Dept. for Speech, Music and Hearing
Quarterly Progress and Status Report

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KTH Computer Science and Communication

journal: TMH-QPSR
volume: 45
number: 1
year: 2003
pages: 047-059

http://www.speech.kth.se/qpsr

Musician's and computer's tone inter-onset-interval in Mozart's Piano Sonata K 332, 2nd mvt, bar 1-20

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Abstract

The Director Musices generative grammar of music performance is a system of context dependent rules that automatically introduces expressive deviation in performances of input score files. A number of these rules concern timing. In this investigation, the ability of such rules to reproduce a professional pianist's timing deviations from nominal note inter-onset-intervals is examined. Rules affecting tone inter-onset-intervals were first tested one by one for the various sections of the excerpt, and then in combinations. Results were evaluated in terms of the correlation between the deviations made by the pianist and by the rule system. It is found that rules reflecting the phrase structure produced high correlations in some sections. On the other hand, some rules failed to produce significant correlation with the pianist's deviations, and thus seemed irrelevant to the particular performance analysed. It is concluded that phrasing was a prominent principle in this performance and that rule combinations have to change between sections in order to match this pianist's deviations.

Introduction

Our research in music performance started in the 1970s as a co-operation with the late musician Lars Frydén. Our research strategy was first analysis-by-synthesis only. In this way, a number of simple rules were developed, that Anders Friberg later converted into the Director Musices performance grammar, henceforth DM (Friberg, 1995a).

DM rules contain two elements. The *context* defines in what context the rule should be applied. The *quantity* parameter states how great effects it should produce in the performance. For example, the target notes of the Double Duration rule are pairs of tones, the first of which has twice the inter-onset-interval (henceforth IOI) of the second. In this context, some duration is subtracted from the longer note and added to the subsequent shorter note. The quantity parameter k determines how much duration should be transferred. By choosing different quantity values, differing performances of the same piece are obtained.

More recently we have applied also the analysis-by-measurement strategy for the further development of the system. Thus Friberg (1995b) made successful attempts to match the tone IOIs in three pianists' renderings of Robert Schumann's *Träumerei* by tuning DM rules.

Some characteristics of DM appear to shed some light on the basic aspects of music communication. The rules apparently serve different purposes, such as grouping, differentiation, and emphasis. The grouping rules mark which tones belong together and where the structural boundaries are. The differentiation rules increase the differences between tone categories such as pitch classes, intervals, and note values. The emphasis rules emphasise unexpected notes.

Interestingly, these three purposes seem relevant also to speech. Thus, grouping would be the main principle underlying the phenomenon of final lengthening in speech prosody, i.e., the slowing down of the syllable rate towards the end of a phrase. Similarly, the differentiation principle, seems capable of explaining certain phenomena in phonetics, where it has been termed auditory enhancement (Diehl, 1991); for example, the difference in vowel duration between long and short vowels is enhanced by a difference in formant frequencies in some languages, e.g., Swedish. These considerations suggest that the principles underlying the expressive deviations in music performance are important from a perceptual/cognitive point of view.

The DM system has been found to be capable also of adding various emotional colours to a performance (Bresin & Friberg, 2000; Juslin & al., 2002). This result was achieved by varying the quantity parameter of various rules. For example, it was demonstrated that the performance of two examples could sound happy, solemn, angry, sad, tender or scared, depending on the quantity chosen for the various performance rules.

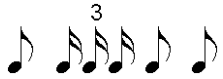
The DM system has been developed on the basis of effects generated on a limited number of music examples. This method obviously has important limitations. If a combination of rules improves the musical quality of the performance of a given example, the only conclusion is, strictly speaking, that the combination could be successfully applied to that example, leaving the general applicability of the rule an open question. In this investigation, we ask how closely our present version of the DM system can match a given professional performance of a piece that was not used in the development of the system.

Method

The basic idea was to compare the real pianist's performance with different performances produced by the DM system (version 2.4). The example used was bars 1-20 in the second movement of Mozart's Piano Sonata F major, K 332. As departures from nominal IOI are highly relevant to musical expressivity, analysis was limited to this parameter.

Gerhard Widmer, Vienna, kindly supplied data of a professional pianist's rendering of this excerpt. The pianist played on a Bösendorfer SE290 computer-monitored concert grand piano provided with MIDI output. The data comprised score time in beats, MIDI-note numbers, inter-onset-interval in ms, played duration in ms, sounded duration in ms and dynamics in MIDI velocity.

