained in example 2. The final chord fifth in example 2 was always carried by an inner voice. This systematic manipulation of the presence or absence of the fifth in the final chord was stimulated by the observation that whenever there was a perfect cadence in the original chorales, the fifth was invariably present even if its omission was harmonically acceptable. We were curious to find out whether inclusion of the fifth in the final chord affected judgements of modulation, perhaps by influencing the establishment of the final key.

Immediately before each presentation, the initial key of the sequence was established by presenting an arpeggiated tonic triad. (Previous work in our laboratory had determined that a broken tonic triad is as effective in establishing a tonal center as is the tonic chord and chord cadences (Cuddy, 1986).) Sequences were realized by a DMX-1000 signal processor under control of a host computer PDP 11/23. All tones were complex, containing the first five partials of the overtone series with the amplitudes of each partial inversely proportional to the partial number. All tones were 350 msec in duration and had rise and decay times of 22 msec each.

Experimental studies

A. Identification of key changes

In two main studies, highly trained listeners were asked to identify the type of key change implied by the sequences. In one study, listeners heard each of the four voices of each sequence in isolation and attempted to identify the key change from the information provided by a single voice. In the second study, listeners heard the full harmonic sequence before recording a judgement. The order of presentation of the sequences in both studies was random. Identification proceeded in two steps. First, the distance of the modulation was judged, and second, the direction of the modulation was judged. For judgements of distance of modulation, listeners were allowed four categories of choice: nonmodulating and ending on the tonic chord; nonmodulating and ending on the dominant chord; modulating one step on the circle of fifths; and modulating two steps on the circle of fifths. For judgements of direction of modulation, listeners were allowed three categories of choice: non-modulating; modulating in the clockwise direction on the circle of fifths; and modulating in the counterclockwise direction on the circle of fifths.

In the description of the results that follows, all differences reported are statistically significant. The statistical analyses, however, will not be discussed; details are available in Thompson (1986) and Thompson & Cuddy (in preparation).

B. Overall identification accuracy

A comparison of the accuracy of distance and direction judgements for the first and second study revealed that judgements following single voices, in most cases, were as accurate as those following the full chorale sequence. The one exception to this finding concerned the single voice in the second example that ended on the fifth of the final chord: here, correct identification appeared to be quite difficult. All other voices, however, were judged as accurately as the full harmonic sequences. Table 2 reports the average accuracy scores for these voices and for the full chorale sequences. The data suggest that a single voice is able to establish a sense of key and key change that is as strong as that produced when four voices combine to form harmony.

C. Influences on judgements of key change

Although single voices usually produced identification scores comparable to those obtained with full harmonic sequences, similar scores need not reflect identical psychological operations. This possibility was considered by examining the effect of musical factors on judgements in the two conditions.

1. Construction of the final chord.

The mean accuracy of distance and direction judgements for the first and the second example of the sequence types are shown in Table 2. Identification accuracy for harmonic sequences was consistently and significantly higher for the first of the two examples. In the first example of each sequence type under harmonic presentation, the fifth of the final chord was omitted, while in the second example it was retained. The difference in identification accuracy between the two examples may suggest an important role for the construction of the final chord.

We considered the possibility that the difference in identification accuracy between the two examples

Table 2. Mean percent accuracy of identifying modulation distance and modulation direction for harmonic and single voice presentations.

Judgements of modulation distance

	example 1	example 2	X
Harmonic sequences	73.34	42.00	57.67
Single voice sequences*	59.00	59.60	59.30

Judgements of modulation direction

	example 1	example 2	X
Harmonic sequences	68.66	53.32	60.99
Single voice sequences*	62.20	62.00	62.10

of each sequence type reflected consistent differences in the harmonic progression. Initially, this explanation seemed unlikely. If harmonic progression were important to this effect, one would expect a similar effect to emerge for judgements of melody or bass voices presented in isolation: these voices should imply the harmonic progression. However, no difference between the two examples was found in the single voice data.

