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C. BEAT THEORIES OF MUSICAL CONSONANCE

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Abstract

Helmholtz' beat theory of consonance has recently been given a new lease on life, mainly through the theory of Plomp and Levelt ("Tonal consonance and critical bandwidth", J.Acoust.Soc.Am. 38, pp. 548-560, 1965) based on the critical-bandwidth concept. Their experiments on pure tone pairs allowed them to calculate dissonance values for complex tone pairs on the assumption that the roughness produced by beats was additive. In an attempt to test the correctness of this assumption the dissonance values of complex dyads as well as tetrads were rated by eighteen subjects. There was a reasonable agreement for most dyads, but for other dyads and for the tetrads the model clearly did not work. A comparison of two- and four-tone chords showed that consonant intervals mitigate the sharpness of simultaneous dissonances. The periodicity model for pitch perception appears to offer a mechanism for both the roughness and the pleasantness of isolated chords.

Since the early discovery that consonance is related to simple numerical frequency ratios, many suggestions have been put forward to explain the particular effects of the most common musical intervals. The best known, that of Helmholtz, was made in terms of beats. As two pure tones are gradually separated in frequency, the beats become more and more rapid and the sound increasingly rough, reaching a roughness maximum, according to Helmholtz, for a frequency difference of 30-40 Hz. At larger frequency separations the roughness decreases, and the sound becomes consonant for all interval ratios. For complex tones the situation is different. Tones with simple interval ratios, for instance the octave (2:1), the fifth (3:2), and the fourth (4:3) will have harmonics that often coincide, and there will be beats between few adjacent harmonics. In contrast, dissonant intervals, such as the major seventh, will have no or few coinciding harmonics, and many adjacent ones producing beats, Helmholtz' theory was widely acclaimed at the time, but never gained total acceptance in the scientific and musicological community.

Criticism was voiced on some important points. First, many research workers believed that chords made up of pure tones could be set up which were dissonant without having beats between the tones. Second, it was pointed out that the 30 Hz frequency separation producing maximal dissonance only seemed to apply for the medium frequency range. Third, some musicians felt that such a fundamental musical phenomenon as consonance could not possibly be based on the mere absence of disturbing noise among the partials of the tones.

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Recent psychoacoustic studies have greatly reduced the relevance of much of this criticism. It is above all the work by Plomp & Levelt (1965) which has made the beat theory of consonance the most widely accepted today. Plomp and Levelt could confirm that consonance for pure tones was a function of the distance between the tones rather than of frequency ratios. The dissonance assigned by some critics to certain combinations of pure tones the authors attributed to the high degree of musical training the critics had received, which made them identify the intervals heard and classify them according to their preconceived ideas of what the consonance values should be. Like Helmholtz, they found a clear maximum in the frequency separation curve for dissonance, but one which varied systematically according to the frequency range. Plomp and Levelt could show that it was proportional to the critical bandwidth, a psychophysical concept based on data from studies on auditory masking, loudness and the ear's ability to hear out individual components in a complex tone. As a measure of the limit for interaction between tones critical bandwidth is clearly relevant to auditory beats. The maximum value for dissonance being fixed at about 25% of the critical bandwidth it was possible to draw a normalized curve for consonance and dissonance as a function of frequency separation between pure tones. Using this curve one may compute a theoretical value for dyads of complex tones by finding the value for every pair of adjacent harmonics of the two complex tones and adding them up. The assumption is then that dissonance is the sum of contributions from all pairs of interfering harmonics.

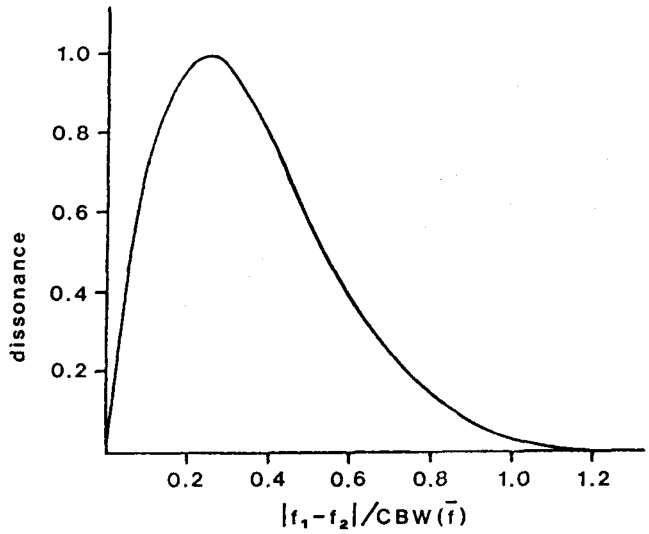

Fig. 1. Normalized curve representing dissonance of pure tone intervals as a function of frequency difference in units of the critical bandwidth (after Plomp & Levelt). The graph, in accordance with Hutchinson & Knopoff, shows the variation in dissonance rather than consonance.