A score file of the MUS format that is used in the DM system was derived from the MIDI version of the excerpt that was downloaded from the Internet. This MIDI version, in which note values were twice as long as in the original score, was loaded into the DM system and then converted into a MUS file. The MUS file thus obtained was then edited such that correct relations between note values were obtained. The first grace notes in bars 7 and 16 were translated into 1/32rd notes and the sequence of four sixteenth notes interspersed by three grace notes as a sequence of six sixteenth notes. Following theory of performance practise (Ferguson, 1975, p 122), the *gruppetto* notes in bars 1, 2, 5, 6, and 8 were represented by an 8th note followed by three triplet 16th notes and another 8th note:



After analysis of harmonic progressions and musical structure, the MUS file was provided with chord symbols and phrase markers at structural boundaries. Two hierarchical levels were used for the phrase markers. In addition, phrase-start and phrase-end markers were introduced at a third, lowest hierarchical level, at the beginning and end of each slur sign in the Urtext score (G. Henzle Verlag, München). However, the note appearing on the beat note following the slur sign was interpreted as the phrase-end note, unless this note initiated another slurred group of tones. With regard to the *gruppetti* in bars 1, 2, 5, 6, and 8, the note appearing on the beat note following the grace notes were interpreted as the end note of these slurs. The phrase markers are shown in Figure 1.

The testing was carried out for the top voice only, since the lower voices serve an accompaniment function mainly. This voice was processed with DM. As the analysis was limited to departures from nominal IOI, only rules affecting this performance parameter were tested. These rules are listed in Table 1 together with a description of their effects. The resulting IOIs were then transported to an Excel file, using DM's Tools/Exporter command.

The results were evaluated by calculating the correlation between the pianist's departures and those produced by DM, with the DM deviations from nominal IOI, in ms, as the independent variable. Thereby, special care was taken to ensure that the notes compared were nominally identical. Mainly because of different realisations of ornament notes, certain tones were excluded from the comparison.

For the comparison, the piece was divided into 16 sections according to our interpretation of the musical structure (Table 2).

Results

In an initial run, the effect of each rule affecting IOI, adjusted to the quantity of k=2.0, was tested one by one. The agreement between the pianist

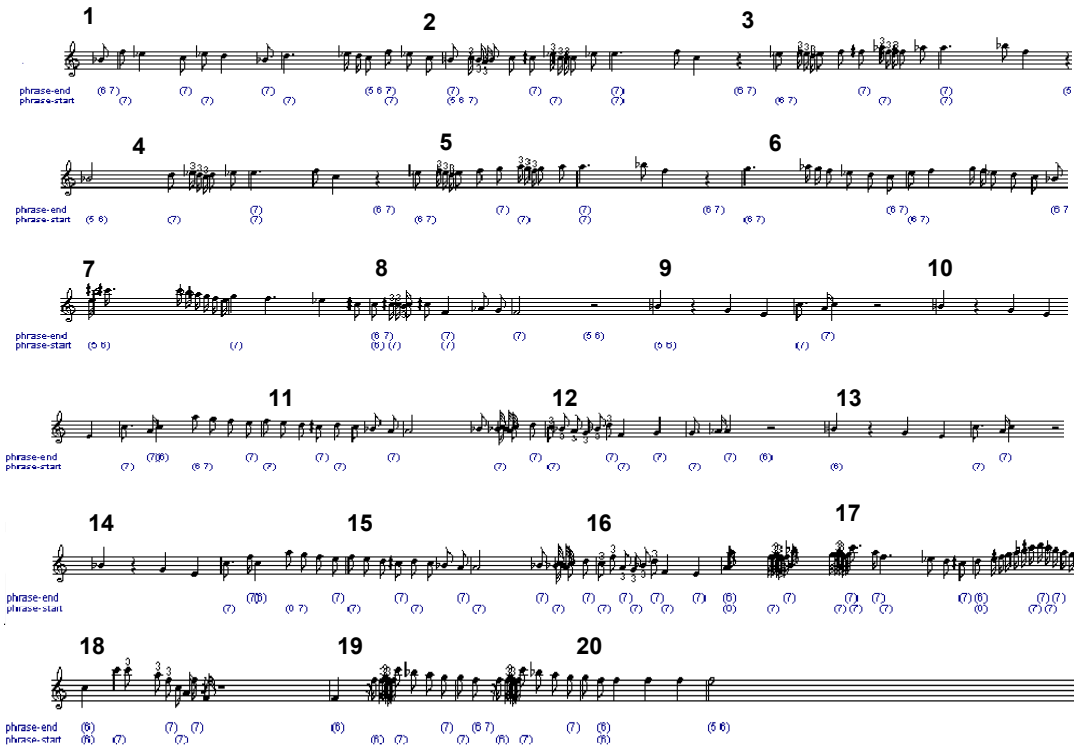


Figure 1. Score used in the experiment. Numbers within parentheses represent the levels of the phrase markers, numbers in the lower row showing the start of phrases and numbers in the upper row marking the end of phrases.