Another possibility is that the reduced identification accuracy in harmonic sequences ending with a triadic chord is comparable to the reduced identification accuracy for individual voices ending on the final chord fifth. This possibility was examined through an analysis of errors. It was found that single voices ending on the fifth tended to be judged as though the final note were the tonic of the final key. Thus, very low accuracy was found for voices from all sequence types except those modulating to the key of the supertonic. In the latter case, there were no choice categories that would satisfy the fifth being mistaken as final tonic: listeners were not given the option of judging the sequence to be modulating to the submediant (i.e., the fifth of II).

In harmonic sequences, errors of judgement also showed a systematic bias, but of a different kind. For all types of modulation, including modulations to the key on the supertonic, listeners tended to underestimate the distance of modulation when the final chord included the fifth. This effect suggests that when a new key is established with a triadic chord, as compared to a chord in which the fifth has been omitted, the perceived tonal change may be reduced. Thus, the precise effect of the final chord

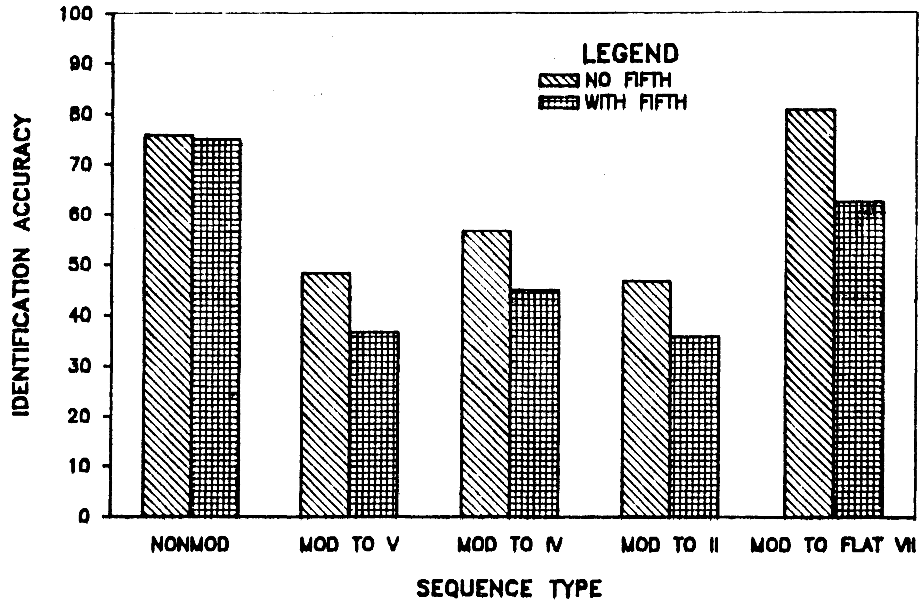

Figure 2. Mean accuracy of identifying modulation distance for two types of final chord construction: either with the fifth of the triad omitted or with the fifth of the triad included.

fifth in harmonic sequences was not predictable from judgements of the individual voices of those sequences.

The data strongly suggest that the construction of the final chord may have a significant influence on judgements of key change in harmonic sequences. Although we had reasoned it unlikely that differences in chord progression could account for the data, this possibility was examined in a supplementary experiment. A new group of musically trained listeners tried to identify key changes in the chorale sequences. In this experiment, sequences in which the fifth had previously been omitted from the final chord were written with the fifth included, and vice versa. Combining the supplementary experiment with the main experiment permitted an examination of the influence of chord construction across sequences that were otherwise identical (i.e., contained the same harmonic progression).

Figure 2 shows mean accuracy of distance judgements for the combined data. For each modulation type, there are two bars--one representing mean accuracy of distance identification when there was no fifth in the final chord, the other representing mean accuracy of distance identification when the fifth was included in the final chord. Differences in accuracy for the two types of final chord construction therefore reflect differences that remain after data for the two examples of each types of modulation were averaged. For nonmodulating sequences, there was no overall effect of final chord construction. However, for all modulating sequences, identification accuracy for distance of modulation was lower when the fifth was included in the final chord than when it was omitted. The findings suggest that chord construction may become important in harmonic sequences when the chord is used in the establishment of a key change.

