Doctoral Course
in
Speech and Speaker Recognition

Language Modeling

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Ch 11 Language Modeling

- Formal Language Theory
- Stochastic Language Models
- Complexity Measure of Language Models
- N-gram Smoothing
- Adaptive Language Models
- Practical Issues

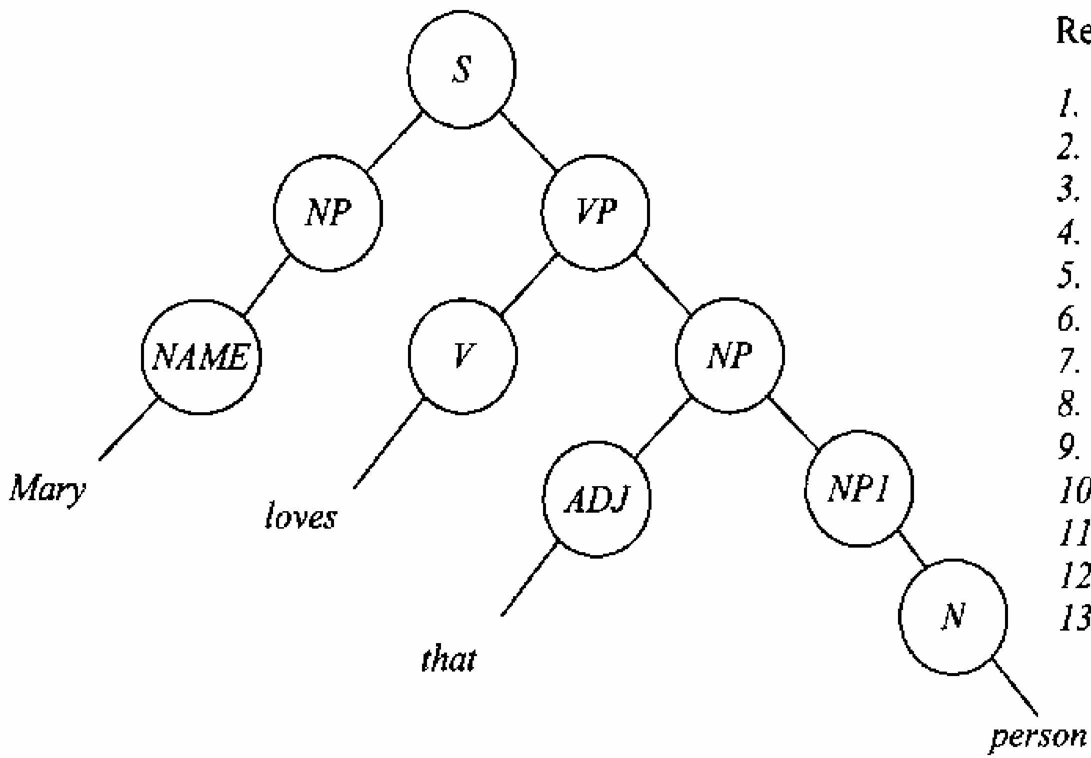
11.1 Formal Language Theory

- Important aspects of syntactic grammar
  - Generality - cover typical sentences for an application
  - Selectivity - distinguish different kinds of intended actions
  - Understandability - easy maintenance and improvement

  - *Grammar*
    - *formal specification of the permissible structures for a language*
  - *Parsing*
    - *Analysis to see if a sentence is compliant with the grammar*
    - *Search through various ways of combining grammar rules*

Tree representation

- The most common way to represent the grammatical structure of a sentence



Rewrite Rules:

1. *S → NP VP*
2. *VP → V NP*
3. *VP → AUX VP*
4. *NP → ART NP1*
5. *NP → ADJ NP1*
6. *NP1 → ADJ NP1*
7. *NP1 → N*
8. *NP → NAME*
9. *NP → PRON*
10. *NAME → Mary*
11. *V → loves*
12. *ADJ → that*
13. *N → person*

11.1.1 Chomsky Hierarchy

- Chomsky's formal language theory
- A grammar is defined as G = (V,T, P, S)
  - V: Non-terminal
  - T: Terminal
  - P: Set of production rules
  - S: Start symbol
- Analysis by sequential application of production rules
- Production rule type $\alpha \rightarrow \beta$, α, β strings of V and T
- Four major language types, hierarchically structured

Chomsky hierarchy and corresponding machines

Types	**Constraints**	**Automata**
Phrase structure grammar	$\alpha \to \beta$. The most general grammar. α, β: strings of non-terminals and terminals	Turing machine
Context-sensitive grammar	Subset of phrase structure grammar. $\|\alpha\| \leq \|\beta\|$	Linear bounded automata
Context-free grammar *Widely applied in NLP* *Often powerful enough*	Subset of context-sensitive grammar $A \to \beta$, A: non-terminal, β: w or BC	Push down automata
Regular grammar	Subset of CFG $A \to w$ and $A \to wB$	Finite-state automata

Push-down automata

- Also called Recursive Transition Network
- Transition Network: nodes and labeled arcs
- Parsing
  - Start at the initial state S
  - Traverse an arc if current word is in the arc category
  - If arc is followed, update current word
  - A phrase is parsed if there is a path from S to a *pop* (final) arc
  - More than one parse is possible

11.1.2 Chart Parsing for Context-Free Grammars

- Vast literature on parsing algorithms
  - Mostly for programming languages
- Chart parsing is the most relevant for spoken language systems
  - Widely used

Top Down or Bottom Up Parsing? Goal- or Data-Directed?

