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III. MUSICAL ACOUSTICS

A. MEASUREMENTS OF THE HEAD-JOINT PERTURBATION AND THE EMBOUCHURE-REACTANCE OF FLUTES

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Significant determinants for the tuning and the scale of the modern flute are the head-joint dimensions, the effective embouchure reactance, and the effective length of the $\lambda/4$ pipe from the embouchure up to the cork. This is well known by the instrument makers and many improvements are due to their great experience. However, a more specific knowledge is needed about (1) the head-joint dimensions and (2) the effective embouchure reactance during appropriate blowing. The need of convenient measuring methods for such studies is apparent.

In a cylindrical flute the fundamental frequencies of the overblown tones, i.e. the tones of the higher modes, are lower than the corresponding multiple frequencies of the first mode. Two useful compensation methods have been employed. The old flute has a cylindrical head-joint and the body has a conical taper towards the end. The modern Boehm-flute has a head-joint tapering towards the cork and a cylindrical body. This latter type will be investigated here.

Measurements were made on flutes with a conical head-joint from a modern flute and a specially made cylindrical head-joint. The body consisted of a cylindrical tube without finger holes. Four tubes of different lengths were used and in this way four flutes with cylindrical ($N^0_1c$ to $N^0_4c$) and four with conical head-joints ($N^0_1p$ - $N^0_4p$) could be tested.

The sine wave modulated Ionophone was used as sound source and the Ionophone electrodes were placed in the center of the flute and at the center of the embouchure. The sound pressure was measured with a 1/2" Brüel & Kjær microphone about 4 cm from the open end. Measurements were made with open as well as closed embouchure without moving the sound source. In order to obtain a value of the effective embouchure reactance, the flutes were blown mezzoforte by two flutists. The first four modes were blown on each flute except on the two shortest (4e and 4p) where the fourth mode was difficult to blow. Four tests by each flutist were recorded and measured. It is known that at moderate wind pressure the fundamental frequency approximately equals a resonance frequency of the instrument. In
the present investigation the effective embouchure reactance is specified as the embouchure reactance calculated from the above-mentioned blown frequencies. In order to calculate a mean value of the sound velocity a thermistor was placed in the center of each flute. The mean value of the temperature was about 30°C inside the flute at 25°C ambient temperature. The Ionophone measurements were correspondingly corrected with respect to temperature.

List of symbols (pertaining to the Figure on p.17 and to Eqs. (1) - (8))

- \( \frac{P_t}{U_i} \) = The transfer function
- \( P_t \) = The sound pressure at the open end
- \( U_t \) = The input volume velocity

\[ \Gamma = \text{The complex propagation constant} = \gamma_1 + \gamma_2 + \gamma_t = \alpha \xi + j\beta \xi \]
where \( \alpha \) is the attenuation constant and \( \beta \) the phase constant

\[ \Gamma(\lambda/4) = \text{The complex propagation constant for the flute with closed embouchure} \]
\[ \Gamma(\lambda/2) = \text{The complex propagation constant for the flute with open embouchure} \]

\( \omega = \text{Angular frequency} = 2\pi f \)

\( z_o = \text{The characteristic impedance} = r_o + jx_o \approx \frac{\rho c}{q} \)

\( z_e = \text{The embouchure impedance} = r_e + jx_e \)

\( \rho = \text{Density of the air} \)

\( c = \text{Velocity of the sound} \)

\( q = \text{Cross-sectional area} \)

\( \xi(\lambda/4) = \text{Equivalent length of the flute with closed embouchure} \]
\[ \xi(\lambda/2) = \text{Equivalent length of the flute with open embouchure} \]
At resonance: $\sinh(\lambda/2) = 0$

For closed embouchure, i.e. $z_e = \infty$:

$$\frac{P_t}{U_i} = z_o \cdot \sinh \Gamma t \frac{\cosh(\Gamma_1 + \Gamma_2 + \Gamma_t)}{\cosh \Gamma_1} + \frac{z_o}{z_e} \cdot \sinh(\Gamma - \Gamma_1)$$

