KTH Computer Science and Communication

2F1120 Spektrala transformer för Media
Solutions to Steiglitz Chapter 3

Preface

This document contains solutions to selected problems from Ken Steiglitz's book: "A Digital Signal Processing Primer" published by Addison-Wesley. Refer to the book for the problem text.

3.1

As the text book states, one bel is ten decibels. Using the definition for the decibels:

$$10\text{dB} = 20\log_{10}(R) \iff R = \sqrt{10} \simeq 3.16$$

3.3

A sinusoid at 330Hz is sampled at 300Hz. As indicated by the text book, the phenomenon of aliasing takes place because after sampling, any signal (phasor) of the kind

$$e^{jnT_s(\omega_0 + k2\pi/T_s)}$$

is equivalent for any integer value of k. In this case, if we impose the quantity in parenthesis for $k = 1$ equal to the real frequency of the signal, and T_s to the sampling frequency we obtain:

$$\omega_0 + 2\pi 300 = 2\pi 330$$

Rearranging we find that the frequency $\omega_0 = 2\pi 30$ rad/s, corresponding to a period of $T = 0.033$ seconds, is equivalent to a frequency of $2\pi 330$ rad/s under this sampling frequency. Check this result against Figure 1.1 page 44 in the book.

3.6

Harmonic number 79 corresponds to the frequency $w_{79} = 79 \times \omega_0$ rad/s, or equivalently in Hz: $f_{79} = 79 \times f_0$ Hz, where as usual $\omega = 2\pi f$. Substituting the value in the text book: $f_{79} = 79 \times 700 = 55300$Hz. This frequency is above the Nyquist frequency $f_s/2 = 20000$Hz and is therefore aliased. By repeating the spectrum at $-f_s$ we see that this contribution appears in the base band at frequency 15300Hz.

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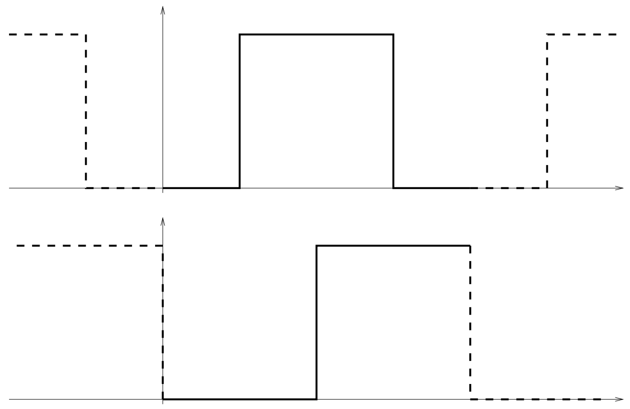

Figure 1.

3.8

Figure 1 shows the waveforms for the shifts indicated by the problem. For a $T/4$ shift (plot above), $f(x) = f(-x)$ and the series does not contain sin terms. Also $f(T/2 - x) = f(T/2 + x)$ thus only even harmonics are present in the series. For a $T/2$ shift (plot below), the waveform is simply the negative of the one shown in the book, thus the series is identical apart from a minus sign.

3.12

The statement in the exercise text can be simply proved using Eurler's formula (we simplify the notation writing $\omega t = x$):

$$\cos(2x) = \frac{e^{j2x} + e^{-j2x}}{2} = \frac{(e^{jx})^2 + (e^{-jx})^2}{2}$$

We know that the square of a binomial is: $(a+b)^2 = a^2+b^2+2ab$, and thus $a^2+b^2 = (a+b)^2-2ab$. Substituting with $a = e^{jx}$ and $b = e^{-jx}$, we obtain:

$$\begin{aligned} \frac{(e^{jx})^2 + (e^{-jx})^2}{2} &= 2\left(\frac{e^{jx} + e^{-jx}}{2}\right)^2 - \frac{2}{2}e^{jx}e^{-jx} \\ &= 2\cos^2(x) - 1 \end{aligned}$$

(a) The same way for the third power, but first note that $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$ and that for $a = e^{jx}$ and $b = e^{-jx}$, we have $a^2b = e^{jx}$ and $ab^2 = e^{-jx}$. We have:

$$\begin{aligned} \cos(3x) &= \frac{e^{j3x} + e^{-j3x}}{2} = \frac{(e^{jx})^3 + (e^{-jx})^3}{2} \\ &= 4\left(\frac{e^{jx} + e^{-jx}}{2}\right)^3 - 3\frac{e^{jx} + e^{-jx}}{2} \\ &= 4\cos^3(x) - 3\cos(x) \end{aligned}$$

(b) It should be clear by now that the same procedure can be repeated for any power where we use the binomial expansion for that power and the fact that $a^n b^m = e^{j(n-m)x}$ when $a = e^{jx}$ and $b = e^{-jx}$. The result is a set of polynomials called Chebyshev after the mathematician that discovered them. Just to have an idea, this is a few of those polynomials:

$$
\begin{aligned}
T_0(x) &= 1 \\
T_1(x) &= x \\
T_2(x) &= 2x^2 - 1 \\
T_3(x) &= 4x^3 - 3x \\
T_4(x) &= 8x^4 - 8x^2 + 1 \\
T_5(x) &= 16x^5 - 20x^3 + 5x \\
T_6(x) &= 32x^6 - 48x^4 + 18x^2 - 1
\end{aligned}
$$

Notice that $T_2(x)$ and $T_3(x)$ correspond to the solutions in the previous points.
(c) Since the Chebyshev polynomials correspond to different harmonics, we can define the non-linearity as a mixture of those polynomials where the mixing coefficients are the given amplitudes for each harmonic.