The wolf tone in the cello: Acoustic and holographic studies

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III. MUSICAL ACOUSTICS

A. THE WOLF TONE IN THE CELLO: ACOUSTIC AND HOLOGRAPHIC STUDIES

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Abstract

The frequency spectrum of the wolf tone in the cello has been investigated by Sonograph analysis and computer calculated F. F. T. Odd harmonics of the tone are shown to be split into pairs of vibration whereas even harmonics remain single confirming the suggestion that the wolf tone is caused because of beating of pairs of equally forced vibrations in the string.

Interference holographic studies of the motion of the top plate of the cello show that such pairs of vibration are caused because of strong coupling between string and top plate at the frequency of the first top plate mode. Deflection of top plate under a static force has been measured and has given an absolute value for top plate impedance.

Parameters of the cello have been measured by bowing the instrument at the wolf note and determining the ranges over which it will wolf. Vibrating mass, stiffness and impedance of the top plate thence have been calculated. The wolf note criterion introduced by Schelleng is shown to be a valid one in describing the wolfing characteristics of the cello.

Introduction

The violin family of instruments suffers from a very famous defect, the wolf tone. In many instruments of this family, violin, viola and cello, there is one particular tone elicited by the bow which sounds quite different from the rest, but the wolf tone is most well known, or notorious, in the cello. In general this tone is classified as unmusical and in most instruments is found at the frequency of the first top plate resonance; in celli it is found at about $F_3$ or $F_4$ on the G string, or at the same note on the C string, that is on the two heaviest strings. The wolf tone has been described as a jarring note, an impure and wheesy sound, a note of rapid fluctuation in loudness, and as a cyclic stuttering response to the bow. In some instruments a wolf tone is found at the octave above the first top plate resonance and in some others wolf tones are associated with air resonances of the air cavity. This study is concerned with the well known wolf tone in the cello which occurs at the pitch of the first top plate resonance. Fig. III-A-1 shows the waveform of a wolf tone studied later in this report.

In previous studies the occurrence of the wolf note in the cello has been shown by direct frequency analysis to be caused by the beating of two equally forced vibrations in the string\(^1\). The fundamental sinusoidal component of vibration of the string has been shown to be split at the wolf note into a pair of sinusoidal oscillations separated by an interval equal to that of the stuttering frequency of the wolf note. In most celli higher partials are often also split into pairs, their separation being related to that of the fundamental pair.

This report describes experiments undertaken in this laboratory while on Study Leave from the University of St. Andrews. The work was carried out in order to gather more data about (a) the frequency content of the wolf tone, (b) the mechanical action of the cello in the region of the wolf note, and (c) about the parameters of the cello, in particular about those which have been suggested as useful in describing the wolfing behaviour of this instrument. Such measurements were made using equipment not available in St. Andrews but forming the complement of the Department of Speech Communication in Stockholm. In particular the technique of interference holography, which can be used for recording the modes of vibrating surfaces, has been applied to the cello for the first time.

There have been previous investigations of the cello\(^2,3,4\). The explanation frequently accepted for the physical occurrence of the wolf tone is that of Raman, that there is a cyclic alternation between vibrations at fundamental and octave of the string. In a detailed analysis of the action of the instruments of the violin family in terms of elementary electrical circuit theory Schelleng\(^5\) has considered the wolf tone in the cello. By transforming the cello at the wolf tone into an equivalent electrical circuit comprising a resonant transmission line (the string) coupled through a transformer (the bridge) to a series resonant circuit with resistive losses (the body), Schelleng argues that as the two parts have nearly the same frequency at the wolf tone they will act in the same well known fashion as other coupled electrical resonant circuits. With excitation of the equivalent circuit by a generator (the bow) placed in the transmission line there should be three frequencies at which the reactive part of the impedance presented to the generator is zero, and that at two of these steady oscillations are possible, each of which will be equally forced.
It is only by exciting these two together that the bow can elicit a note at this particular position on the stopped string, usually $F_8$ on the G string. By a consideration of the harmonic content of the string motion Schelleng suggests that not only should the fundamental vibration of the string be split but also all odd harmonics. Secondly, a wolf note criterion is introduced and it is suggested that ratio of the characteristic impedance of the string to that of the body together with the Q of the body of the instrument at the first top plate resonance gives a measure of the propensity of an instrument to wolf.