- Top-down
  - Goal-directed search
  - Start from the root of the tree, successive rewrites into terminal symbols matching the input text
  - Example “Mary loves that person”
    - S
    - → NP VP
    - → NAME VP (rewrite S using S→NP)
    - → Mary VP (rewrite NP using NAME→Mary)
    - …
    - → Mary loves that person (rewrite N using N→person)

Rewrite Rules:

1. *S → NP VP*
2. *VP → V NP*
3. *VP → AUX VP*
4. *NP → ART NP1*
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Top Down or Bottom Up Parsing?

- Bottom-up
  - Data-directed search
  - Start with the words in the input text
  - Use the rewrite rules backwards
  - Example “Mary loves that person”
    - → NAME loves that person (rewrite Mary using NAME →Mary
    - → NAME V that person (rewrite loves using V →loves
    - …
    - →NP VP
    - →S (rewrite NP using S →NP VP)

Rewrite Rules:

1. *S → NP VP*
2. *VP → V NP*
3. *VP → AUX VP*
4. *NP → ART NP1*
5. *NP → ADJ NP1*
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Top Down or Bottom Up Parsing?

- Top-down parsing features
  - Very predictive
  - Only considers grammatical combinations
  - Predicts constituents that do not have a match in the text
  - Infinite recursion possible
- Bottom-up parsing features
  - Checks input only once
  - May build trees that can’t lead to full parse
  - Suitable for robust language processing (see Ch. 17)
- Similar performance

Bottom-Up Chart Parsing

- Basic principle: Store partial parsing results in a *chart* to eliminate duplicate work
- Parsing does not need to be left-to-right
- The chart maintains derived constituents and partially matched rules (*active arcs*)
- *Active constituents* represent subparts of the sentence according to the rewrite rules
- Active constituents are stored in an *agenda*

Bottom-Up Chart Parsing cont.

- Operation
  - Identify rules starting with the active constituent or rules that are partially identified and extend these
  - Combine partially matched records with completed constituent to form a new completed constituent or a new partially matched consitutent
  - Depth-first or breadth-first search
    - Breadth-first better if probabilities are used

ALGORITHM 11.1: *A BOTTOM-UP CHART PARSER*

Step1: Initialization: Define a list called chart to store active arcs, and a list called an agenda to store active constituents until they are added to the chart.

Step 2: Repeat: Repeat Step 2 to 7 until there is no input left.

Step 3: Push and pop the agenda: If the agenda is empty, look up the interpretations of the next word in the input and push them to the agenda. Pop a constituent *C* from the agenda. If *C* corresponds to position from w_i to w_j of the input sentence, we denote it $C[i,j]$.

Step 4: Add *C* to the chart: Insert $C[i,j]$ into the chart.

Step 5: Add key-marked active arcs to the chart: For each rule in the grammar of the form $X \rightarrow C\ Y$, add to the chart an active arc (partially matched constituent) of the form $X[i,j] \rightarrow {}^{\circ}CY$, where $^{\circ}$ denotes the critical position called the key that indicates that everything before $^{\circ}$ has been seen, but things after $^{\circ}$ are yet to be matched (incomplete constituent).

Step 6: Move $^{\circ}$ forward: For any active arc of the form $X[1,j] \rightarrow Y \ldots {}^{\circ}C \ldots Z$ (everything before w_i) in the chart, add a new active arc of the form $X[1,j] \rightarrow Y \ldots C^{\circ} \ldots Z$ to the chart.

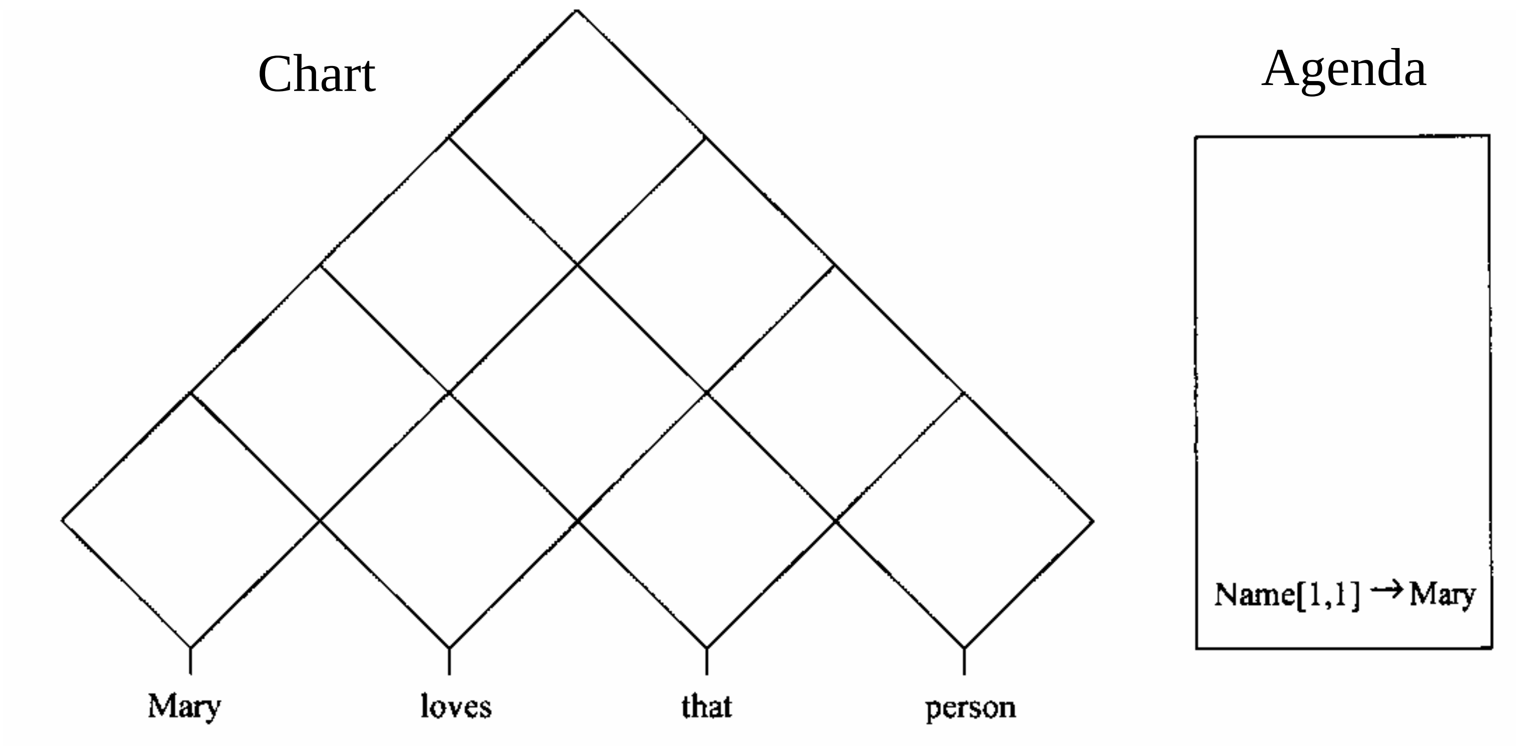
Step 7: Add new constituents to the agenda: For any active arc of the form $X[1,I] \rightarrow Y \ldots {}^{\circ}C$, add a new constituent of type $X[1,j]$ to the agenda.