The curve we computed for complex tones consisting of six harmonics (Fig. 2) agrees with that of Plomp and Levelt with sharpness clearly related to consonance of the intervals. The dissonance maxima are broader and not quite in agreement with the common rank ordering. In particular the major seventh is noticeably less dissonant according to this curve than earlier experiments and musical practice have led us to expect. In view of these discrepancies we decided to extend this method of calculating dissonance to other combinations of complex tones.

1. Method

A. Stimuli

The signals used as stimuli in the experiment were generated in real time by a custom-built digital signal processor controlled by a personal computer. The signal processor can be programmed by microcode to act as a filter, generator or some other device. In the present experiment it was used to generate a bank of sine-wave oscillators. The two DA-converters used are 16-bit special audio converters. A sampling frequency of 25 KHz was used throughout the experiment.

The set-up was designed, built and programmed by one of the authors.

B. Subjects and procedure.

Eighteen subjects, mostly staff and students at the Dept. of Speech Communication and Music Acoustics took part in the experiments. All reported an interest in music and almost everyone had experience of performing music. About half the group had some knowledge of the theory of music, but only two had studied harmony systematically.

The listening conditions were chosen so as to correspond as closely as possible to those in the Plomp and Levelt experiment. The subjects judged each stimulus on a 7-point scale, 7 corresponding to most dissonant, and 1 to most consonant (actually the reverse of the Plomp and Levelt scale). No definition of dissonance was given on the instruction sheet, and none of the subject asked for one. The listening took place in a sound proof booth at a sensation level of approximately 60 dB. The stimuli were all tones with six partials of equal amplitude. They were presented for about four seconds, with an eleven second interval in which the subjects had to record their ratings on a prepared sheet. The stimuli were presented on two separate occasions in a different order. A test session with the fourteen dyads preceded the first experiment.

2. Experiment 1

In the first experiment the subjects listened to fourteen intervals. The lower tone was always 240 Hz. Twelve were the intervals within an octave, and two were minor ninths, an interval not plotted in the Plomp and Levelt study. One of these was similar to the other twelve in being based on complex tones with six partials, the other consisted of tones where that partial in each tone which contributed most to the total dissonance value had been removed. The computed dissonance value was thereby reduced from 2.35 for the two six-partial tones to 0.68 for the two five-partial ones.

The results are shown in Figs. 2 and 3. In the first figure the rated dissonance values have been added to the computed curve for the dissonance values for two six-partial tones over a range of just over an octave. The second figure plots the rated values as a function of computed values.