2. Chord progression

Although judgements of modulation distance in harmonic sequences were clearly influenced by the construction of the final chord, judgements of modulation direction were not affected by this factor. An analysis similar to that conducted for distance judgements reported in Figure 2 showed no effect of chord construction. In both experiments accuracy of direction judgement favored example 1 over example 2--by 15.3% in the main experiment and by 14.0% in the supplementary experiment. It may be suspected that the specific nature of the chord progressions was primarily responsible for the differences between examples. This effect of chord progression on direction judgements, however, was evident for harmonic sequences only. There was no evidence that the implied chord progression or any melodic factor affected the two examples differentially in the single-voice data, either in the average data (as can be seen in Table 2) or for any isolated instance of a single voice condition.

3. Direction of modulation.

In addition to the specialized effects of chord construction and chord progression found for harmonic sequences, our data suggest that perceived key relationships for harmonic sequences are asymmetric with respect to the circle of fifths. In Figure 2, it can be seen that key changes in the clockwise direction on the circle of fifths (modulation to V and II) tended to be judged less accurately than corresponding key changes in the counterclockwise direction (modulation to IV and VIIb). An analysis of errors revealed that distance judgements for clockwise modulations tended to be underestimated.

A difficulty arises with the interpretation of the difference in accuracy for modulations one step around the circle of fifths. The lowered performance for modulations one step clockwise could be attributable to confusion of this modulation with the control condition - no modulation ending on the dominant chord. However, a supplementary experiment reported below renders this interpretation unlikely. Moreover, the larger asymmetry between modulations two steps around the circle of fifths cannot be attributed to confusions with control sequences. This asymmetry was not found in single-voice judgements.

The asymmetry for modulation judgments was confirmed for harmonic sequences in a final supplementary experiment in which listeners without formal training in music theory rated the perceived distance between the first and final keys of each sequence. (No formal reference was made to the

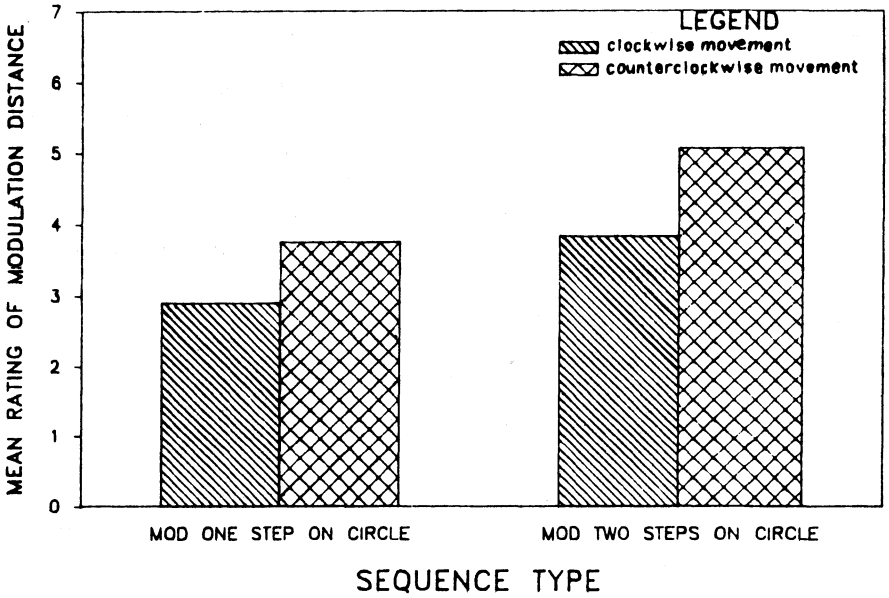

Figure 3. Mean ratings of perceived modulation distance.

circle of fifths with this sample of listeners.) In this investigation, the control sequences were excluded from the presentations.