Table 1. Rules included in the experiment.

Rule name	Short description
Double-duration	Decreases the IOI contrast for two adjacent notes having the nominal IOI ratio 2:1, e.g. a quarter note followed by an eighth note*.
Duration-contrast	Long notes are lengthened and short note shortened.*
Faster-uphill	Decreases IOI of notes in uphill motion of melody.*
Harmonic-charge	Emphasises chords harmonically remote from the current key.*
Leap-tone-duration	Shortens the first note of an ascending leap and lengthens the first note of a descending leap.*
Melodic-charge	Emphasises notes that are far away on the circle of fiths from the root of the current chord.*
Phrase-arch	Each phrase is performed with an arch-like tempo curve: starting slow, faste middle, and ritardando towards the end. The sound level is coupled so that a tempo is associated with a low sound level. Phrase boundaries must be mark in the score.
Punctuation	Automatically locates small tone groups and marks them with a lengthening the last note and a following micropause.**

* Friberg A (1991) "Generative Rules for Music Performance: A Formal Description of a Rule System", Computer Music Journal, 15 (2), pp. 56-71.

** Friberg A, Bresin R, Frydén L, Sundberg J (1998) "Musical punctuation on the microlevel: Automatic identification and performance of small melodic units", Journal of New Music Research, 1998, Vol. 27, No. 3, pp. 271-292

Table 2. Notes included in the 16 sections. The first and last notes of the section are given in terms of their pitch names with the note's order number in the bar within parenthesis.

Section	Start		End	
	Bar	Pitch	Bar	Pitch
1	1	B4	1	C5
2	2	Eb5	2	F5(2)
3	3	G5(1)	3	Bb4
4	4	F5(1)	4	C5(2)
5	5	Bb4	5	Rest
6	6	Eb5	6	Rest
7&8	7	E5	8	Rest
9&10	9	Bb4	10	C5(2)
10&11	10	A5	11	Bb4(2)
11&12	11	D5(3)	12	Rest
13&14	13	Bb4	14	C5(2)
14&15	14	A5	15	Bb4(2)
16	16	C5	16	E4
17	17	D5(2)	17	D5(3)
18	18	C5(1)	18	Rest
19&20	19	C6(1)	20	F5(1)

and the DM version for the various sections are shown in terms of the correlation coefficients for the various sections in Figures 2 and 3. The single rules produced quite different effects, as expected. Duration Contrast, Faster Uphill, Leap Tone Duration, and Melodic Charge yielded low correlation for most sections. Some rules yielded high positive correlation with the real performance in certain sections. Thus, Punctuation produced high correlation for sections 9&10, 12, 13&14, and Harmonic Charge yielded high correlations at the end of the excerpt, for sections 16, 17, and 18. Among the Phrase Arch rules, the one operating at level 5 mostly gave low correlations, while those at levels 6 and particularly 7 gave several positive and reasonably high correlations, except for the last sections, which showed high correlations for Harmonic Charge.

In a second run, combinations of rules were tested. Combining the Phrase Arch rules at levels 5, 6, and 7 increased the correlation for several sections as compared with those obtained for one Phrase Arch rule alone, see Figure 3. The Phrase Arch rules with k=0.5 at levels 5 and 6 and k=1.0 at level 7 were then combined with three other rules. Somewhat higher mean correlation means across sections were obtained when the Harmonic Charge and, in negative quantity, Duration Contrast were added to the Phrase Arch rules, Figure 4. A high mean correlation across all sections was obtained for

