KTH Computer Science and Communication

2F1120 Spektrala transformer för Media
Solutions to Steiglitz Chapter 8

Preface

This document contains solutions to selected problems from Ken Steiglitz's book: "A Digital Signal Processing Primer" published by Addison-Wesley. Refer to the book for the problem text.

8.1

We have to prove that

$$\langle e^{j2\pi mt/N}, e^{j2\pi nt/N}\rangle = \sum_{t=0}^{N-1} e^{j2\pi(m-n)t/N} = 0 \quad \text{if } m \neq n$$

This is a geometric series of the kind

$$\sum_{t=0}^{N-1} a^t$$

where $a = e^{j2\pi(m-n)/N}$. We know that the partial sum of this series can be written (see also Exercise 4.3) as:

$$\sum_{t=0}^{N-1} a^t = \frac{a^N - 1}{a - 1}$$

In our case, substituting the value of a we have:

$$\sum_{t=0}^{N-1} e^{j2\pi(m-n)t/N} = \frac{e^{Nj2\pi(m-n)t/N} - 1}{e^{j2\pi(m-n)t/N} - 1} = \frac{e^{j2\pi(m-n)t} - 1}{e^{j2\pi(m-n)t/N} - 1}$$

Since $(n-m)t$ is integer, the exponential in the numerator is always equal to 1 and the inner product is zero.

8.2

From equations 2.5 and 2.6 in the text book:

$$X_k = \sum_{t=0}^{N-1} x_t e^{-j2\pi kt/N} \tag{1}$$

$$x_t = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi kt/N} \tag{2}$$

Substituting 1 into 2 we have[1]:

$$x_t = \frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{u=0}^{N-1} x_u e^{-j2\pi ku/N}\right] e^{j2\pi kt/N}$$

For the associative property ($a(b+c) = ab + ac$), we can take the last term into the inner sum:

$$\begin{aligned}
\cdots &= \frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{u=0}^{N-1} x_u e^{-j2\pi ku/N} e^{j2\pi kt/N}\right] \\
&= \frac{1}{N}\sum_{k=0}^{N-1}\sum_{u=0}^{N-1} x_u e^{-j2\pi k(u-t)/N}
\end{aligned}$$

We can now invert the two sums (they are linear combinations, and the order of summing does not matter), and take the term x_u out of the sum over k.

$$\begin{aligned}
\cdots &= \frac{1}{N}\sum_{u=0}^{N-1}\sum_{k=0}^{N-1} x_u e^{-j2\pi k(u-t)/N} \\
&= \frac{1}{N}\sum_{u=0}^{N-1} x_u \sum_{k=0}^{N-1} e^{-j2\pi k(u-t)/N} \qquad (3)
\end{aligned}$$

We call the inner sum $f(u-t)$:

$$f(u-t) = \sum_{k=0}^{N-1} e^{-j2\pi k(u-t)/N}$$

Now we note that if $t \neq u$ this function is always zero (see Exercise 8.1). If $u = t$, instead, $f(0) = N$ because we sum N times the unity. So Equation 3 becomes:

$$\cdots = \frac{1}{N}\sum_{u=0}^{N-1} f(u-t)x_u$$

When we sum over u we see that the only non-zero term is the one for $u = t$ leading to:

$$\cdots = \frac{1}{N} f(0) x_{u=t} = \frac{1}{N} N x_t = x_t$$

as we wanted to prove.

[1] note that the index t in 2 stands for a fixed time point at which we want to compute the signal x, while in 1 it is a mere index used to loop over all the samples of the signal to compute the DFT coefficients. To avoid confusing between the two indexes, we changed name of the second to u