Step 8: Exit: If $S[1,n]$ is in the chart, where *n* is the length of the input sentence, we can exit successfully unless we want to find all possible interpretations of the sentence. The chart may contain many *S* structures covering the entire set of positions.

Algorithm: A Bottom-Up Chart Parser

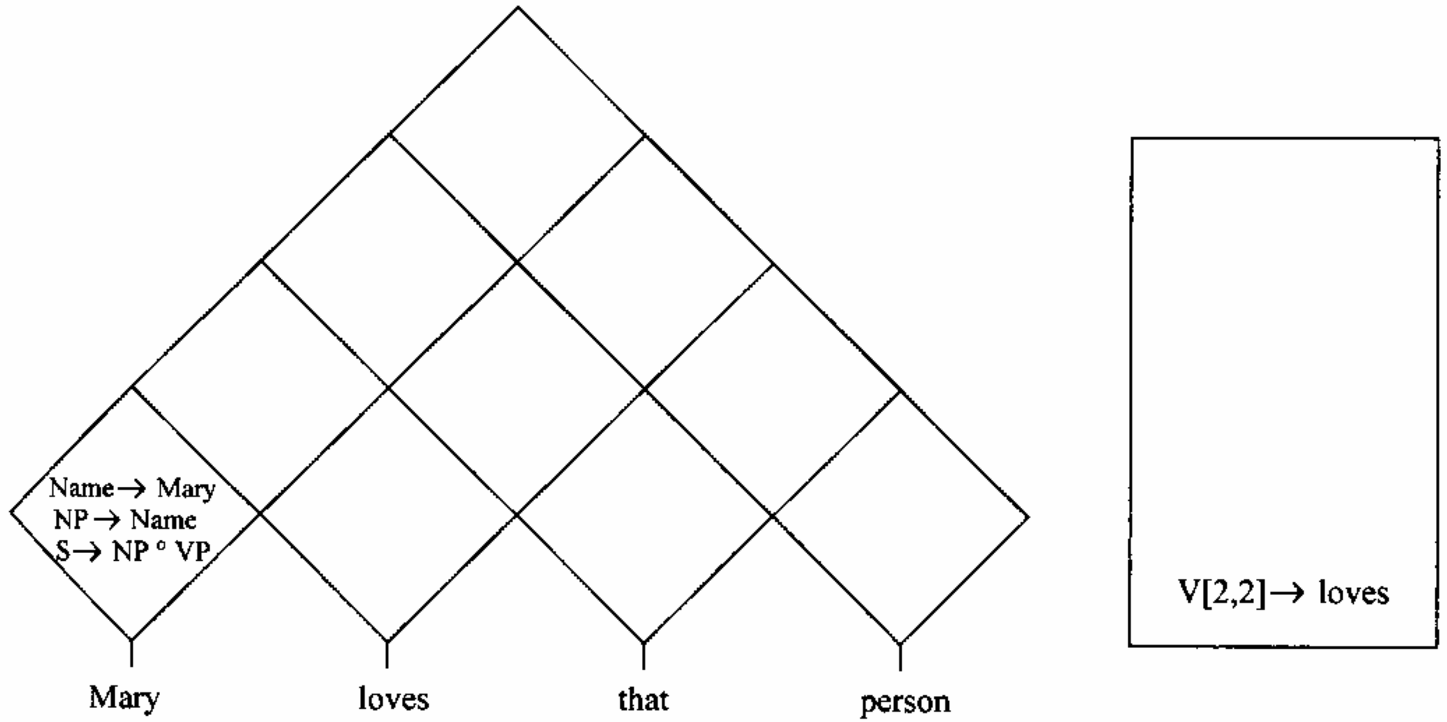
- 1. Initialization
- 2. Repeat 2 to 7 until all input words are processed
- 3. Push input word interpretation to, pop constituent from the agenda
- 4. Add the constituent to the chart
- 5. Find and add partial matches (key-marked) to the chart
- 6. Extend partial matches (Move the keys forward)
- 7. Put the partial matches to the agenda
- 8. Exit, successfully if the whole sentence is interpreted
  - continue if all sentence interpretations are required