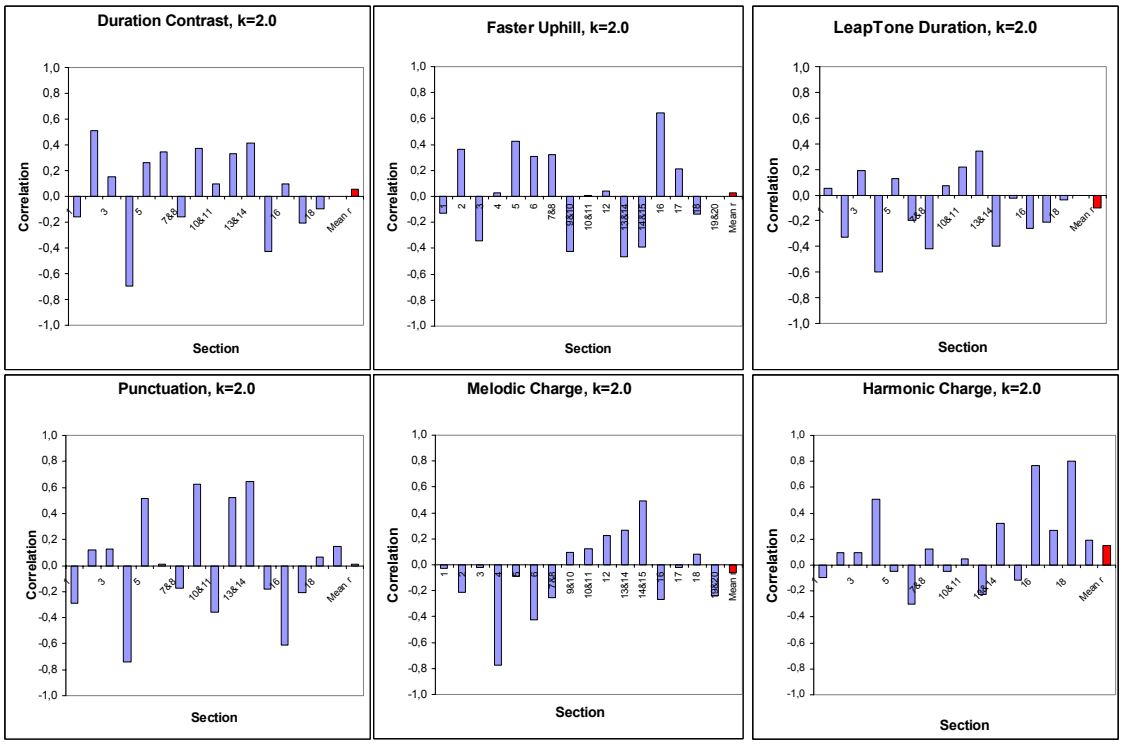


Figure 2. Correlation between the pianist's and the DM deviations from nominal duration obtained for the different sections by applying one rule at the time. The rightmost bar indicates the average across sections of the correlations.

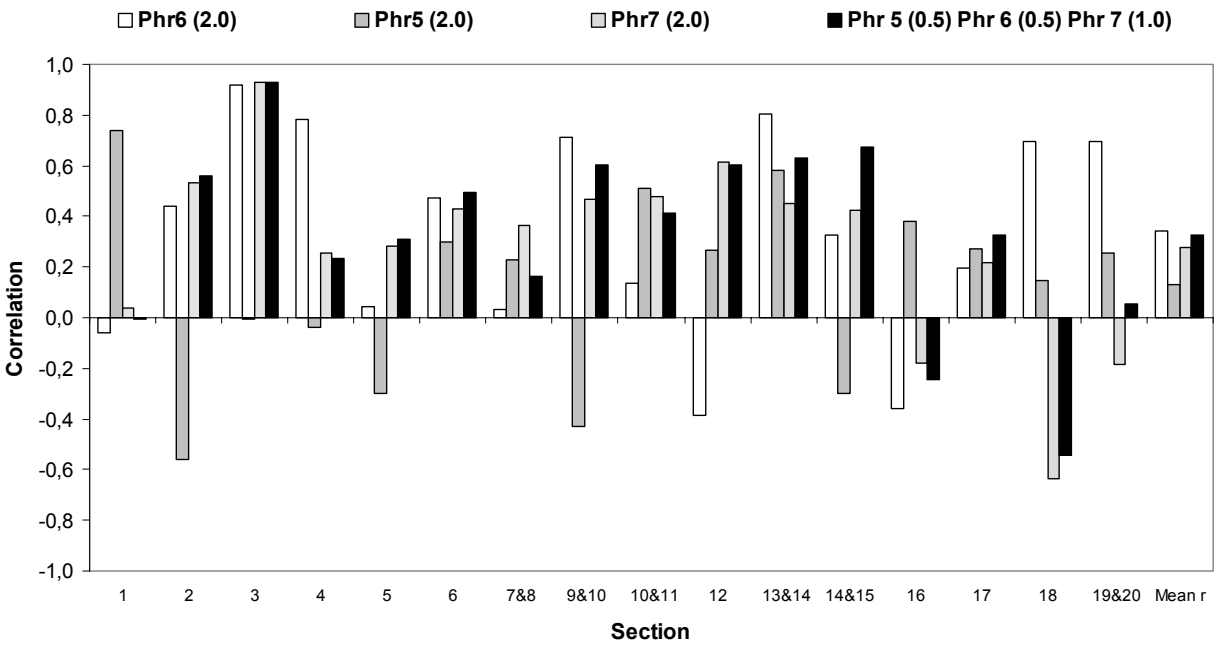


Figure 3. Correlation between the pianist's and the DM deviations from nominal duration obtained for the different sections by applying the indicated phrasing rule combinations. The rightmost bar indicates the average across sections of the correlations.