Figure 3 shows mean ratings of perceived distance between the first and final keys of modulating sequences. When key changes suggested movement in the clockwise direction on the circle of fifths, the judged distance between the two keys of the sequence was less than when the key change suggested movement in the counterclockwise direction. These results suggest that while the circle of fifths may be a valid representation of the way listeners understand key relationships, there are clearly other factors that affect the assessment of key relationships in a harmonic sequence.

We considered two explanations of the asymmetry of perceived modulation distance. First, it is possible that in these experiments the level of key relationships was accessed through the hierarchy of scale tones as represented by the tonality profile for major keys (Krumhansl & Kessler, 1982). Within the hierarchy of scale tones, the dominant is more stable than the subdominant, and the supertonic is more stable than the flattened seventh. The asymmetry may thus be a consequence of the fact that for modulations equivalent in distance on the circle of fifths, the tonic note of a key in the clockwise direction is more stable with respect to the initial key than is the tonic note of a key in the counterclockwise direction. Thus, perceived key distance may be explicitly encoded as the extent to which the tonic of the second key is assimilated into the scale of the first key. Although this explanation may be reasonable, it is strained by the fact that sequences modulating to the subdominant and sequences modulating to the supertonic were given similar average ratings (mean ratings of 3.75 and 3.83, respectively) even though the supertonic is a less stable scale tone than the subdominant.

A second explanation is that the level of key relationships is engaged by a sensitivity to the structure of the overtone series. According to Rosen (1971), the circle of fifths has a basis in the overtone series, each key implicating its clockwise neighbor by its second overtone. Since the overtone series projects immediate neighbors in the clockwise direction only, the structure is asymmetric. Thus, key changes in the clockwise direction may be heard as conveying less tonal movement because they are consistent with the natural structure of the overtone series.

Neither explanation can readily explain the finding that asymmetry was evident for harmonic sequences but not for single-voice sequences. Clearly, greater elaboration of the possible reasons for an asymmetry of perceived distance is needed. What is particularly important about the possible explanations advanced here, however, is that both imply that structural relationships suggested by the initial key provide an abstract framework within which the tonic of the second key is evaluated.

The asymmetry of perceived key relationships may play a significant role in the performance of music. According to Sundberg, Frydén & Askenfelt (1983), an important factor affecting the acceptability of musical performance is the variation in loudness that marks the musical distance of tones from the tonal center. In their model of musical performance, the distance from the tonic along the circle of fifths is determined for each note by considering the chord to which the note belongs and its musical relation to the root of that chord. The amplitude given to a note is increased in proportion to its distance from the tonic. However, the authors also assume that the tonic distance of tones and chords on the counterclockwise side of the circle of fifths is greater than the tonic distance of tones and chords on the clockwise side of the circle of fifths. Thus, their model of musical performance assumes an asymmetry similar to the one reported in the present paper.

It is interesting to note that the model proposed by Sundberg & al. concerns rules applied in the performance of melodies, whereas we found evidence for an asymmetry in harmonic sequences only. Perhaps by introducing an asymmetry in the performance of a melodic sequence, the performer is more strongly able to convey the underlying harmony.

Discussion

The simple hierarchical model proposed earlier predicted that a single voice would convey less information about key and key change than would a harmonic sequence. An analysis of mean identification accuracy did not support this prediction. Judgement accuracy was usually as high following single-voice sequences as it was following harmonic sequences.

Several of the findings suggest that harmony and melody operate somewhat differently in their implications for perceived key structure. First, inclusion of a final chord fifth in harmonic sequences had the distinct effect of reducing the perceived distance of modulation. However, single voices ending on the fifth of the final chord were merely misleading, probably because the final note was taken to be the tonic of the final key. Second, judgements of harmonic sequences were influenced by the precise chord progression, which contributed to a large difference in identification accuracy between the two examples of each sequence type. In contrast, there was no overall difference in identification accuracy between examples when individual soprano and bass voices were presented. Evidently, the harmonic progression implied in individual melodic lines does not have the same impact on perceived key relationships as does the explicit harmonic progression conveyed in a harmonic sequence. Third, the perceived distance between keys as they occur in harmonic sequences is asymmetric with respect to direction of modulation. This asymmetry was not evident when the single lines of those harmonic sequences were presented.