On the basis of these parameters the electrical analogue predicts the heavier strings of an instrument to be in greater danger of wolfing, and that the cello is more likely to have a wolf tone than viola or violin, predictions both in accord with players' experience. Measurements made in these studies have been aimed at obtaining a better frequency analysis of the wolf tone and testing the adequacy of the wolf note criterion. The two celli which have been used in these studies are described in Table III-A-I and the measurements undertaken on each indicated.

**Table III-A-I. Descriptions of Celli and Experiments**

| Cello 1 | Made and owned by Herr Stig Källhager, Stockholm  
|        | Date: 1974  
|        | Steel Covered  
|        | Strings by Prim  
|        | Tested in-the-White  
|        | Frequency analysis by  
|        | (a) Sonograph, and  
|        | (b) FFT.  
|        | (c) Measurements of normal mode frequencies by holography, and  
|        | (d) Air tones by ionophone.  
| Cello 2 | Maker - unknown  
|        | Owner - I.M. Firth  
|        | Age > 40 years  
|        | Flexocor, heavy, Steel covered  
|        | Strings by Pirastro  
|        | Frequency analysis by  
|        | (a) Sonograph.  
|        | (b) Measurements of normal frequencies by holography, and  
|        | (c) Air tones by ionophone.  
|        | (d) Mechanical action of top plate at wolf tone.  
|        | (e) Measurement of parameters of cello, and  
|        | (f) Evaluation of wolf note criterion.  

Frequency Analysis: Cello 1

The most steady and easily played wolf in this cello was found to occur when the sound post side of the bridge was loaded with a metal weight of 20 g. Bowed in an anechoic chamber with this adjustment and C, D, A, strings open, but damped and the G string stopped to 39.5 cm the wolf of Fig. III-A-1 was obtained. In order to investigate the fine structure of the harmonic content of the wolf tone the tape recording was replayed at 8 and 16 times recording speed through a Sonograph equipment so that the normal bandwidth of the Sonograph was effectively reduced to 5 and \( \frac{21}{2} \) Hz respectively. Sonographs of the wolf tone are shown in Figs. III-A-2 and III-A-3. Fig. III-A-2 shows that the fundamental and 3rd harmonic of the tone are split, but that the 2nd harmonic remains single. Fig. III-A-3 shows the splitting of fundamental in greater detail. Although only three harmonics can be investigated with sufficient resolution by this method it is noted that only the odd harmonics become doublets.

Further information can be obtained about the detailed structure of the harmonic content by considering the instantaneous spectrum at different times during a repetition period of the wolf, especially at the times of beat maximum and beat minimum, see Fig. III-A-1 and refer to Schelleng. The instantaneous spectrum for time intervals of 20 ms were computed with the tape recording replayed at half speed (hence the intervals are equivalent to 10 ms in real time) by a Fast Fourier Transform computation using a standard programme for the Control Data Computer of the Department of Speech Communication. Instantaneous spectra at two adjacent beat maximum and beat minimum are shown in Fig. III-A-4, and for a series of time intervals spanning a few such periods the spectra are shown in perspective view in Fig. III-A-5. The repetition rate of the wolf beat is here 10 Hz.

Fig. III-A-4 shows that between beat maximum and minimum there are significant differences in the intensity of harmonics numbered 1, 5, 7, 9, 11 and smaller differences between those numbered 2, 4, 6, 8, 10, 12. Even harmonics are less altered in intensity than odd between the time of beat maximum and minimum, and odd harmonics decrease in intensity at the beat minimum. According to Schelleng’s explanation of the wolf tone all harmonics should be present at beat maximum, whereas at beat minimum odd harmonics should be absent. The results of Fig. III-A-4 follow qualitatively this pattern.
Fig. III-A-1. Waveform of wolftone of cello 1. Part of tone analysed by sonograph.

Fig. III-A-2. Sonograph of wolftone of cello 1 on G string. Effective bandwidth of analyser = 5 Hz.