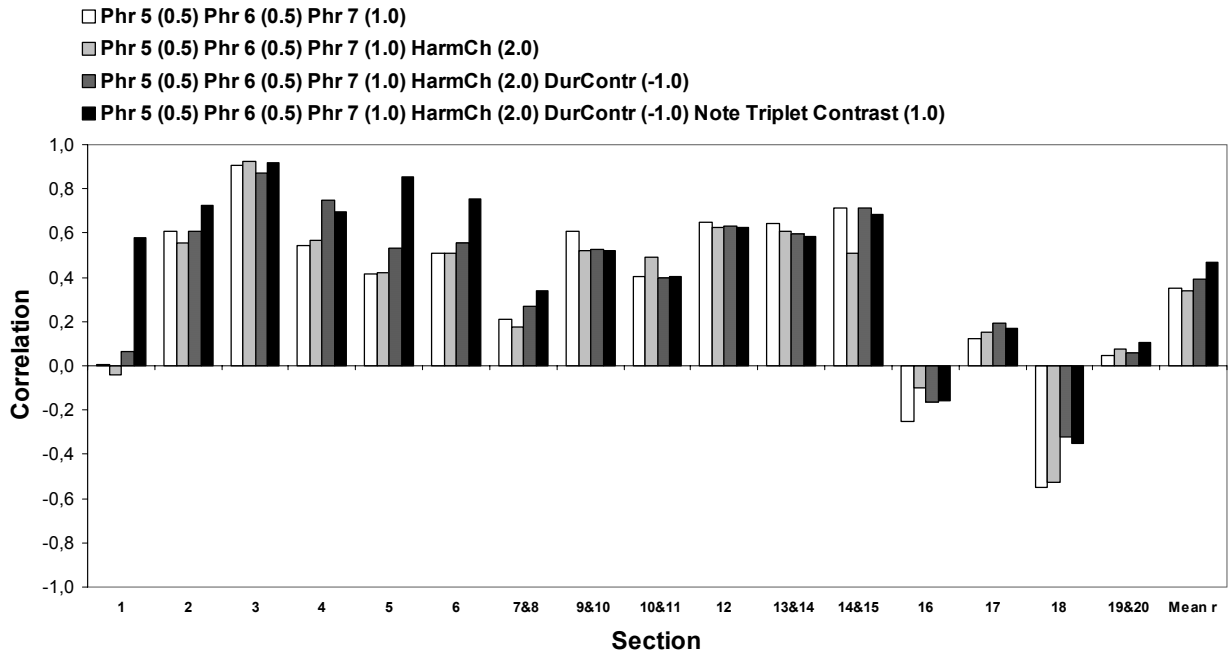


Figure 4. Correlation between the pianist's and the DM deviations from nominal duration obtained for the different sections by applying rule the combinations indicated at the top. Numbers within parentheses show the k values. The rightmost bar indicates the average across sections of the correlations.

Table 3. Max r and Mean r show the highest and the mean correlation coefficients (best linear fit) obtained for the indicated sections between the departures from nominal IOI made by the pianist and produced by the listed combinations of DM rules. The columns Phr. 5, Phr. 6, Phr. 7, HarmCha, DurContr, Grupp, LeapToDr, and Pctn, representing the rules Phrase Arch at levels 5, 6, 7, Harmonic Charge, Duration Contrast, Gruppetto, Leap Tone Duration and Punctuation, specify the k-values of the respective rule used for obtaining the indicated correlation values.

Best rule Section	**Phr 5**	**Phr 6**	**Phr 7**	**Harm Cha**	**Dur Contr**	**Grupp**	**Leap ToDr**	**Pctn**	**Max r**	**Mean r**
1	2.0	0	0	0	0	0	0	0	0,740	0,129
2	0.5	0.5	1.0	2.0	-1.0	1.0	0	0	0,727	0,467
3	0.5	0.5	1.0	0	0	0	0	0	0,929	0,325
4	0	2.0	0	0	0	0	0	0	0,781	0,341
5	0.5	0.5	1.0	2.0	-1.0	1.0	0	1.0	0,845	0,447
6	0.5	0.5	1.0	2.0	-1.0	1.0	0	0	0,756	0,467
7&8	0	0	2.0	0	0	0	0	1.0	0,370	0,447
9&10	0	2.0	0	0	0	0	0	0	0,714	0,341
10&11	2.0	0	0	0	0	0	0	0	0,513	0,129
12	0.5	0.5	1.0	2.0	-1.0	0	1.0	0	0,657	0,390
13&14	0	2.0	0	0	0	0	0	0	0,803	0,341
14&15	0.5	0.5	1.0	0	-1.0	0	0	0	0,779	0,359
16	0	0	0	2.0	0	0	0	0	0,722	0,149
17	0.5	0.5	1.0	0	-1.0	0	0	0	0,336	0,359
18	0	0	0	2.0	0	0	0	0	0,840	0,149
19&20	0	2.0	0	0	0	0	0	0	0,696	0,341