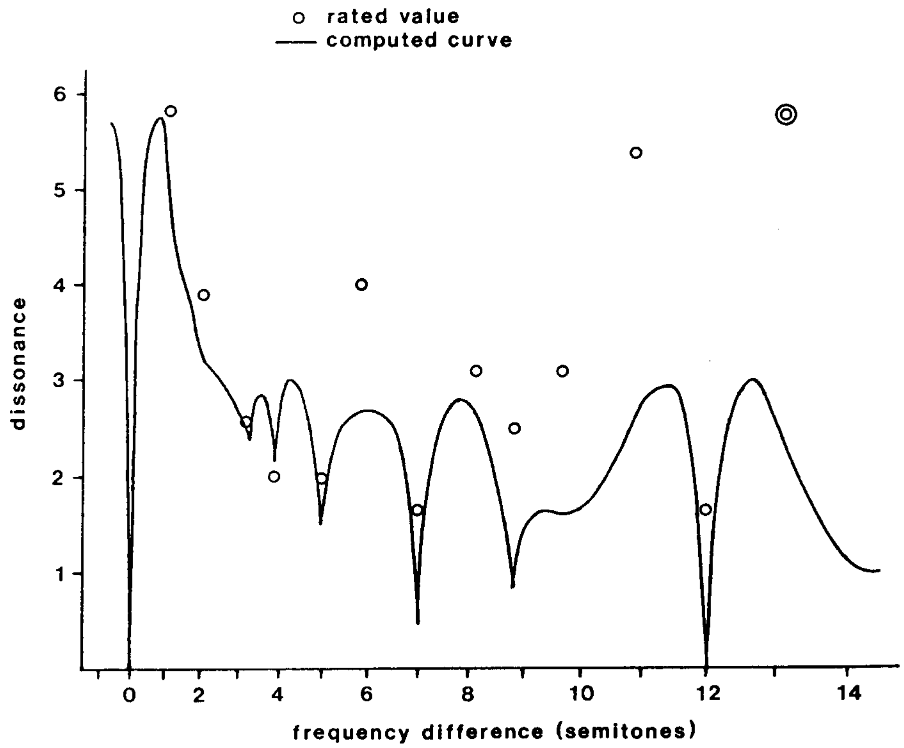

Fig. 2. Computed and rated values of dissonance as a function of frequency difference.

There is for most intervals a reasonably good agreement between the computed and rated dissonance. The rank order for rated consonance is the traditional one, except for the minor seventh, which received the same rating as the minor sixth, which is commonly considered more consonant. The major seventh and the minor ninth with no partial removed, however, were rated considerably more dissonant than we would expect from the computed value. But much more remarkable is the case of the minor ninth with one partial removed from each complex tone. Here, in spite of a difference in computed value of more than 70% the two ninth intervals were judged equally dissonant. It is difficult to reconcile this result with a beat theory of consonance.

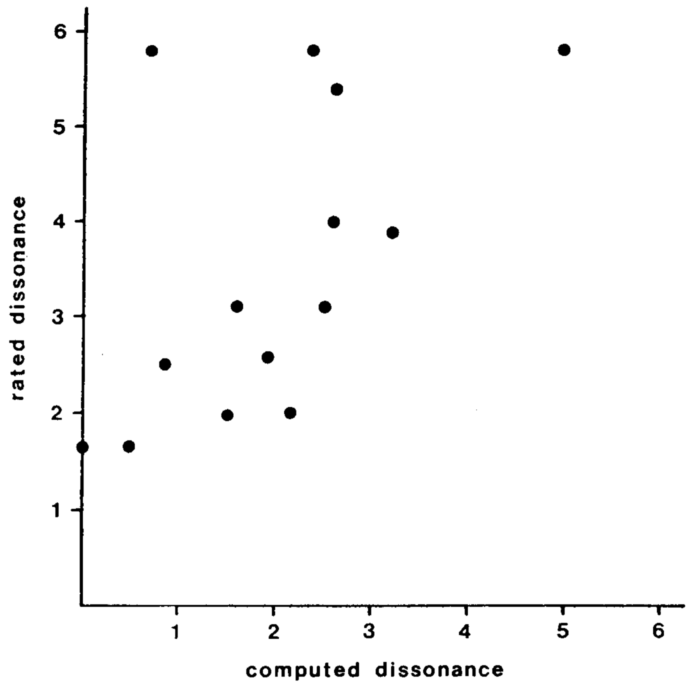

Fig. 3. Rated dissonance values for dyads as a function of computed values.

Experiment 2

The nineteen chords in the second experiment were all combinations of four complex tones with six partials. A chord was sometimes used more than once in different arrangements (inversions) or with different tuning. The major triad occurred with added major sixth (chord 12), minor seventh (8, 10, 15, 16), major seventh (7, 14), and octave (1, 18). The minor triad was used with added major sixth (11, 13, 17), minor seventh (4, 9), and octave (2). All the commonly used chords within one octave can be found, with the exception of the diminished and the augmented chords. In addition there were two chords exceeding an octave, one a major triad with the third above the octave in pythagorean tuning, the other a non-traditional chord (5). The narrowly spaced cluster chord is, of course, also a non-traditional one. The results can be seen in Table 1. (The notes are about one whole-tone higher than the actual frequency in order to refer the chords to C rather than B flat.) The just major (1) and minor (2) chords were rated predictably low in dissonance, while the cluster chord (19) and the irregular chord (3) were rated correspondingly high. The minor seventh chords were generally felt to be fairly consonant, with the exception of No. 10, where the natural seventh (7:4) probably contributed to a slightly unpleasant effect. The tempered seventh chords were judged quite consonant, the third inversion (16) even more so than the root position (15). Among the other chords the major ninth chord without the third (6) stands out as being considered surprisingly dissonant, as did the pythagorean third chord (3).

