

Dept. for Speech, Music and Hearing  
**Quarterly Progress and  
Status Report**

**A new anti-resonance circuit  
for inverse filtering**

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journal: STL-QPSR  
volume: 2  
number: 4  
year: 1961  
pages: 001-006



**KTH Computer Science  
and Communication**

<http://www.speech.kth.se/qpsr>



## I. SPEECH ANALYSIS

## A. A NEW ANTI-RESONANCE CIRCUIT FOR INVERSE FILTERING

The technique of "inverse filtering", also referred to as "anti-resonance filtering", is based on a cancellation of the formant structure of speech in order to regenerate a signal which has the properties of the primary sound source, e.g. the larynx source. By this technique applied to selected samples of human speech it is possible to determine formant frequencies with great accuracy and the residue signal at the output of the system can be regarded as representative of the source providing the transfer function of the anti-filter is appropriately supplemented with a higher pole correction and an integration stage to compensate for the radiation transfer characteristics of the speaker.

These features were all included in the instrumentation underlying the measurements reported on earlier <sup>(1)</sup><sup>(2)</sup><sup>(3)</sup>. The basic anti-resonance circuit of this design was a simple voltage divider employing RLC-elements, see Fig. I-1.A. Providing the net is realized as in Fig. I-1.B. or C. the zero frequency is entirely determined by the series connected elements  $C_2$   $r_2$   $L_2$  of the shunt arm. The voltage transfer functions of these two nets are

$$H_B(s) = \frac{s^2 + s \cdot \frac{R_2}{L_2} + \frac{1}{L_2 C_2}}{s^2 + s \left( \frac{R_1 + R_2}{L_2} \right) + \frac{1}{L_2 C_2}} \quad (\text{Eq. 1})$$

$$H_C(s) = \frac{s^2 + s \cdot \frac{R_2}{L_2} + \frac{1}{L_2 C_2}}{s^2 + s \left( \frac{R_1 + R_2}{L_2} \right) + \frac{C_1 + C_2}{L_2 C_1 C_2}} \quad (\text{Eq. 2})$$

Providing the resistance  $R_1$  of net B is sufficiently great there will occur a pole of low frequency. Each of the two nets B and C have two poles in addition to the two zeros. In net B, providing  $R_1$  is large, one of these poles occurs at