the combination Phrase Arch 5, k=0.5, Phrase Arch 6, k=0.5, P Phrase Arch 7, k=1.0, Harmonic Charge, k=2.0, Duration Contrast, k=-1.0. This inclusion of Duration Contrast improved the correlation in some sections, particularly section 4 and 14&15. Also it reduced the negative correlation for sections 14-16 and 18. This was not entirely unexpected. The Phrase Arch rules alone yielded rather high mean correlation in sections 2 to 14 and correlations close to zero in sections 15 to 19, while the Harmonic Charge rule produced reasonably positive correlation in these same sections 15 to 19. Overall, this version sounded reasonably acceptable.

Table 3 shows the rule combination that produced the highest correlation in each of the various sections together with the maximum and mean correlations obtained with that rule combination for that section. In about half of the sections, Phrase Arch rules produced the highest correlation. In sections 16 and 18, however, the highest correlation was obtained by the Harmonic Charge rule, even though in these cases the DM deviations were considerably smaller than those of the pianist. The Punctuation rule contributed to maximum correlation only in section 5 and 7&8. For most sections, the mean correlation across all sections were, however, generally rather low, ranging between 0.129 and 0.467. This indicates, once again, that there was no particular rule combination that yielded a high correlation for all sections.

Obviously, correlation represents only a limited aspect of the agreement between the pianist's and the DM deviations from nominal IOIs. A detailed analysis of the various individual notes' deviations from nominal duration in the real and in the rule-generated performances may offer complementary information on the mechanisms underlying the performance analysed.

A note that lowered the correlation values for all combinations of rules was the second note of the first bar. As mentioned, this note and the following four note *gruppetto* was nominally notated as an 8^{th} note followed by three triplet 16^{th} notes and another 8^{th} note in the MUSfile. In the excerpt seven other examples of *gruppetto* occur in analogous contexts, in bars 2, 5, 6 and 8. The pianist lengthened the first of these notes

by about 100 or 200 ms, shortened the subsequent triplet notes and shortened the final note somewhat less or lengthened it, as illustrated in Figure 5. Mostly, the duration of the entire group tended to remain reasonably unaffected by these modifications, such that the lengthening of the first note was compensated by the shortening of the subsequent notes. Among the DM rules, only Phrase Arch at level 7 managed to lengthen the first 8th note in these *gruppetti* appreciably. The significance of this failure of the performance grammar to replicate this detail in the real performance is reduced by the fact that this sequence of notes is a rhythmic cliché that perhaps was not accurately represented by the note values used in the comparison. Yet, a special rule was formulated, Note-Triplet-Contrast, to reflect this performance principle, lengthening the first note and shortening the subsequent short notes in this particular context. The rule successfully matched the departures from nominal IOIs when applied at k=2.0 (Figure 5). The rule increased the correlation in section 1 by no less than 0.64 and in the other sections containing gruppetti by about between 0.23 in section 5 and 0.16 in section 7&8.

In some cases, the note preceding an apparently stressed note was substantially lengthened. This could be observed for the Eb5 note preceding the E5 on bar 1 beat 3 and for the corresponding notes in bars 2, 5, and 6. Likewise, substantial lengthenings were observed on the G4 note preceding the F4 in bar 8 beat 3, on the 16th A4 preceding the ¼ note A4 in bar 11 and 15, and on C#6 preceding the D6 on bar 17 beat 4. Figure 6 shows three of these examples. Lengthening in such positions seems to serve the purpose of emphasising the subsequent note. This *emphasis by delayed arrival* has been observed previously, also in singing and in emphatic speech (see e.g. Drake & Palmer, 1993; Sundberg, 2000). In many cases, the correlation would have improved greatly, had the rule system been able to produce the pianist's large lengthening in these contexts. It seemed, however, difficult to formulate a rule for this particular case, as the pianist did not choose to lengthen all notes appearing in a corresponding context.

Discussion

We used the correlation coefficient as a measure of the agreement between the pianist's deviations from nominal and those produced by the DM system. This shows the agreement with regard to the sign of the overall deviations, not their quantity. In many cases, however, the quantity of the deviations is of less interest than the sign7, since different players make departures that tend to vary more in size than in direction. Yet, some high correlations, such as that obtained for Harmonic Charge in sections 16, 18, and 19&20 were associated with very great differences in quantity, thus indicating that the pianist's deviations in these sections could not be described by this rule.

Another limitation of the correlation as a measure of the agreement is that the correlation is highly sensitive to extreme values. Thus, correlation will increase considerably in cases where most data points of the pianist and of the DM are close to zero and one or few are great and of identical sign. On the other hand, large departures would be important to the musical quality of the performance.