Bottom-Up Chart Parsing example (1)



Look up interpretations of the next input word → push to Agenda
Pop constituent from Agenda, insert in the chart

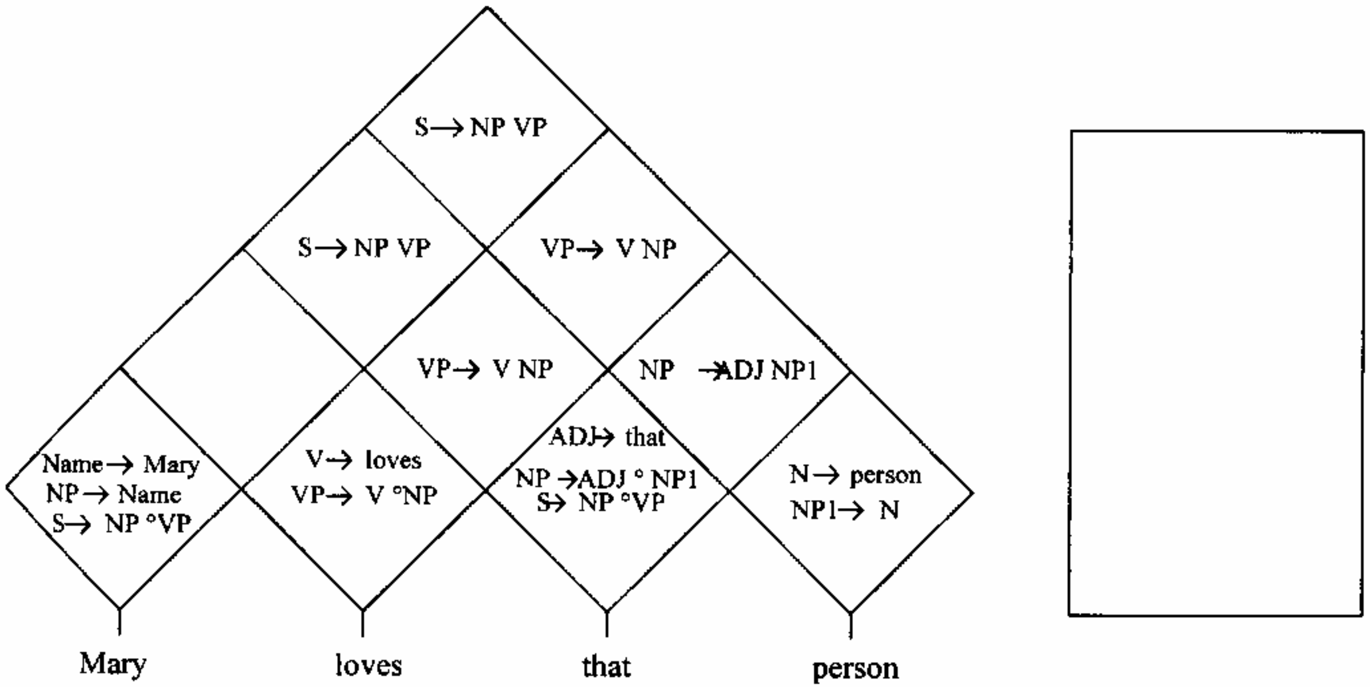
Bottom-Up Chart Parsing example (2)



(b) After Mary, the chart now has rules *Name→Mary*, *NP→Name*, and *S→NP°VP*.

Find partially matched rules

Bottom-Up Chart Parsing example (3)



(c) The chart after the whole sentence is parsed. S→ NP VP covers the whole sentence, indicating that the sentence is parsed successfully by the grammar.

11.2 Stochastic Language Models (SLM)

- In formal languages, P(**W**) = 1 or 0 for accept/reject
  - Inappropriate for spoken language since
  - Incomplete grammar coverage
  - Speech is often ungrammatical
- Probabilistic Context-Free Grammars (PCFG)
- N-gram Language models

11.2.1 Probabilistic Context-Free Grammars (PCFGs)

- Bridge between formal and n-gram grammars
- Each rule is assigned a probability
- Recognition problem
  - What is the probability that the language generates the word sequence **W**, P(S $\Rightarrow$ **W**|G)
- Training problem
  - Determine a set of rules and estimate their probabilities
  - With fixed rule set, count the number of times each rule is used
  - If annotated corpus use ML estimation

$$P(A \rightarrow \alpha_j \mid G) = C(A \rightarrow \alpha_j) / \sum_{i=1}^{m} C(A \rightarrow \alpha_i)$$

  - Else use EM algorithm (here also known as inside-outside)

The inside-outside algorithm

- Analogous to Forward-Backward algorithm, main difference:
  - F-B is time sequential, chart parsing is hierarchical
- PCFG rule format $A_i \rightarrow A_m A_n$ and $A_i \rightarrow w_l$
- Inside probability *inside(j,* A_i*, k)* *(~ forward prob.)*
  - The probability of A_i generating the word sequence $w_j w_{j+1} \ldots w_k$
  - Computed bottom-up
- Outside probability *outside(s,* A_i*, t)* *(~ backward prob.)*
  - The sum of probabilities of all partial parses outside the word sequence $w_s \ldots w_t$, which is covered by A_i
  - Computed top-down after the inside probabilities are computed
- Sentence prob. is the sum of all products of inside and outside probs to each node