$$\hat{s}_1 = - \frac{R_1 + R_2}{L_2} \quad (\text{Eq. 3})$$

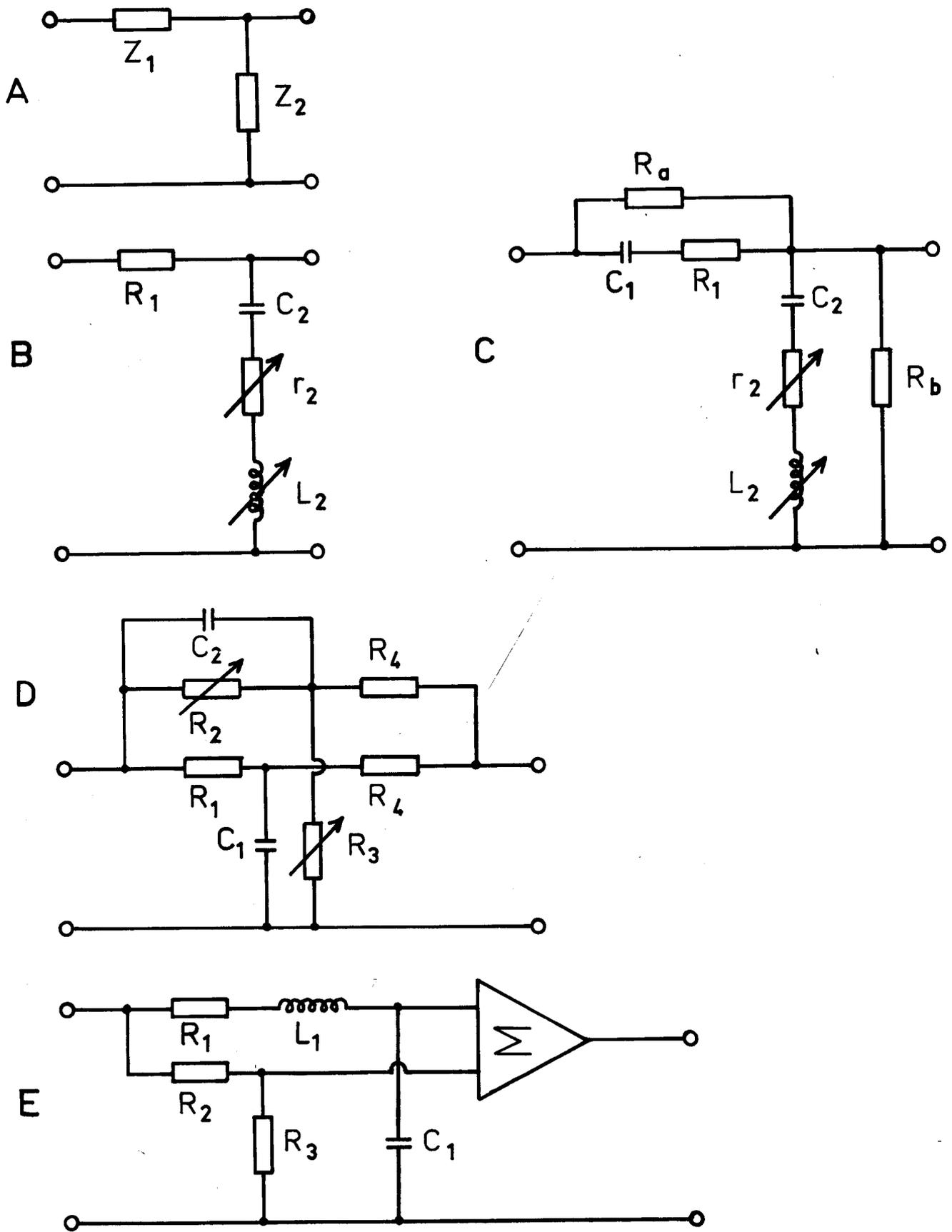


Fig. I-1. Anti-resonance circuits A. Basic net B. and C. Series resonance in the shunt arm, alternative B. intended for additional integration. D. RC-network of the same function as B. E. net for approximating the function  $kH$  of the higher pole correction.

which, in view of its high frequency, will determine the upper limit of the usable frequency range. The other pole on the real axis

$$\hat{s}_2 = -\frac{1}{(R_1+R_2)C_2} \quad (\text{Eq. 4})$$

will be of a very low frequency and may be regarded as an integrator.

In the case of small  $R_2$  the two zeros are conjugate complex

$$\bar{s}_1, \bar{s}_1^* = -\frac{R_2}{2L_2} \pm j\sqrt{\frac{1}{L_2C_2} - \left(\frac{R_2}{2L_2}\right)^2} \quad (\text{Eq. 5})$$

The anti-frequency is approximately

$$F = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L_2C_2}} \quad (\text{Eq. 6})$$

and the bandwidth of the anti-resonance is

$$B = \frac{R_2}{2\pi L_2} \quad (\text{Eq. 7})$$

In net C the poles are conjugate complex providing  $C_1$  is small. They will occur at a higher frequency than the conjugate zero.

$$\hat{s}_1, \hat{s}_1^* = -\left(\frac{R_1+R_2}{2L_2}\right) \pm j\sqrt{\frac{C_1+C_2}{C_1C_2L_2} - \left(\frac{R_1+R_2}{2L_2}\right)^2} \quad (\text{Eq. 8})$$

The integration inherent in net B is useful when the radiation transfer from the mouth of the speaker is to be compensated. This type of net has generally been used in F1-units and net C for other formants.

There are, however, definite draw-backs of both circuits B and C. It is impossible in these circuits to satisfy simultaneously the following demands.