A third limitation of the correlation measure is that it is much more sensitive to the agreement for single notes when the number of notes compared is small as compared to when it is large. In section 1, 16 and 18, the number of notes compared were 10, 8 and 7, respectively, while in the remaining sections they were 12 or more. Therefore, the degree of agreement in sections 1, 16, and 18 is of less importance.

Some rules applied in isolation failed to produce any substantial positive correlation in any of the sections. For example Leap Tone Duration, which shortens the first note of an ascending leap and lengthens the first note of a descending leap, produced correlations smaller than 0.34, and negative correlations in 6 sections. Duration Contrast, Faster Uphill, and Melodic Charge produced almost equally poor results. The reason for this is not clear. Thompson and collaborators (Thompson & al., 1989) found that Melodic Charge and, under certain conditions, Faster Uphill improved the musical quality of performances of music examples, and Gabrielsson & Juslin (1996) and Bresin & Friberg (2000) noted that Duration Contrast was important in emotional colouring of performances. Sundberg et al. (1991) found support for Duration Contrast and Melodic Charge. It is possible that these rules produce desirable effects only when combined with other rules and/or only in certain types of music.

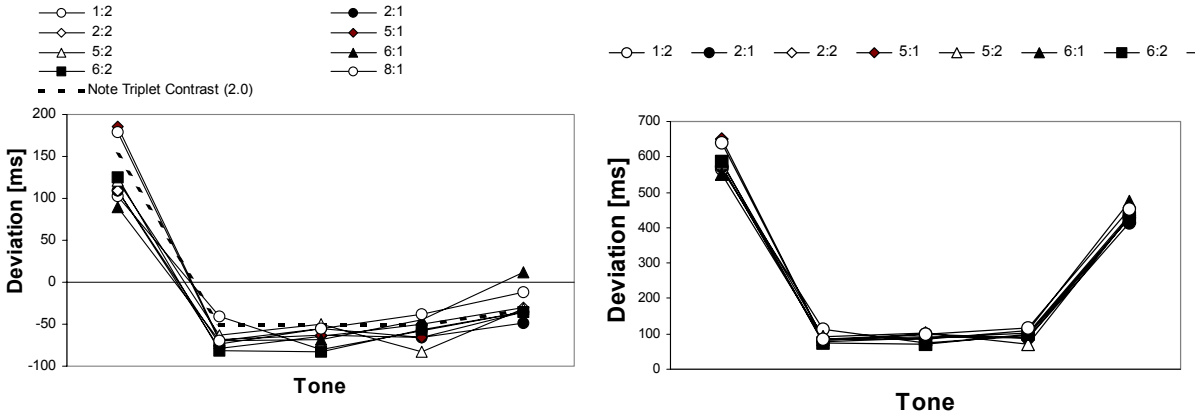


Figure 5. Pianist's performance of gruppetti in the indicated bars and beats. The left and right panels show the deviation from nominal duration in ms and the right panel shows the inter-onset intervals in ms. The heavy dashed lines in the left panel refers to the deviations produced by the new Note Triplet Contrast rule, applied with a quantity of k=2.0.