The inside algorithm

$$inside(j, A_i, k) = P(A_i \Rightarrow w_j w_{j+1} ... w_k) = \sum_{n,m} \sum_{l=j}^{k-1} P(A_i \to A_m A_n) P(A_m \Rightarrow w_j ... w_l) P(A_n \Rightarrow w_{l+1} ... w_k)$$

$$= \sum_{n,m} \sum_{l=j}^{k-1} P(A_i \to A_m A_n) inside(j, A_m, l) inside(l+1, A_n, k)$$

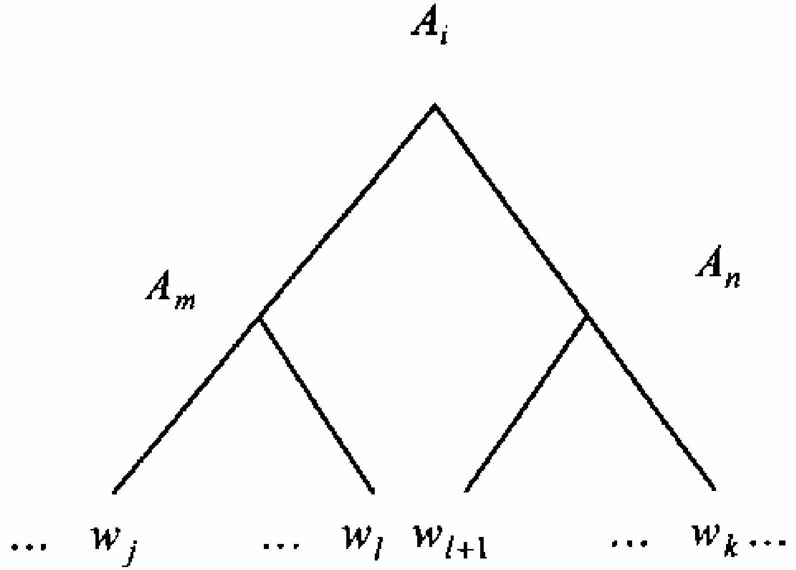
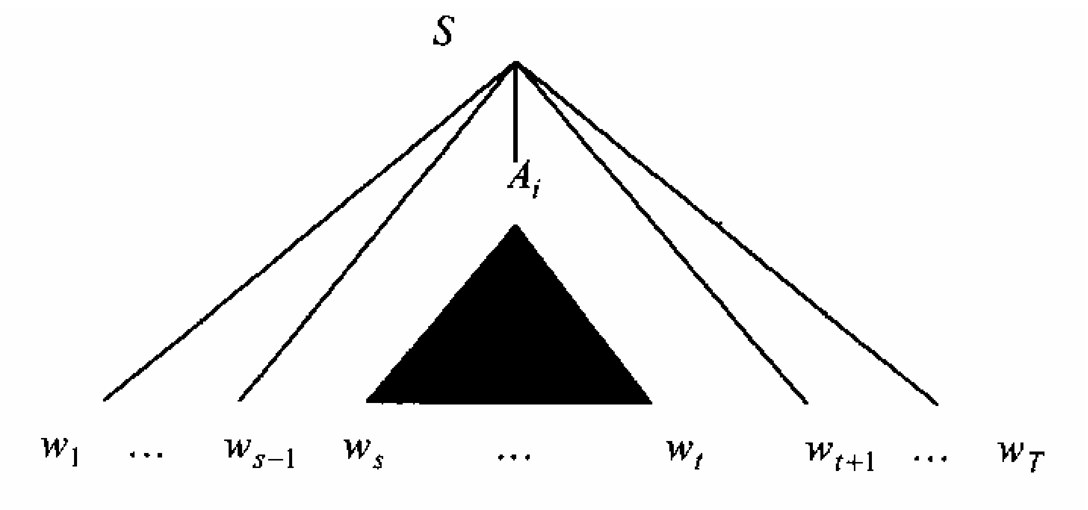


Figure 11.3 Inside probability is computed recursively as sum of all the derivations.

The outside algorithm

- Outside probability *outside(s, A_i, t)*
  - The sum of probabilities of all partial parses outside the word sequence $w_s \ldots w_t$, which is covered by A_i

$$outside(s, A_i, t) = P(S \Rightarrow w_1 ... w_{s-1} A_i w_{t+1} ... w_T) = ...$$



PCFG Rule probability

- Probability of rule $A_i \rightarrow A_m A_n$ covering words $w_s \ldots w_t$

$$\xi(i,m,n,s,t) = P(A_i \Rightarrow w_s \ldots w_t, A_i \rightarrow A_m A_n \,|\, S \Rightarrow \mathbf{W}, G)$$

$$= \frac{1}{P(S \Rightarrow \mathbf{W} | G)} \sum_{k=s}^{t-1} P(A_i \rightarrow A_m A_n | G) inside(s, A_m, k) inside(k+1, A_n, t) outside(s, A_i, t)$$

- Probability on all word spans in the sentence

$$P(A_i \rightarrow A_m A_n \mid G) = \frac{\sum_{s=1}^{T-1} \sum_{t=s+1}^{T} \xi(i,m,n,s,t)}{\sum_{m,n} \sum_{s=1}^{T-1} \sum_{t=s+1}^{T} \xi(i,m,n,s,t)}$$