Although the findings present difficulties for a simple hierarchical model of musical organization, one cannot reject a hierarchical model on the basis of the present evidence alone. One problem is that the apparent similarity between scores for melodic and harmonic sequences may be more a result of the presence of the three musical factors that influenced judgements in the harmonic conditions rather than the result of two identical processes of key abstraction. An alternative worth considering, however, is a partially hierarchical model in which key information is encoded explicitly at both the level of notes and the level of chords. Listeners may represent single voices, chords, and keys hierarchically, but some kinds of information may be stored in a redundant manner throughout the hierarchy. At a given level of abstraction, relations that can be derived from other levels in the hierarchy may be stored explicitly at that level as well (e.g., see McNamara, 1986). This type of model would allow key relationships to be abstracted at both the levels of chords and of single voices. It would also allow for structural differences at different levels of the hierarchy so that judgements of key change in a harmonic sequence are not always predictable from judgements of key change in a single voice of that harmonic sequence.

Finally, the general finding that several musical factors may influence judgements of key change in harmonic sequences raises an important issue regarding the relationship between key structure and musical aesthetics. It is likely that each of the harmonic effects reported here contribute to the quality and richness of harmonic music. Nonetheless, our data suggest that these factors need not improve the clarity of the precise key change as defined by the circle of fifths. Lerdahl (personal communication, February, 1986) has supplied us with revisions of the harmonic sequences that we used in the studies. The revisions are intended to establish the modulation within specific sequences more explicitly, and should lead to greater identification accuracy than that reported here. But it is far from clear that easily identified key changes are musically pleasing. The experiments that we have reported suggest that harmonic music may involve a wealth of cognitive and acoustical principles, not all of which may serve to facilitate an analytic judgement of key change.

Acknowledgements

We thank Carol Krumhansl for her valuable comments on this work, and Dan Scheidt and Sue Becker for programming assistance. The research and writing was supported by the Natural Sciences and Engineering Research Council of Canada, the Ontario Graduate Scholarship Council, and the Swedish Institute.

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SOUND EXAMPLES

Example for Sundberg: Harmony and Harmonic Spectra.

A tritone interval followed by a minor sixth. First these intervals are played with harmonic spectra, thereafter with inharmonic spectra in which neighbor partials are separated by a tritone interval, as shown in Fig. 3 of this article.

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List of Sound Examples for Krumhansl: Tonal and Harmonic Hierarchies

Sound Example 1. Illustrates the probe tone method with a sampling of trials using C major and C minor contexts.

Context	Probe Tones
Ascending C major scale	G, F#, C, D#, E, A, D
IV V I in C minor	Eb, E, G, D, C, F#, A

Sound Example 2. Illustrates the methodology used in the scaling study on pairs of tones.

Context	Tone Pairs
Descending C major scale	GC, C#G#, FE, CC#, CE, AC#

Sound Example 3. Illustrates the methodology used in the scaling study on pairs of chords.

Context	Chord Pairs
Ascending C major scale	G major (V) C major (I)
	E minor (III) A minor (VI)
	F major (IV) G major (V)
	A minor (VI) B diminished (VII)

Sound Example 4. Illustrates the technique for tracing the developing and changing sense of key.

Context chords	Probe tones
F major	C, F#
F major G major	G, D#
F major G major A minor	E, A#

The following sequences are played twice:
Sequence in C major: F major G major A minor F major C major
A minor D minor G major C major (Figure 8)
Sequence modulating from C major to G major: F major G major
C major A minor E minor B minor E minor D major G major (Figure 9)
Sequence modulating from C major to Bb major: F major G major
C major A minor F major G minor Eb major F major Bb major (Figure 10)
Sequence modulating from C minor to C# minor: D diminished G major
C minor G major Ab major A major F# minor G# major C# minor (Figure 11)