Fig. III-A-3. Sonograph of wolftone of Fig. III-A-2 with analyser bandwidth = $2 \frac{1}{2}$ Hz.
Fig. III-A-4. Computer analysis of (a) beat maximum and (b) beat minimum of wolftone of cello 1 (same as Fig. III-A-1) by Fast Fourier Transform calculation. (c) Harmonic content of beat maximum (X) and beat minimum (■) versus harmonic number.
Fig. III-A-5. Computer calculated Fast Fourier Transform analysis for three periods of the wolftone in cello 1. Harmonics 1, 3 show most regular fluctuations, and 2, 6, 8, 10 greatest constancy over a period of wolf.
Over three repetition cycles of the wolf, Fig. III-A-5 indicates that harmonics numbered 2, 6, 8, 10 show the greatest consistency in intensity, and those numbered 1 and 3 the most regular fluctuations. In this particular wolf, produced in cello 1, it appears that the 1st and 3rd harmonics determine the periodicity of the wolf; they show the same repetition rate (that of the intensity fluctuation of the tone) and appear to change in phase with one another. It is to be stressed that investigating the wolf through measurements of the fine structure of its harmonic content (i.e. searching for pairs of vibrations), or through calculating the instantaneous spectrum at different times during the repetition period of the wolf tone (i.e. searching for differences in harmonic content) are analogous measurements. They both lead to the same conclusion, that the perceived wolf tone results from the creation in the string by the bow of pairs of equally forced vibrations. The components of each pair act together to give an instantaneous frequency which is acceptable to the string, and each component remains substantially unchanged through a whole beat cycle. These investigations indicate further that for an easily elicited wolf, such as that of cello 1, it is the odd harmonics which are split into pairs of vibrations whereas the even remain practically unchanged.

Holographic studies were made to ascertain the normal plate modes of top and back plate of cello 1, and acoustical measurements using the STL-Ionophone were made to determine resonance frequencies of air modes of the cello cavity (6) and these are listed in Table III-A-11. It is noted that the frequencies of T1, B1, A1 are all close to the wolf tone region but further holographic studies with plate tone testing and string excitation testing, similar to those reported below for cello 2, showed that the wolfing action of cello 1 occurs because of coupling between string and top plate at the frequency of the T1 mode.

**Frequency Analysis: Cello 2**

Measurements were made on the most easily bowed and most regular wolf tone which could be obtained on this cello. Such a wolf was obtained by loading the plates of the cello with 34 g added to the back plate at the position of the sound post and 5.6 g added to the top plate between f-hole and foot of the bridge on the bass bar side. These additions are justified
Table III-A-II. Normal Mode Frequencies of Plates and Air of Celli

<table>
<thead>
<tr>
<th>Mode</th>
<th>Cello 1 Frequency, Hz</th>
<th>Q</th>
<th>Cello 2 Frequency, Hz</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top plate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ti</td>
<td>165</td>
<td>23</td>
<td>185</td>
<td>37</td>
</tr>
<tr>
<td>T2</td>
<td>290</td>
<td></td>
<td>292</td>
<td></td>
</tr>
<tr>
<td>T3</td>
<td>380</td>
<td></td>
<td>330</td>
<td></td>
</tr>
<tr>
<td>T4</td>
<td></td>
<td></td>
<td>440</td>
<td></td>
</tr>
<tr>
<td>Back plate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>178</td>
<td></td>
<td>205</td>
<td></td>
</tr>
<tr>
<td>B2</td>
<td>295</td>
<td></td>
<td>283</td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>320</td>
<td></td>
<td>308</td>
<td></td>
</tr>
<tr>
<td>B4</td>
<td>390</td>
<td></td>
<td>390</td>
<td></td>
</tr>
<tr>
<td>B5</td>
<td></td>
<td></td>
<td>460</td>
<td></td>
</tr>
<tr>
<td>B6</td>
<td></td>
<td></td>
<td>595</td>
<td></td>
</tr>
<tr>
<td>Air cavity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A0 (Helmholtz)</td>
<td>90</td>
<td></td>
<td>104</td>
<td>17</td>
</tr>
<tr>
<td>A1 (1st Air</td>
<td>188</td>
<td>47</td>
<td>203</td>
<td>17</td>
</tr>
</tbody>
</table>

later. The Sonograph of Fig. III-A-6 shows the detailed structure of the fundamental of the wolf tone, the analysis being made on a tape recording of the wolf tone speeded up by 16 times by Sonograph. The wolf tone has a complicated frequency content at fundamental, four strong peaks being present close to 187 Hz. Other Sonographs showed the 2nd harmonic to split into two components. This wolf has clearly a different character to that of cello 1.