PCFG Rule estimation aspects

- Only select rules with sufficient probabilities
  - Reduce risk that low probability rules generate too many greedy symbols
- Only local maximum guaranteed (as in F-B)
- Problems
  - Assumes independence between the expansion of non-terminals
  - Lack of word sensitivity within word class

11.2.2 N-gram Language Models

- A stochastic language model gives the probability $P(\mathbf{W})$ that a word string **W** occurs as a sentence

$$
\begin{aligned}
P(W) &= P(w_1, w_2, \ldots, w_n) \\
&= P(w_1)P(w_2 \mid w_1)P(w_3 \mid w_1, w_2)\ldots P(w_n \mid w_1, w_2, \ldots, w_{n-1}) \\
&= \prod_{i=1}^{n} P(w_i \mid w_1, w_2, \ldots, w_{n-1})
\end{aligned}
$$

- Theoretically, every word depends on all previous words
  – Huge number of possible unique preceding strings
  – Very low occurrence in training data
- Assume dependence only on recent words
  – unigram, bigram, trigram, …, n-gram

Unigram, bigram, etc., estimation

- Unigram: $$P(\mathbf{W}) = \prod_{i=1}^{n} P(w_i)$$
- Bigram: $$P(\mathbf{W}) = \prod_{i=1}^{n} P(w_i | w_{i-1})$$
- Trigram: $$P(\mathbf{W}) = \prod_{i=1}^{n} P(w_i | w_{i-2}, w_{i-1})$$

- Probability estimation is simple occurrence count
  - (why not EM algorithm?)

$$P(w_i | w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i)}{C(w_{i-2}, w_{i-1})}$$

11.3 Complexity Measure of Language Models

- Test-set perplexity
  - Evaluates the generalization capability of the language model
- Training-set perplexity
  - Measures how the language model fits the training data
- Typical perplexity values
  - Digit strings: 10
  - n-gram on English text 50 - 1000
  - Wall Street Journal test set
    - trigram 128
    - bigram 176

11.4 N-Gram Smoothing

- Problem
  - Many very possible word sequences may have been observed in zero or very low numbers in the training data
  - Leads to extremely low probabilities, effectively disabling this word sequence, no matter how strong the acoustic evidence is
- Solution: smoothing
  - produce more robust probabilities for unseen data at the cost of modeling the training data slightly worse

N-gram Smoothing - simple technique

- Add constant (often 1) to all word sequence counts
- Example for bigrams:

$$P(w_i \mid w_{i-1}) = \frac{1 + C(w_{i-1}, w_i)}{\sum_{w_i} (1 + C(w_{i-1}, w_i))} = \frac{1 + C(w_{i-1}, w_i)}{V + \sum_{w_i} C(w_{i-1}, w_i)}$$

Interpolation and Backoff Smoothing

- Interpolation models
  - Linear combination with lower order n-grams
  - Modifies the probabilities of *both* non-zero and zero count n-grams
- Backoff models
  - Use lower order n-grams when the requested n-gram has zero or very low count in the training data
  - Computes models with zero count from lower order n-grams.
  - Nonzero count n-grams not updated by lower order n-grams
  - *Discounting*
    - Reduce the probability of seen n-grams and distribute among unseen ones

11.4.1 Deleted Interpolation Smoothing

- Interpolation between n-grams of different length
- Example on combination of unigrams and bigrams

$$P_I(w_i \mid w_{i-1}) = \lambda P(w_i \mid w_{i-1}) + (1-\lambda) P(w_i)$$

- The optimal λ is specific for each word history
  – A high-frequent context generally gets higher weight
  – Requires enormous amount of training data
- Cluster into moderate number of weights

11.4.2 Backoff Smoothing

- **Good-Turing Estimate (1953)**
  - Better estimate of correct n-gram frequency
  - Partition n-grams into groups depending on their frequency in the training data
  - Change the number of occurrences of an n-gram according to

$$r^* = (r+1)\frac{n_{r+1}}{n_r}$$

    - where r is the occurrence number
    - n_r is the number of n-grams that occur r times
- The **Katz smoothing** extends the Good-Turing estimate by combining higher and lower order models
- Bigram example:

$$C^*(w_{i-1}w_i) = \begin{cases} d_r r & \text{if } r > 0 \\ \alpha(w_{i-1})P(w_i) & \text{if } r = 0 \end{cases} \qquad d_r \approx r^*/r$$

$\alpha(w_{i-1})$ is computed to satisfy the probability constraints

- Discount non-zero bigrams and distribute among zero-count bigrams

Motivation for Good-Turing Estimate

Example (not from book)

- Estimate how common various species of birds are in your garden. You log the first 1000 birds you see; perhaps you see 212 sparrows, 109 robins, 58 blackbirds, and lesser numbers of other species, down to one each of a list of uncommon birds.
- What is the probability that the next bird seen will be, say, a blackbird?
- Most people would surely say that the best guess is 58 ÷ 1000, i.e. 0.058.
- Well, that's wrong.
- Consider an uncommon species which didn't occur in the thousand-bird sample, but which does occasionally visit your garden: say, nightingales. If the probability of blackbirds is estimated as 58 ÷ 1000, then the probability of nightingales would be estimated as 0 ÷ 1000, i.e. nonexistent. Obviously this is an underestimate for nightingales; and correspondingly 58 ÷ 1000 is an overestimate for blackbirds.
- Gale and Sampson "Good–Turing frequency estimation without tears", *Journal of Quantitative Linguistics*, vol. 2 pp. 217–37
- G. Sampson http://www.grsampson.net/RGoodTur.html