Sound Example 5. J. S. Bach C minor Prelude, Book II
Measures one through four (Figure 12)
Measures five through nine (Figure 13)
Measures nine through twelve (Figure 14)
Measures thirteen through eighteen (Figure 15)
Measures nineteen through twenty-two (Figure 16)
Measures twenty-three through twenty-eight (Figure 17)

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List of Sound examples for Mathews, Pierce & Roberts: Harmony and New Scales

Sound example 1. Intonational sensitivity of 3:5:7 triads and 4:5:6 (major) triads. A 3:5:7 chord is played three times first in just intonation, then with center note 15 cents sharp, then with center note 30 cents sharp. Next a 4:5:6 chord is played three times with the same intonation changes. The entire sequence of six chords is repeated.

Sound example 2. A Pierce nine-tone scale is played twice including repeating the tritave. The two-step and one-step intervals in the scale are easy to hear.
Sound example 3. A major (0 6 10) and a minor (0 4 10) Pierce triad are played. These two chords are repeated. The minor chord sounds "minor" relative to the major chord.

Sound example 4. A harmonized Pierce scale is played twice. The scale is the upper voice. The 5th and 7th steps are harmonized with minor triads. The 9th step is harmonized with the first inversion of a major triad. The rest of the steps are harmonized with major triads.

Sound example 5. A Canon 2 by Alyson Reeves This is a short simple melodious canon. Except for the last measure, the second and third voices are repetitions of the first voice delayed respectively by two and four measures. The canon ends with a IV V I cadence which arises naturally from the melodic structure of the first voice. Different timbres were used for each voice to aid in their perceptual separation.

Sound example 6. Fugue 1 by Alyson Reeves The first fugue composed in the Pierce scale experiments with analogies of many traditional fugue elements. The subject is exposed in all three voices (measures 1 to 12) and the exposition ends with a VI V I cadence (measures 12 and 13). A three measure episode follows (measures 13 to 15). The episode is translated downward one step in measures 16 to 18 and downward yet another step in measures 19 to 21, thus achieving a transition to a presentation of the subject in the subdominant key (starting at measure 22). The rest of the fugue includes an inverted subject, the subject in the tonic key, and finally the subject is augmented. For the most part these elements are audible. The fugue ends with another VI V I cadence.

Sound example 7. Min 2c by Alyson Reeves This minuet, written in the style of a Mozart minuet, is an exercise in establishing a key and in modulation. The first section ends in a half cadence with a VI V I chord progression. The second section ends in a full cadence with another VI V I cadence.

The trio, which starts 24 seconds after the beginning, modulates to the subdominant key and makes heavy use of minor chords. The key change is clearly audible. It ends with a minor VII chord in the subdominant key, which contrasts clearly with the tonic in the final repetition of the first section. The recapitulation and return to the original key starts 49 seconds after the beginning.

Sound example 8. Ragged Rag by Alyson Reeves This rag combines the very characteristic rhythms of the rag style with the Pierce scale. It is a fun piece.

Sound example 9. Excerpt from "Duo for Oscar" by Jon Appleton This piece contains a short section in the Pierce scale played on the Synclavier, which is a commercial synthesizer that can be tuned to the Pierce scale. The excerpt consists of 90 seconds from the beginning of the piece in the diatonic scale, a 60 second transition passage formed from glissandos, and the Pierce scale section which lasts 60 seconds. The Pierce scale section can be heard as a transformation of material from the opening section.

Sound example 10. A well-known hymn played first in the diatonic scale and then in a stretched scale (A = 2.4). Some melodic lines are perceivable in the stretched version, but the harmony sounds very different.

Sound example 11. An AXBX key sensing test. A and X are in the same key.

Sound example 12. Cadences
1) Four chord diatonic anticadence
2) Four chord diatonic cadence
3) Four chord stretched anticadence
4) Four chord stretched cadence
5) Four chord anticadence with timbres having equally spaced overtones
6) Four chord cadence with timbres having equally spaced overtones. Note especially the strong contrast in dissonance between the last two chords.

Sound example 13. A simple melody, "Are You Sleeping, Brother John", is played only in a stretched (A = 2.4) scale. Most listeners can identify the tune.