The main purpose of the experiment, however, was to compare computed and rated values for the dissonance. Even with scales that are not strictly comparable the discrepancies are noticeable. They are brought out more clearly in Fig. 4. The simple relationship predicted by the Plomp and Levelt theory is nowhere to be seen.

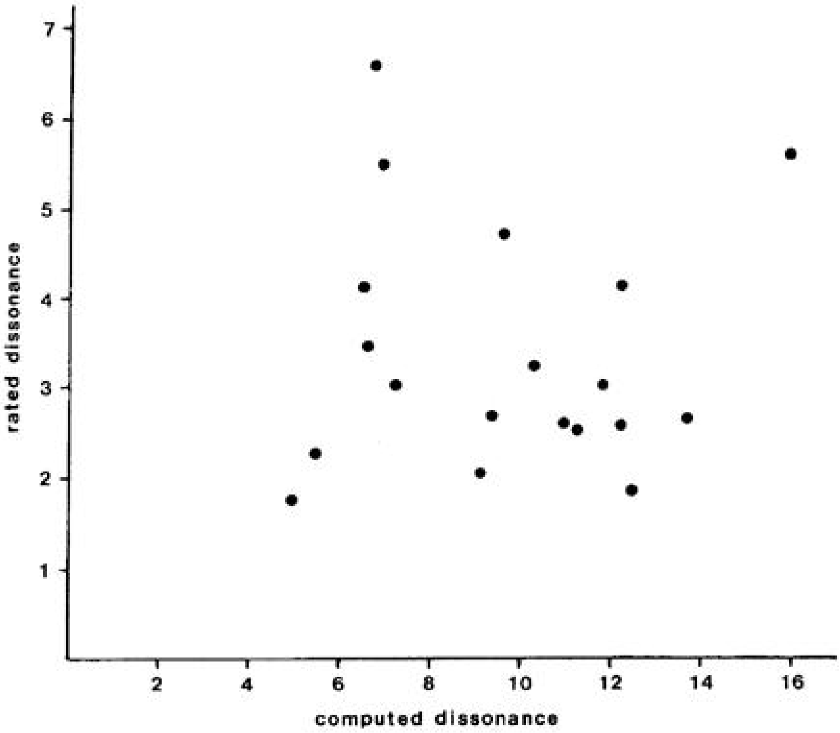

Fig. 4. Rated dissonance values for tetrads as a function of computed values.

3. Discussion

Before considering the reasons for the failure of the Plomp and Levelt model, we should ask if other versions of the beat theory might more successfully predict the perceived dissonance of musical chords.

Kameoka & Kuriyagawa (1969a,b) made calculations for complex dyads based on their own study of the dissonance of pure tone pairs. Their results differed in some details from those of Plomp and Levelt, but for the frequency range we are concerned with here, the differences are unimportant. Their values for complex dyads calculated according to an extension of the Plomp and Levelt model show similar discrepancies to those noted earlier. The interval of the major seventh is thus computed to be far less dissonant than we would expect. This discrepancy has been commented on by Terhardt (1984), who, considering the fact that the major seventh in the Kameoka and Kuriyagawa study was less dissonant

chord no	frequencies and names of tones in chords				tuning	computed dissonance	rated dissonance
1	240 'C	300 'E	360 'G	480 'C	just	4.94	1.76
2	240 'C	288 'Eb	360 'G	480 'C	just	5.45	2.28
3	240 'C	360 'G	480 'C	611 'E	pythagorean third	6.51	4.12
4	240 'C	288 'Eb	360 'G	432 'Bb	mixed just	6.61	3.45
5	240 'C	440 'B	640 'F	880 'B	irregular	6.71	6.61
6	240 'C	360 'G	420 'Bb	540 'D	just	6.9	5.5
7	240 'C	300 'E	360 'G	450 'B	just	7.21	3.02
8	240 'C	270 'D	337.5 'F#	405 'A	mixed just	9.09	2.06
9	240 'C	270 'D	320 'F	400 'A	just 3rd and 4th	9.33	2.68
10	240 'C	300 'E	360 'G	420 'Bb	just	9.61	4.72
11	240 'C	288 'Eb	343 'Gb	432 'Bb	mixed just	10.3	3.22
12	240 'C	305 'E	358 'G	403 'A	tempered	11.24	2.52
13	240 'C	306 'E	339 'F#	400 'A	irregular	11.8	3.03
14	225 'B	240 'C	300 'E	360 'G	just	12.18	4.15
15	240 'C	302.5 'E	359.5 'G	428 'Bb	tempered	12.19	2.57
16	240 'C	269 'D	339 'F#	403 'A	tempered	12.45	1.85
17	240 'C	270 'D	320 'F	384 'Ab	mixed just	13.67	2.65
18	240 'C	305 'E	358 'G	403 'A	mistuned	10.96	2.59
19	240 'C	270 'D	300 'E	320 'F	just	15.9	5.62