Alternative Backoff Models

- **Absolute discounting**
  - Subtract constant from each non-zero count
- **Kneser-Ney smoothing**
  - Background
    - Lower order n-grams are often used as backoff model if the count of a higher-order n-gram is too low (e.g. unigram instead of bigram)
  - Problem example
    - Some words with relatively high unigram probability only occur in a few bigrams. E.g. *Francisco*, which is mainly found in *San Francisco*. However, infrequent word pairs, such as *New Francisco,* will be given too high probability if the unigram probabilities of *New* and *Francisco* are used. Maybe instead, the *Francisco* unigram should have a lower value to prevent it from occurring in other contexts.
  - Method
    - Instead of counting the occurrences of a unigram, count the number of *word identities* that it follows.
    - $P_{KN}(w_i)$ = (The number of *word identities* that it follows) / (The vocabulary size)
    - Discount and interpolate to estimate smoothed bigrams from KN unigrams and low-frequency bigrams

11.4.3 Class N-grams

- Group words into semantic or grammatical classes and build n-grams for class sequences

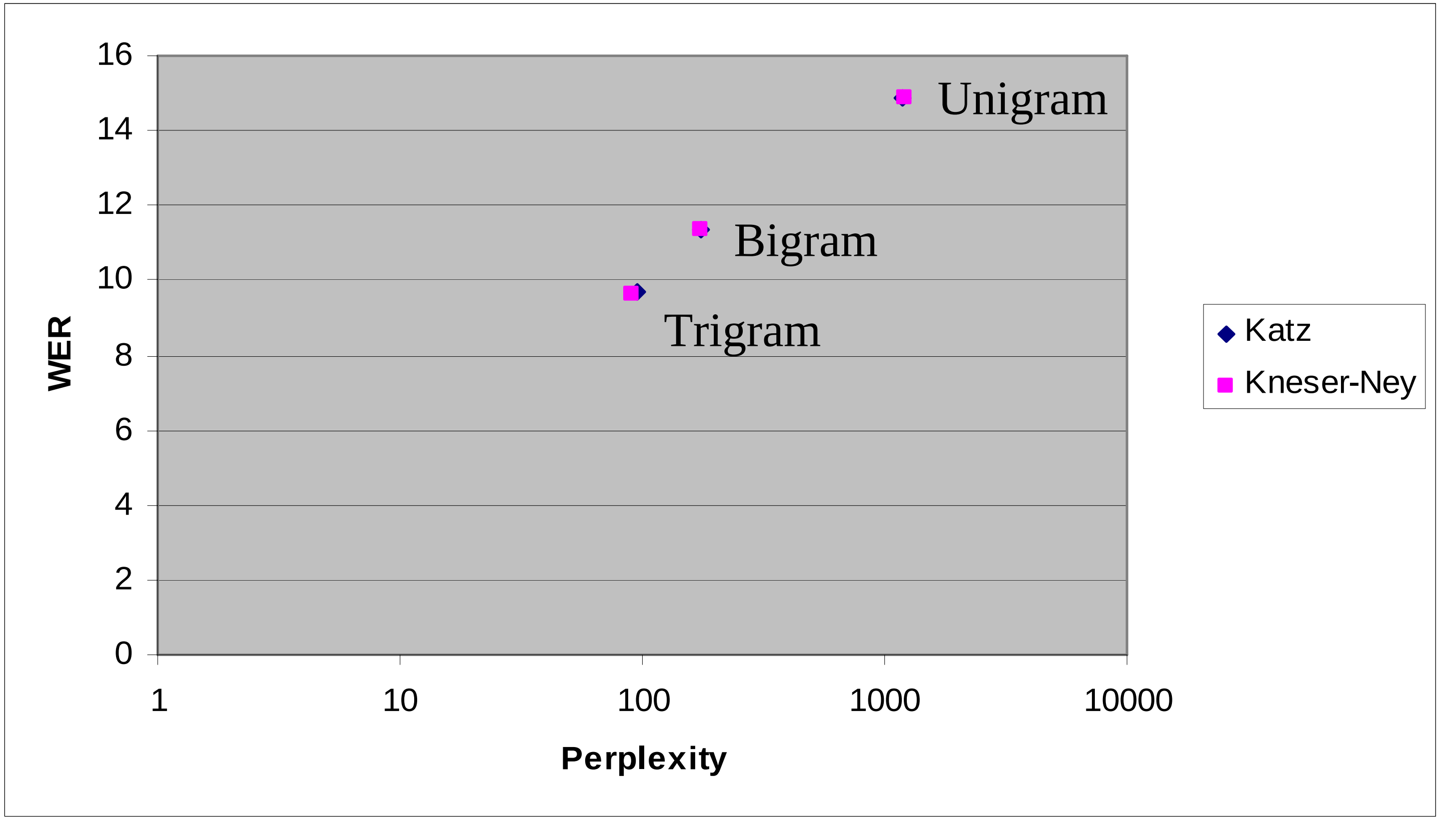
$$P(w_i \mid c_{i-n+1} \dots c_{i-1}) = P(w_i \mid c_i) P(c_i \mid c_{i-n+1} \dots c_{i-1})$$

- Benefits
  - rapid adaptation, small training sets, reduced memory requirement
- Very helpful for limited domain recognition
- Classes can be rule-based or data-driven
  - Rule- and knowledge-based classes useful in domain-specific systems
  - Data-driven in general-purpose systems
    - EM algorithm for clustering