(1)

$$\Gamma_t = \text{arc tanh} \frac{z_t}{z_o}$$

$$\frac{\cosh \Gamma}{\cosh \Gamma_1} + \frac{z_o}{z_e} \cdot \sinh(\Gamma - \Gamma_1) = \sinh(\lambda/2)$$

(2)

At resonance: $\sinh(\lambda/2) = 0$

From Eqs. (2) and (4)

$$z_e = z_o \cdot \frac{\sinh(\Gamma - \Gamma_1)}{-\cosh(\lambda/4)}$$

(5)

or

$$z_e = z_o \cdot \frac{\sinh[\text{arc cosh}(\cosh(\Gamma_1) \cdot \cosh(\lambda/4)) - \Gamma_1]}{-\cosh(\lambda/4)}$$

(6)

Assuming low losses:

$$x_e = z_o \cdot \frac{\sin[\text{arc cos}(\cos(\lambda/4)) - \beta \Gamma_1]}{-\cos\beta(\lambda/4)}$$

(7)

From Eq. (4) at resonance:

$$\cos \beta(\lambda/4) = 0 \quad \frac{\beta \Gamma(\lambda/4)}{\omega} = \frac{\lambda(\lambda/4)}{c} = \frac{2n-1}{4f_n} \quad \text{or} \quad \lambda(\lambda/4) = c \cdot \frac{2n-1}{4f_n}$$

(8)
The equivalent length $l(\lambda/4)$ with the sound source at the embouchure is calculated using Eq. (8) and resonance frequencies measured with a gliding sine wave for closed embouchure $z_e = \infty$, Fig. III-A-1.A. The results are shown in Fig. III-A-2.A. Extrapolation for intermediate values was made by

$$l(\lambda/4)_x = l(\lambda/4)_{f_n} (1 + k \cdot f_{x})^m$$

(9)

The equivalent length of the quarter wave pipe above the embouchure may be calculated from the frequency of the first zero in Fig. III-A-1. This length was found to be 1.82 cm and

$$\beta l_1 = \frac{w}{c}$$

(10)

The reactance for open embouchure and the effective reactance at blowing are calculated for respective resonance frequencies, $\sin \beta \ell(\lambda/2)=0$ by Eq. (7). Values of $\beta \ell(\lambda/4)$ and $\beta l_1$ for corresponding frequencies are calculated from Fig. III-A-2.A and Eq. (10). The results are shown in Fig. III-A-3.A and Fig. III-A-3.B. The reactance for the open embouchure, Fig. III-A-3.A, is simply $x = k \cdot f$. The importance of this fact will be evident later. The effective embouchure reactance, Fig. III-A-3.B, increases monotonically but not linearly with the frequency. The change in the slope of this curve is to a great extent due to the well known fact that the embouchure aperture is more covered by the flutist's lips for high tones than for low tones. This means that a more covered aperture will cause an increased reactance. In an earlier study, the frequencies of blown tones were measured on three modern flutes. Each flute was played by three professional flutists. No systematic difference in frequencies was found. This means that no systematic difference in the effective embouchure reactance exists although the difference in the open embouchure reactance was considerable.

The effective embouchure reactance might be independent of the embouchure dimensions, i.e. the player adjusts his covering to some optimal value independent of the embouchure dimensions. However, this should not be interpreted as indicating that the embouchure design is of no
Fig. III-A-1. Transfer function for a cylindrical flute.
Fig. III-A.2. Equivalent lengths of experimental flutes.
A. Cylindrical head joint
B. Conical head-joint, closed emb. \( z_e = \infty (\lambda/4 \text{ pipe}) \)
C. Conical head-joint, open cmb. \( z_e = \infty (\lambda/2 \text{ pipe}) \)
Fig. III-A-3. Embouchure reactance.
A. Open emb. reactance
B. Eff. reactance at blowing
importance to the ease of blowing. More extensive investigation of the embouchure is required.\