Action of the Top Plate at the Wolf Tone

The motion of the top plate can be inferred from two types of measurement: plate excitation or string excitation testing with microphone pickup, and time average interference holography of the top plate vibrations. Measurements using the first technique show that with plate tone testing, in which the plate is driven near the f-hole on the bass bar side with a signal swept in frequency, the usually single peak of the first top plate mode (T1) changes to a double peak when the G string is stopped in the range of the wolf tone, Fig. III-A-7. The mutation of sound emission in this type of test at the wolf tone has been recorded previously (1) in another cello and shows the extensive coupling between body and string. The recorded dip corresponds to a zero of the input admittance at the driving point and occurs in this type of test because the string is absorbing energy at this frequency.
Fig. III-A-6. Sonograph of wolftone of cello 2 on G string. (Top plate + 5.6 g, back plate + 34 g.) Analyzer bandwidth = 2 Hz. Intensity section shown.

Fig. III-A-7. Plate excitation testing of cello 2. CDA strings open, but damped, with G string stopped in range of wolf. (a) Single peak at 192 Hz with G string damped becomes (b) a double peak (183, 196 Hz) with sharp dip in SPL at 191 Hz.
When the string is vibrated by placing a horseshoe magnet about the G string at the point of bowing and passing an a.c. current, swept in frequency, through the metal string emission of sound can likewise be measured in the range of the wolf tone and it is found that sound emission takes place over a broad frequency band showing two peaks, but no zero between. The acoustical measurements of these two means of excitation can be compared with results which would be obtained by measuring the secondary current in the equivalent electrical circuit proposed by Schelleng[5] when the generator is placed first in the secondary circuit (the body) and second, in the primary (the string). Diagrams of Fig. III-A-8 indicate the action of the equivalent circuit; here current \( \propto \) velocity of top plate \( \propto \) sound pressure output.

The action of the top plate can be studied by using time average interference holography when the instrument is plate excitation, or string excitation tested at the wolf. By the use of an additional speckle interferometer in the holography equipment, the vibrational frequency can be adjusted easily to peaks or dip in plate tone testing, for as Fig. III-A-9 demonstrates the peaks and dip measured acoustically correspond to large motion or no motion of the plate respectively. The peaks correspond to the first top plate mode, \( T_1 \), see Fig. III-A-11 and demonstrate that only a single mode of vibration of the top plate takes part in the production of the wolf tone. Damping the G string, which is stopped to a length sounding at 185 Hz the frequency of the dip, reinstates the vibration of this mode to its normal value and the vibration of the plate then only shows a single peak. With string excitation testing the top plate mode \( T_1 \) remains present over a wide frequency range 180 to 195 Hz, that is, there is no appreciable dip. Fig. III-A-10 shows the holographic record over this frequency range; the motion of the plate is in accord with the broad flat frequency sound output which is recorded in string excitation testing. That the fringe pattern obtained by holography is that of the mode \( T_1 \) confirms the coupling between string and this mode of vibration. For reference, the frequencies of the normal modes of vibration of cello 2 are given in Table III-A-II; the frequencies of the modes \( T_1, B_1, A_1 \) are significantly different in this cello. Holographic and acoustical studies on both celli indicate, therefore, that it is the coupling between string and the mode \( T_1 \) which is important for the production of the wolf in the cello.
Fig. III-A-8. Elements of the equivalent electrical circuit of the cello at the wolf note. (a) plate excitation testing, (b) string excitation testing. Velocity of plate, measured as SPL at microphone shown diagramatically according to electrical circuit theory for coupled resonant circuits.
Fig. III-A-9. Reconstructed images of the top plate of the cello by time average interference holography when plate excitation tested in the wolf tone region. CDA strings open, but damped, with G string in range of wolf.