11.4.4 Performance of N-gram Smoothing

- Best: Kneser-Ney (small difference)
- Next: Katz and Deleted Interpolation
- All three significantly better than No Smoothing
  - Regardless of the amount of training data
    - If all parameters can be accurately trained, then switch to a higher order n-gram and sparsity becomes an issue again

Relation n-gram length and perplexity vs. word error rate



MS Whisper results

11.5 Adaptive Language Models

- Dynamic adjustment of the language model
  - Conversation topic is unstationary
  - Topic remains for some period of time
- Techniques
  - Cache Language Models
  - Topic-Adaptive Models
  - Maximum Entropy Models

11.5.1 Cache Language Models

- Basic idea
  - Accumulate n-grams spoken so far
  - Use these to create local (low-order) dynamic n-gram models
  - Interpolate with static n-gram

$$P_{cache}(w_i \mid w_{i-n+1} \dots w_{i-1})$$
$$= \lambda_c P_{static}(w_i \mid w_{i-n+1} \dots w_{i-1}) + (1-\lambda_c) P_{cache}(w_i \mid w_{i-2} w_{i-1})$$

  - Accounts for the fact that many words tend to be repeated during e.g. a conversation or dictation
  - But doesn't account for higher probability of words in the same category (topic-specific words)

11.5.2 Topic-Adaptive Models

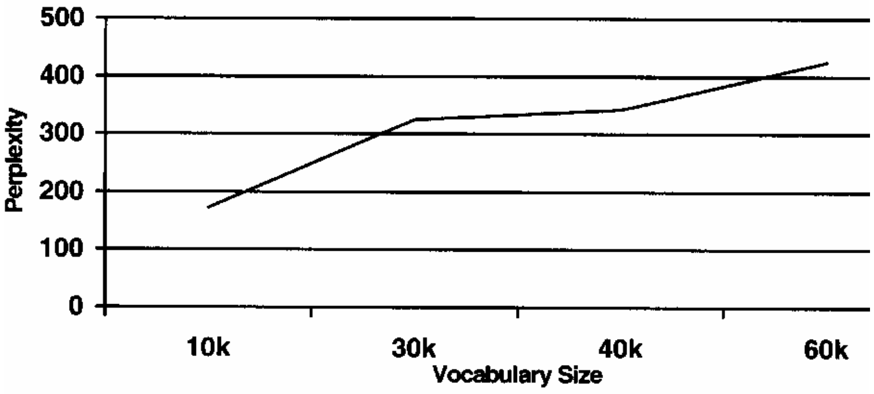
- Topic information can improve the static language model
  - Example
    - The most probable word after “*the operating*” in a hospital is different from that in an office
- Topic-clustered language models
  - Manual or data-driven (better)
  - Use information retrieval techniques to find the appropriate documents in the training database
    - Step 1: Use what is recognized so far to find similar documents
    - Step 2: Adapt the topic-independent model to these documents
    - Retrieval measure TFIDF (Term Frequency - Inverse Document Frequency) can be used to locate similar documents in the training database

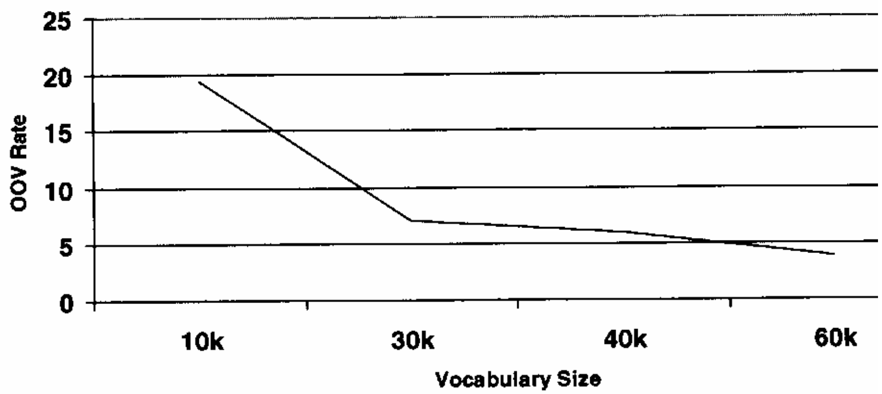
11.5.3 Maximum Entropy Models

- Combine n-gram models with another method than linear interpolation
- … … ?
- Has not offered significant improvement in comparison to linear interpolation

11.6 Practical Issues

- Vocabulary size
  - Conflict confusion rate vs. out-of-vocabulary (OOV) rate
  - For 99.5% English coverage 200 000 word vocabulary is required
  - Larger for inflectional languages (e.g. Swedish, German)
  - Combine fixed and personal vocabularies
  - Increase coverage 93% => 98% by adding 1000-4000 personal words





11.6.2 N-gram Pruning

- The n-gram model size becomes easily too large for practical applications
  - Pruning necessary
    - Remove low-count n-grams (those with lowest effect on entropy)
    - The remaining probabilities are unchanged
    - The backoff weights are recomputed
  - Pruning is effective
    - Trigrams can be compressed 25% with no performance degradation
    - Pruned 4-gram model better than unpruned (much larger) trigram model

11.6.3 CFG vs. N-gram Models

- Combine the portability of n-grams with the domain-specificity of CFG
  - Similar to class n-grams but the categories can be CFGs

How large training data to reach human listening performance?

Extrapolated word error rates of a state-of-the-art system for increasing quantities of training data
(Moore, Eurospeech 2003)