1. Essentially constant reference gain.
2. Essentially independent variation of anti-resonance frequency and bandwidth by means of one circuit element for each of these two parameters.
3. Phase correct response at low frequencies.

Thus in net B a variation of the anti-frequency by means of  $L_2$  affects the bandwidth of the anti-resonance. A variation of the anti-frequency by means of  $C_2$  in net B will affect considerably the integrating pole and thus the low frequency phase response and gain.

In circuit C there is a necessary requirement to compensate the phase effects of the unavoidable loading resistance  $R_b$  by a resistance  $R_a$  over-bridging the condenser  $C_1$  of the  $Z_1$  arm

$$R_a = R_b \cdot \frac{C_2}{C_1} \quad (\text{Eq. 9})$$

This correction is valid only for fixed values of  $C_2$ . In addition there is a variation in the reference gain with varying  $C_2$  which is objectionable providing  $C_1$  is not much smaller than  $C_2$ . If, as an alternative, the anti-frequency is varied by means of  $L_2$  there is an associated bandwidth change through  $r_2$  to take into account.

A bandwidth variation by means of a resistor paralleling  $C_2$  will give rise to phase errors according to Eq. 9. The best alternative is to vary the bandwidth by a resistor  $R_2$  paralleling  $L_2$ . This requires, however, that the frequency variation by means of  $L_2$  be supplemented by a change in  $r_2$  in order to maintain constant bandwidth, or that  $r_2$  may be neglected.

The same negative results as to the possibility of simultaneously satisfying the demands 1, 2, and 3 above are found when the basic circuit Fig. I-1.A. is realized by means of a parallel resonance branch in  $Z_1$ .

These disadvantages are not found in circuit D - a new variant of the double T RC-network which we have developed specifically for use in inverse filters. This circuit has the further advantage that the frequency and the bandwidth are each varied by means of a resistor. The voltage transfer function  $H_D(s)$  of net D is similar to that of net B, i.e., it contains one very low frequency pole,  $\hat{s}_1$ , and one very high frequency pole,  $\hat{s}_2$ . Ordinarily, the circuit D is thus combined with a differentiating network.

$$H_D(s) = \frac{(s-\bar{s}_1)(s-s_1^*)}{2(s-\hat{s}_1)(s-\hat{s}_2)} \quad (\text{Eq. 10})$$

$$\left\{ \begin{aligned} \bar{s}_1, \bar{s}_1^* &= -\left(\alpha_0 + \frac{\gamma}{2}\right) \pm j \sqrt{\alpha \left[ \beta + \gamma \left(1 + \frac{R_1}{R_4}\right) + \frac{1}{R_4 C_2} \right] - \left(\alpha_0 + \frac{\gamma}{2}\right)^2} \\ &\approx -\left(\alpha_0 + \frac{\gamma}{2}\right) \pm j \sqrt{\alpha \beta} \\ \hat{s}_1 &\approx -\alpha_0 = -\alpha(1 + R_1/2R_4) \\ \hat{s}_2 &\approx -\alpha_0 = -1/R_3' C_2 \end{aligned} \right. \quad (\text{Eq. 11})$$

where

$$\left\{ \begin{aligned} \alpha &= 1/R_1 C_1 \\ \beta &= (R_2 + R_3)/R_2 R_3 C_2 \approx 1/R_3 C_2 \\ \frac{1}{R_3'} &= \frac{1}{R_3} + \frac{1}{2R_4} + \frac{1}{R_2} \\ \gamma &= 1/R_2 C_2 \end{aligned} \right. \quad (\text{Eq. 12})$$

The insertion loss derived from the voltage transfer at zero frequency is

$$G_0 = -20 \log_{10} \left( 2 + \frac{R_1}{R_4} \right) \text{ dB} \quad (\text{Eq. 13})$$

Providing  $R_3$  is small compared with  $R_1$ ,  $R_2$  and  $R_4$  the anti-resonance frequency is approximately

$$F = \frac{1}{2\pi \sqrt{R_1 R_3' C_1 C_2}} \quad (\text{Eq. 14})$$

which is conveniently controlled by  $R_3$ . The bandwidth is

$$B = \frac{1}{2\pi} \left[ \frac{1}{R_2 C_2} + \frac{2}{R_1 C_1 (1 + R_1 / 2R_4)} \right] \quad (\text{Eq. 15})$$

which is controlled by  $R_2$ .