(a) Single peak at 185 Hz with G string damped becomes (b) a double peak (172, 201 Hz) with sharp dip at 185 Hz showing little vibration (see Fig. III-A-7).

(Top plate + 5.6 g, Back plate + 34 g.)
Fig. III-A-10. Vibration of top plate of cello 2 when string excitation tested. CDA strings open, but damped, with G string in range of wolf and excited with a current through string and magnet at bowing position. Series of reconstructed images in the range of the wolf. (Top plate + 5.6 g, Back plate + 34 g.)
Fig. III-A-11. First top plate mode $T_1$ of cello 2 with various additional masses placed at f-hole on bass bar side. (Back plate + 34 g.)
Top and Back Plate Modes of Cello

In these studies it has been found useful to add masses to top and back plate of the cello in order to (a) adjust the instrument so that a regularly pulsating wolf could be easily elicited by bowing, and (b) so that measurements could be made of effective vibrating mass and stiffness of the top plate. In view of the importance of the mode T1 for the production of a wolf tone a series of holographic measurements were carried out to ascertain the influence on vibrational patterns of the first few modes of the cello and on their vibrational frequencies. Masses were added to the top plate between f-hole and bridge foot on the bass bar side (the position of maximum displacement of the mode T1) and on the back plate directly above the sound post. The best wolf was obtained in cello 2 with the addition of 5.6 g to the top and 34 g to the back.

The records to Fig. III-A-11 show (a) that a mass added to the back plate at the sound post enhances and sharpens the top mode T1 through making the bridge foot at the sound post side less moveable, and (b) the subsequent addition of mass to the top plate does not change the vibrational pattern of this mode, there being only a reduction in vibrational amplitude with increased mass. It is concluded that a point mass in this position on the top plate adds to its effective vibrating mass changing its frequency accordingly. Frequency measurement in this series of experiments was accurate to ±5 Hz so that the vibrating mass could not be found from these measurements.

These mass added to the top plate or back plate affects higher modes of the top plate to a lesser extent. In general it has been found that the mass added to back plate (34 g) sharpens modes but does not change vibrational pattern or frequency. Mass added to top plate changes the frequency of the modes T2, T3, T4 to a very small extent, for it is fixed to the plate at inactive areas for these modes. Fig. III-A-11 demonstrates the effect of added mass for these higher modes.

Investigations were further carried out on the back plate. The modes of vibration were ascertained by driving the back plate at positions of high vibrational amplitude (antinodal regions) and modes were explored firstly with the speckle interferometer and photographed using time average interference holography. The effect of the additional mass of 34 g placed on the back plate at the position of the sound post was assessed, Fig. III-A-13. The mass produced little change in vibrational frequency, even
for the modes B1 and B2 for which it is at a position of considerable motion, and only for B1 was the mode pattern changed. At other back modes, the mass again served to sharpen the modes making them more distinct (that is, enhancing the definition of the mode, or in other words reducing the inactive areas between regions of motion).

From these investigations, Figs. III-A-11, III-A-12, III-A-13, three conclusions can be made: (a) Mass added at the position of the sound post on the back plate (and this means perhaps also to the front plate at the sound post) does not influence mode patterns or frequencies to any significant extent but serves to sharpen the definition of the lowest modes. In this study such a mass has been used to obtain a 'better' wolf. (b) A point mass added to the top plate between f-hole and bridge foot on the bass bar side adds to the vibrating mass of the first top plate mode and can be used therefore to mass load this plate (i.e. lower its frequency) without altering the vibrational character of this mode. Such a point load is a useful adjunct in measuring certain parameters of the cello. (c) These experiments indicate that the vibrational patterns of modes is dictated more by boundary conditions imposed at the edges of the plates where they are glued to the ribs, and by the position of the sound post than by the distribution of mass (and perhaps stiffness) of the wood of the plates. Further evidence for this last conclusion comes from Table III-A-II in which the Q's of similar modes of vibration of wood and air are noted for two radically different celli; one in-the-white with new glued joints, and the other, old and with very poor joints, not only loose but leaky: the Q of T1 rises with age of joint whereas that of A1 falls.