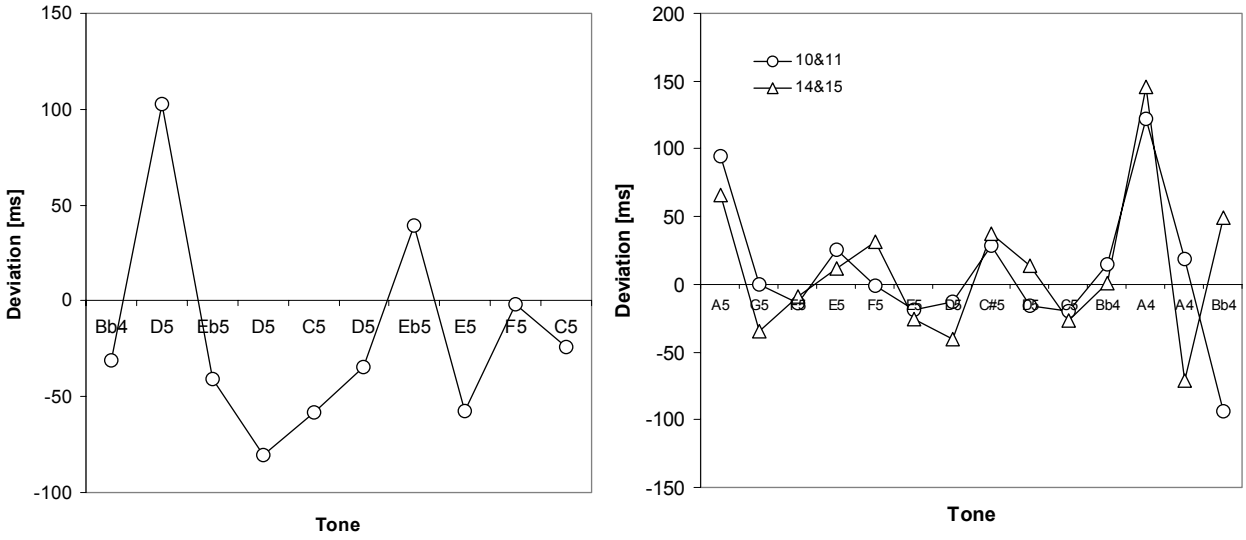


Figure 6. Examples from the pianist's performance of emphasis by delayed arrival (circled). The left panel refers to section 1 and the right panel to sections 10&11 and 14&15. In the right panel the pitch names refer to section 10&12.

Of the rules applied in isolation, two produced reasonably high positive correlations in one or more sections. Punctuation yielded correlations exceeding 0.5 in sections 5, 9&10, 12, and 13&14. The Phrase Arch rules at levels 6 and, in particular, 7, on the other hand, produced positive correlations in all sections except in the sections toward the end of the excerpt. Recalling that phrase level 7 reflected the legato markings in the score, these observations would suggest that the pianist tended to reflect the legato markings in his performance of the first 15 bars and possibly more emphasised the harmonic progressions toward the end of the excerpt. One might argue that the final bars of the excerpt have a somewhat different, more extravagant character than the preceding part of the excerpt, which rather seem to establish main melodic ideas. In any event, these observations support the assumption that better performances would emerge if rules were not applied with a fixed, constant quantity throughout a piece. Rather, rule quantity should change depending on the musical character of the composition. Zannon and collaborators present an interesting practical solution to this problem (forthcoming).

It is also noteworthy that the Phrase Arch rules alone produced high correlations, in the range of 0.41 to 0.90, in a majority of the sections, except for the four last bars of the excerpt. In his attempt to match three pianists' performances of Robert Schumann's *Träumerei*, Friberg (1995b) found that much of the deviations from nominal IOI could be produced by tuning the Phrase Arch rules. This indicates that phrasing is a truly basic aspect of music performance, an idea also supported by the fact that musicians tend to reproduce phrasing patterns of timing even when asked to deliberately perform without any musical expressivity (Sundberg & al., 1995; Palmer 1989).

Phrase Arch rule at level 7, which we used to reflect the legato signs in the score, were particularly efficient in matching the pianist's departures from nominal IOIs. This suggests that this pianist paid particular attention to the legato signs and tended to perform legato groups of tones as phrases. On the other hand, it seems unlikely that tones united by a legato sign should invariably be regarded as a phrase, and such cases would not necessarily be reflected as exceptionally low correlation in our study.

The pianist played the *gruppetto* grace notes such that their IOI was 100 ms or somewhat shorter. Data published by Timmers (2002) showed that the IOI of grace notes followed by its adjacent scale tone was shorter than 100 ms over a wide range of tempi. Friberg & Sundström (2002) observed that the minimum IOI of notes shortened by the swing ratio in jazz music performances was about 90 ms. Windsor & al. (2001) observed that the IOI of grace notes were not scaled in accordance with the overall tempo. Indeed, our experiences of synthesising grace notes suggest that the 100 ms IOI appears to be a magical limit for whether or not tones are perceived as autonomous pitches; notes shorter than 100 ms appear to loose their autonomy.

Applying the Duration Contrast rule with a negative quantity contributed to a better matching with the pianist's performance. Bresin & Friberg (2000) found that a negative quantity of this rule induced a special character of performances of examples that listeners identified as "sad" or "tender". A positive quantity, on the other hand, contributed to inducing a different character, which listeners classified as "happy" or "angry". The emotional colour of the Mozart Adagio examined in the present study is certainly tender/sad rather than happy/angry. Therefore, the positive result reached by applying a negative quantity of the Duration Contrast rule seems to be in accordance with the Bresin & Friberg findings.

Conclusions

Our investigation has shown that some of the deviations from nominal IOI that a professional pianist made when performing bars 1-20 of the second movement of W A Mozart's *Piano Sonata* in F, K 332 could be approximated by the DM rule system. The Phrase Arch rules successfully reproduced much of the pianist's deviations, thus offering another confirmation of the dominating influence of the phrase structure in the timing of music performance. Particularly the phrasing reflecting the legato signs in the score explained much of the pianist's deviations, thus indicating that tones joined by a legato sign in the score were interpreted as low level phrases. To match the pianist's deviations in the entire excerpt the rule combination and the quantities of the rules applied need to change between sections. Therefore, time varying rule palettes would be a worthwhile target for future attempts to develop the DM system.

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