The head-joint perturbation

The Eqs. (1)-(8) are valid for a flute with cylindrical head-joint and cylindrical body. The equation can be applied to a flute with perturbed head-joint if the effective length of the head-joint is known. According to Lord Rayleigh (Theory of Sound, Vol. II, 2nd edition, London 1926, p. 66) and A.H. Benade (3,4) the length corrections for a \( \lambda/4 \) pipe are

\[
\Delta \ell = + \int_0^{\ell} \frac{\Delta s}{s_o} \cos \left( \frac{2(n-1)\pi x}{\lambda} \right) \, dx
\]

and for a \( \lambda/2 \) pipe \( \Delta \ell = - \int_0^{\ell} \frac{\Delta s}{s_o} \cos \left( \frac{2nmx}{\lambda} \right) \, dx \)

By substitution of the wavelength \( \lambda \) for the length \( \ell \):

\[
\Delta \ell = \frac{1}{4} \int_0^{\ell} \frac{\Delta s}{s_o} \cos \left( \frac{3nmx}{\lambda} \right) \, dx, \quad (11)
\]

where \( \Delta s \) is the change in the cross-sectional area and \( s_o \) is the unperturbed cross-sectional area. The + sign pertains to the \( \lambda/4 \) pipe, the - sign to the \( \lambda/2 \) pipe. \( \ell_p \) is the length of the perturbed head-joint. Resonance measurements are now performed in the same way as for the flutes with a cylindrical head-joint. The difference \( \Delta \ell \) between the effective length of the flute with perturbed head-joint \( \ell_p \) and of the corresponding flute with cylindrical head-joint \( \ell_c \) at the same frequency is

\[
\Delta \ell = [\ell_p - \ell_c]
\]

and is shown in Fig. III-A-4.A. In the same way open embouchure measurements were made, Fig. III-A-4.B. From these measurements it is found

* An old Japanese flute (not a recorder) with extremely large embouchure area [\( \sim 4.2 \, \text{cm}^2 \)] has to be blown from the end. The embouchure is the larger open end of the tube \( \Phi = 2.3 \, \text{cm} \) and at one place the rim is cut so as to form a small sharp edge which one blows against. The lower lip must cover most of the embouchure during the blowing, so that the effective embouchure aperture is about the same as of an ordinary flute during blowing. The fundamental frequency of a blown tone was 323.3 Hz and the resonance frequency with open embouchure 360.7 Hz, i.e. a difference of 11.6 %. Similarly derived data for an ordinary flute was 307.9 Hz and 315.1 Hz, i.e. a difference of only 2.3 %. This indicates that the effective embouchure reactance is independent of the embouchure –dimensions (but not of frequency).
Fig. III-A-4. Conical head-joint. Correction of length due to perturbation.
A. $\lambda/4$ pipe measured values
B. $\lambda/2$ " "
C. $\lambda/2$ " calculated "
that the length correction $\Delta l$ is, within measuring accuracy, the same for the $\lambda/4$ and the $\lambda/2$ pipes but with reversed signs. A simple method, called the "tuning-slide method" was tried. It is a method for direct measurement of the length correction without computation. The cylindrical head-joint was provided with a tuning slide and was adjusted so that each corresponding resonance frequency was the same for both flutes. Thin walled tubes with only 0.5 mm thickness were employed in order to minimize the influence of the cavity in the tuning slide. The length correction $\Delta l$ could now be measured by a ruler at the tuning slide. The results from these measurements showed the same values as shown in Fig. III-A-4. The calculated values from Eq. (11) are drawn by a broken line in Fig. III-A-4. C. The agreement between calculated and measured values is reasonably good. The Rayleigh Eq. (11) is only valid if no discontinuities exist. In this case there is a slight discontinuity of the first derivative at the joint between the conical head-joint and the cylindrical tube. Another important fact, not accounted for in the formula, is that the sound source is placed at the embouchure. The solution of the Rayleigh integral for a more complicated head-joint shape is facilitated by means of a computer. A computer program for this purpose has been written so that the influence of different perturbations can conveniently be studied in advance of measurements.