Parameters of Cello: Wolf Note Criterion, Static Deflection and Absolute Measure of Body Impedance

In order to evaluate the usefulness of the wolf note criterion introduced by Schelleng it is necessary to measure parameters of the cello, especially impedance of string and body. By applying a static force to the bridge of the cello a static deformation results. By double exposure holography the deflection of the top plate can be measured (7). Fig. III-A-14 shows a set of reconstructed images of cello 2 which record static deflection of the plates as a set of fringes. As this method of recording is most sensitive to motion perpendicular to the top plate it can
Fig. III-A-12. Top plate modes T2, T3, T4 of cello 2 with different masses added to top and back plates.
Fig. III-A-13. Back plate modes B1, B2, B3 of cello 2 with different masses added to back plate.
Fig. III-A-14. Deformation of plates of cello 2 under a static force applied at the G string notch on bridge.
(a) Top plate deformation with 50 g applied transverse to plate to the right looking at the figure.
(b) Top plate deformation with 50 g applied transverse to plate to left, and
(c) Back plate deformation with 50 g applied as in (a).
be assumed that one fringe corresponds to a deflection of the plate of 
\( \frac{1}{2} \lambda \) laser = \( \frac{1}{2} \times 640 \) mm. The fringe pattern of Fig. III-A-14 corresponds closely to the vibrational pattern of mode T1, so that this method of static deflection affords a means of measuring in absolute terms the impedance of the top plate at the G string notch on the bridge when vibrating in mode T1. The impedance can be calculated from (applied force/velocity of centre of vibrating mass) which is here equal to (transverse applied force/displacement of centre of mass in static deformation \( \times \) angular frequency of mode T1) and has a value of \( 8.7 \times 10^4 \) cgs ohms.

From Fig. III-A-11 the centre of mass is judged to be very close to the f-hole on the bass bar side.

Of interest in Fig. III-A-14 are the recordings of the static deflection of back plate for a force applied as before to the bridge. By counting fringes it is seen that the back plate has a maximum deflection of only 1/4 of that of the top plate for the same applied force. Static deflection is maximum at the position of soundpost, as expected, and is about equal for forces applied in either transverse direction to top plate. A further time average interference hologram made of the back plate when the top plate was driven at the frequency of the T1 mode showed the whole back plate to be stationary, i.e. no fringes were observed on the back. The transconductance of soundpost at the frequency of T1 in this cello appears to be very small.

**Effective Vibrating Mass of Top Plate in Mode T1**

The equivalent series stiffness and mass, \( S \) and \( M \), can be measured by noting the frequency of the mode T1 with different masses, \( m \), added to the top plate. As already described this standard method\(^{(5)}\) appears to be justified in the present experiments because the T1 mode of vibration is not greatly altered by their addition. The wolf tone can be used as an indicator of T1. Since \( w = \left[ \frac{S}{M + m} \right]^{\frac{1}{2}} \), it follows by differentiation that at \( m = 0 \)

\[
M = -\frac{1}{3} f/(df/dm) \quad \text{and} \quad S = -2\pi^2 f^3/(df/dm)
\]

from which the body impedance \( \sqrt{SM} \) can be calculated.
The wolf tone can be used as an indicator of the frequency of the mode T1; the cello is bowed at a fixed position on the string and the range of string length over which a wolf can be obtained measured. Both the length of the G string and the frequency of the string at the extremities of the range can be noted for added mass, m. Fig. III-A-15 shows a plot of the frequencies of the extremities of the range of the wolf tone versus applied mass, m. The peak frequency of the mode T1 can be considered as centred in the range of wolfing so that \( \frac{df}{dm} \) at \( m = 0 \) can be obtained. In cello 2 \( \frac{df}{dm} \) \( m=0 = -1.0 \text{ Hz/g} \) at the frequency of T1, 189 Hz, so that \( M = 94.0 \text{ g} \), \( S = 1.3 \times 10^9 \text{ dyn cm} \) and the impedance of the body \( \sqrt{SM} = 1.11 \times 10^5 \text{ cgs } \Omega \). From acoustical measurements the Q of this mode is 37 (Table III-A-II). This value of impedance is to be compared with that measured absolutely by static deflection, \( 8.7 \times 10^4 \text{ cgs } \Omega \).