This circuit has proved to be reliable within a very large range of  $R_3$  values.

The theory of higher pole corrections was originally outlined in Ericsson Technics, No. 1, 1959 <sup>(4)</sup>, p. 43-45. Laplace transforms suitable for network approximations were also provided in this article (p. 99-100). This correction which is of considerable practical importance in cascade connected formant synthesizers is of high-pass type. For the specific case of a 17.5 cm long tube the correction for poles above the fourth is 0 dB at zero frequency, 2 dB at 1000 c/s, 9 dB at 2000 c/s, 21 dB at 3000 c/s, and 41 dB at 4000 c/s.

In the design of an anti-filter the higher pole correction is the inverse and thus a low-pass function! Substituting poles for zeros in Eq. A-1 of ref. <sup>(4)</sup> the desired higher pole network will be

$$K_{r4}(s) = \frac{[(s + \pi \cdot 400)^2 + (2\pi \cdot 4000)^2]}{(2\pi \cdot 4005)^2 \cdot (s - \hat{s}_1)(s - \hat{s}_1^*)} \cdot \frac{(s + \pi \cdot 4000)^2 + (2\pi \cdot 4000)^2}{(s + 2\pi \cdot 2000)^2} \quad (\text{Eq. 16})$$

The first factor, to which a high frequency limiting pole factor has been added, conforms with an anti-resonance  $F5'$  which may be realized in terms of the network C or in terms of the network D and an associated differentiating stage. The former solution has been adopted. The second factor is realized in terms of a network of type E. Providing the elements  $R_1$ ,  $L_1$  and  $C_1$  of E are selected for critical damping, i.e., a double pole on the real axis, the voltage transfer function will contain a conjugate zero with the same real part, i.e. damping constant, as this double pole.

$$H_E(s) = \frac{(s + \alpha)^2 + \frac{1}{k} \alpha^2}{(s + \alpha)^2} \quad (\text{Eq. 17})$$

where

$$k = R_3 / (R_2 + R_3)$$

$$\alpha = R_1 / 2L_1 = 1 / \sqrt{L_1 C_1} \quad (\text{Eq. 18})$$

$$\bar{s}_1, \bar{s}_1^* = -\alpha \pm j \sqrt{\frac{1}{k}} \alpha$$

In our design the higher pole correction may be changed in steps of 250 c/s from a reference frequency of 3000 c/s to 5500 c/s corresponding to vocal tract lengths of 23.4 cm to 12.7 cm. This is accomplished by corresponding scale factor changes of the poles and zeros of the higher pole net. The F4 frequency is varied in 50 c/s quantal steps from 2000-6000 c/s. The F3 frequency is quantized in 25 c/s steps from 1200-4000 c/s, F3 in 25 c/s steps from 400-3200 c/s, and F1 in 12.5 c/s steps from 100-1500 c/s.

Each anti-formant unit has a 21-position switch for bandwidths ranging from 50-1600 c/s in the F4 and F3 units, 30-800 c/s in the F2 unit, and 20-400 c/s in the F1-unit.

The entire anti-formant unit is DC coupled in order to avoid low frequency phase distortion affecting the pulse shape of re-generated glottal pulses. The units are normally connected in the order F2 F3 F4 F5' KH F1 for minimizing noise and overload troubles, but the units may be used independently and in different orders. A continuously variable band-pass filter and a differentiating network are available at the output of the system for enabling frequency selective studies of the residue signal in various frequency regions.

G. Fant

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- (4) Fant, G.: "Acoustic Analysis and Synthesis of Speech with Applications to Swedish", Ericsson Technics 15, No. 1 (1959) pp. 3-108.