The effective length of the $\lambda/4$ flutes with conical head-joint and closed embouchure ($z_e = \infty$) $\cos \beta \ell (\lambda/4) = 0$, are obtained from Fig. III-A-4. A or can be measured in the same way as by the cylindric head-joint. These effective lengths are shown in Fig. III-A-2. B. The effective lengths for the $\lambda/2$ pipe and $z_e = \infty$ are obtained from Fig. III-A-4. B or from the difference A-B in Fig. III-A-2.

The embouchure reactance for a flute with a perturbed head-joint can be obtained from Eq. (7) by using the curve C in Fig. III-A-2 for the effective length $\ell (\lambda/2) z_e = \infty$, Eq. (8) will then be modified and

$$x_e = z_o \cdot \frac{\sin[\arccos(\cos \beta \ell \cdot \cos \beta \ell (\lambda/2)) - \beta \ell]}{-\cos \beta \ell (\lambda/2)} \] (12)$$

$$\beta \ell (\lambda/2) = [\omega \cdot \frac{\omega (\lambda/2)}{c}] z_e = \infty$$
The necessity of using an auxiliary cylindrical head-joint for measurements on flutes is a disadvantage. It can, however, be seen from Fig. III-A-4 that the length corrections for $\lambda/4$ and $\lambda/2$ pipes (curves A and B) are very nearly the same and $\approx 0$ at high frequencies $f \geq 1700$ Hz. For these measurements a cylindrical tube is attached to the head-joint. Measurements are made with closed and open embouchure.

At resonance frequencies between 1600 and 2000 Hz the values of $\beta\lambda(\lambda/2)$ and $\beta\lambda(\lambda/4)$ are almost equal. $x_e$ is obtained from Eq. (12). The earlier mentioned important fact that $x_e = k \cdot f$ now makes it possible to calculate a curve corresponding to C in Fig. III-A-2, $z_e = \infty$. The length correction from the perturbation is then $\Delta z = \frac{C - B}{2}$.

The cross-sectional area $q$ is not constant along the whole length of the flute and the mean value of $q$ depends on the length of the flute, i.e. on how many holes or keys are open. The characteristic impedance $z_0$ will increase when the flute length decreases but only slightly. The correction is in all cases less than 5%.

From Eq. (12):

$$\frac{x_e}{z_0} = -\tan A = \tan (\pi n - A)$$
$$\frac{x_{e\text{ eff}}}{z_0} = -\tan B = \tan (\pi n - B)$$

The resonance frequencies from Ionophone measurements on an actual flute with open embouchure are $f_{in}$ and at blowing $f_{bln}$

$$f_{in} \approx f_{bln} \text{ then:}$$
$$A = \frac{\omega_{in} \cdot \ell}{c} \text{ and } B = \frac{\omega_{bln} \cdot \ell}{c}$$

The frequencies at blowing are then:

$$f_{bln} = \frac{\frac{x_{e\text{ eff}}}{z_0}}{\pi n - \arctg \frac{x_e}{z_0}} \cdot f_{in}$$

$$= \frac{\frac{x_{e\text{ eff}}}{z_0}}{\pi n - \arctg \frac{x_e}{z_0}} \cdot f_{in}$$

(14)
From Ionophone measurements with open embouchure for each stop or stops of interest, the actual resonance frequencies may be obtained from Eq. (14) if the reactance function for open embouchure and the corresponding effective reactance function are known. It is likely that the curve B in Fig. III-A-3 is representative for $x_{e eff}$ but further tests will be made to verify this proposition.

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