For the C G D strings on cello 2 it has been found through bowing that there exists a range of stopped string length in which a wolf can be elicited. Ranges are changed by altering parameters of the cello. The parameters of importance are:

(a) Impedance of top plate. This can be altered by attaching a mass to the top plate (frequency of T1 is lowered);

(b) Impedance of string. Altered by (i) using a string of different linear density, or by (ii) using a different tension in a particular string (the open string pitch can be lowered). The full range on any string can be measured in terms of the range of length of string over which wolfing occurs, the impedance of plate, and the impedance of the string; or more practically, the range of length of the string, the mass added to the top plate and the frequency to which the open string is tuned giving a three dimensional plot of the range.

In order to investigate the nature of wolfing ranges three exploratory measurements were carried out. So that measurements were as reproducible as possible only one string was on the bridge, the one being bowed to obtain the wolf. This string was positioned in the G string notch at nut and bridge and was bowed straight across the top plate at a distance of 3.5 cm from the bridge. The ranges in which the cello could be made to wolf were investigated (a) with a G Pirastro Flexocor string of measured linear density \( \mu_G = 6.8 \times 10^{-2} \text{ g cm}^{-1} \) with an additional mass of 31.1 g on the top plate and 34 g on the back for various tensions of the string, (b) as in (a) but with a mass of 2 g fixed to the top plate, and (c) with a D Pirastro Flexocor string of measured linear density \( \mu_D = 4.6 \times 10^{-2} \text{ g cm}^{-1} \) with a mass of 2 g on top plate and 34 g on
FREQUENCY

and the gradient at \( m = 0 \) drawn (back plate + \( \frac{3\pi}{8} \)).

The mean frequency of the range has been plotted against mass, \( m \), added to top plate at base bar.

Frequency of extreme positions on finger board.

Figure III-4-15. Range of the wolf tone in cello 2, C string showing.

\[ \text{Added mass (m)} \]

\[ \begin{array}{cccccccc}
10 & 20 & 30 & 40 & 50 & 60 & 70 & 80 \\
\end{array} \]

\[ \text{FREQUENCY} \]

\[ \frac{dm}{dt} = 1.0 \]

\[ \text{mean} \]
The wolfing ranges are shown in Fig. III-A-16. In each case there is a large range of string length over which the instrument can be made to wolf easily when the tension is normal, but as the tension of the string is lessened this range reduces to become eventually a single length of the string. With lessened tension there is increased difficulty in producing the wolf by bowing.

The results of Fig. III-A-16 suggest that it might be possible to compare the parameters which have been suggested as describing the wolfing characteristics of an instrument at different body and string impedances. The points on the graph at which the wolfing ranges collapse to a single value of string length might represent equality of wolfing characteristics - the position at which the cello just wolves - and parameters describing these points might be compared. In particular, the wolf note criterion introduced by Schelleng should be equivalent at these points and should have a value which represents the transition from 'safe from wolfing' to 'danger from wolfing' (see ref.\(^{(5)}\), Fig. 9).

The just wolfing points of Fig. III-A-16 can be used to calculate (a) the effective vibrating mass of the\(T_1\) mode, and (b) the ratio of linear densities of G and D strings, quantities which have been measured in other ways. Presume that the stiffness, \(S\), of the top plate remains constant over the range of Fig. III-A-16.

Firstly, consider the just wolfing points for the G string with the additional masses \(m_1 = 31.1\) g, \(m_2 = 2\) g. At these points presume that the ratio of characteristic impedances \(\sqrt{T_1 / SM}\) are equal,

\[
\sqrt{\frac{T_1 \mu G}{S(M+m_1)}} = \sqrt{\frac{T_2 \mu G}{S(M+m_2)}}
\]

As the string tension \(T\) is related to the open string length \(l_0\) and its tuned frequency \(f_0\) by \(\sqrt{T/\mu} = 2\) \(1_{0}{f_0}\)

\[
\therefore \frac{f_{01}}{f_{02}} = \sqrt{\frac{(M+m_1)}{(M+m_2)}},
\]

\[
\therefore \frac{(M+31.1)}{(M+2)} = \frac{74.5}{64.5},
\]

\[
\therefore M = 94.5 \text{ g},
\]

which is in agreement with the value obtained from the data of Fig. III-A-15.
Plate with a string at sound post. Just rolling points are designated T.
With Z & added to top plate, and (c) C string with Z, I & added to top plate. Back
(Fig. III-A-16. Range of roll tone on cello. Z for (a) C string with Z & added to top plate, (b) G string.

LENGTH OF STRING, cm

OPEN STRING TUNING, Hz.

150 140 130 120 110 100 90 80 70 60 50

20

30

40

50

60

LENGTH OF STRING, cm

Fig. III-A-16. Range of roll tone on cello. Z for (a) C string with Z & added to top plate, (b) G string.

LENGTH OF STRING, cm

20

30

40

50

60

Fig. III-A-16. Range of roll tone on cello. Z for (a) C string with Z & added to top plate, (b) G string.
Secondly, compare the ratio of characteristic impedances at the just wolfing points of the G and D strings with additional mass
\[ m = 2 \text{ g}, \]
\[ \sqrt{\frac{T_{G}}{S} \mu_{G}} = \sqrt{\frac{T_{D}}{S} \mu_{D}}, \]
so that
\[ \frac{\mu_{D}}{\mu_{G}} = \frac{f_{oD}}{f_{oG}} = \frac{64.5}{89.0} = 0.72. \]

From direct measurements of the weight and length of these strings
\[ \frac{\mu_{D}}{\mu_{G}} = 0.68. \]
It appears therefore, that (a) the wolf note criterion is relevant to the description of the occurrence of a wolf in an instrument, and that (b) the just wolfing points of Fig. III-A-16 represent an equality of strength in the wolf tones, at which, for one instrument, are the same.

The impedance of the strings at the points of just wolfing with 2 g added to the top plate are for the
\[ \text{G string } \quad \sqrt{T_{G}} = 21 \mu_{G} f_{oG} = 626 \, \Omega, \]
and for the
\[ \text{D string } \quad \sqrt{T_{D}} = 21 \mu_{D} f_{oD} = 581 \, \Omega. \]
An average string impedance at just wolfing for cello 2 is 603 \, \Omega and hence the ratio of characteristic impedances at this adjustment has a value
\[ \sqrt{\frac{T_{G}}{SM}} = \frac{603}{1.1 \times 10^{5}} = 5.5 \times 10^{-3}. \]
Together with the measured Q of 37 for the top plate mode T1 this ratio places this cello on the dividing line between the safe and dangerous regions as calculated by Schelleng (ref. (5), Fig. 9).

Conclusion

The application of time average interference holography together with speckle holography is feasible to instruments the size of the cello using an optical bench of modest dimensions and a laser of low power.
Apart from observing the normal modes of vibration of an instrument, the method is admirably suited to investigating particular aspects of behaviour (or, in this study, misbehaviour) of an instrument over a narrow frequency range.

Double exposure holography can be used to obtain an absolute value of the input impedance of the body of an instrument at its first top plate made.

The addition of masses to the plates of an instrument at selected positions does not radically disturb the lowest modes of vibration, and, in particular, does not alter the mode shape of the first top plate mode.

The wolf tone is caused by splitting of harmonic vibrations of the string into pairs. In simple, easily played, wolf tones odd harmonics are split, whereas even harmonics are not altered significantly in a period of the wolf. Such splitting occurs because of coupling between the fundamental vibration of a stopped string of the cello and its first top plate mode T1.

The wolf note criterion introduced by Schelleng is relevant to describing the wolf note. Its value for an instrument together with the Q of the T1 mode determine whether an instrument will wolf. Instruments can be adjusted to a just wolfing condition by change of tension of strings, or vibrating mass of top plate. Such adjustments are useful to measurements of parameters of instruments.

Parameters of celli can be obtained by simple measurements in which bowing and listening are the important experimental techniques.

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