Table I.

than even the fourth for the kind of complex tones we are dealing with, concludes that the psychoacoustical concept of dissonance has to be modified so as to include the musical harmony aspect. We think, however, that there is not necessarily a conflict between the psychoacoustic data and musical theory in this particular case. Terhardt does not take into account the fact that this calculated value is out of line not only with the rated value in the study, but even more with every earlier study where the interval has been rated. The value in our study is, of course, in complete agreement with the traditional view. It seems more natural to conclude that the model for the calculations cannot account for the results and must, in some respects at least, be considered deficient.

Hutchinson & Knopoff (1978) have also calculated the dissonance of dyads based on a version of the critical bandwidth model modified by means of a change in the shape of the critical bandwidth curve over an extended frequency range. As a consequence the dissonance value for a given interval may differ by a factor of nearly ten over a range of three octaves, which would seem to make the model irrelevant for musical purposes.

If the calculated dissonance by the various proponents of the beat theory at times are strangely at odds with musical experience, our own computed values listed in Table I seem to justify a more sweeping conclusion: performance of music would be impossible if the beat theory , at least in its present form, were correct. Plomp and Levelt point out that the steepness of the curve around the octave and the fifth (Fig. 2) make these intervals much more sensitive to a deviation from the correct frequency ratio than other consonant intervals, such as the third, which may explain why we tolerate impure thirds. But our numerical calculations show that even deviations smaller than those which normally occur in musical performance imply a change in dissonance which may be considerably greater than a change from a chord which we would unquestionably consider being consonant into a dissonant one.

Even if we would be prepared to separate consonance into the two components sensory consonance and harmony, as suggested by Terhardt, it is difficult to see what possible relevance the former concept would have if it is based on the beat theory given the extreme sensitivity of consonance to what, after all, are constantly occurring variations in intonation.

The experimental results in this study merely confirm the inadequacy of the beat model. The change from a seventh chord in just intonation (10) into a tempered one (15) even results in a lower estimate of dissonance in spite of an increase in computed value. It is true that a slightly mistuned major tetrad (6) was judged slightly more dissonant than the chord in just intonation, but the increase is still far smaller than the computed change.

Moreover, these are certainly not changes in intonation which would be uncommon in performance of music. There is also the question whether a slightly mistuned major chord can in any sense be considered dissonant. In this case it is likely that some subjects reacted to the out-of-tuneness rather than what they normally perceived as dissonance.

Still the most important finding is the lack of correspondance between the rank ordering for the computed and the rated consonance of the chords. This poses a problem. Plomp and Levelt have shown beyond any doubt that dissonance for complex tones is related to the presence of harmonics. What then can account for the emergence of dissonance when harmonics are added to pure tones other than beats among adjacent harmonics?

The presumption of additivity for the roughness of beats, tentatively adopted by Plomp and Levelt, seems to be supported by other studies, such as that of Terhardt (1968), which shows that the roughness produced by amplitude-modulated tones in certain circumstances is additive. Yet, the fact that in our first experiment the strongest beats in a dyad could be removed seemingly without affecting the perceived dissonance implies that there can be no simple additivity of roughness produced by beats between adjacent partials.

But if the interaction between adjacent components is not responsible for dissonance, how can the harmonics excercise their influence?

We are led to the conclusion that harmonics achieve the effect of dissonance not primarily from the individual interaction with adjacent harmonics, but by collectively interacting with another group of partials when the interval between the fundamentals is a dissonant one.

Removing the fundamentals of two complex tones, moreover, does not leave just the dissonance unaffected, but also the pitch of the tones, which still corresponds to the absent fundamentals. The latter effect has been called the residue or the periodicity pitch phenomenon.

The term residue was introduced by Schouten (1940) to indicate that the higher harmonics, which cannot be perceived separately, are perceived collectively as one component (the residue) with a pitch determined by the periodicity of the collective waveform and equal to the fundamental tone. It was later shown that lower harmonics are more important than the higher ones in determining the pitch. This is what we would expect if frequency resolution is essential for the perception of low pitch, but it is hardly the proof, as is often maintained, that the hearing out of individual harmonics is necessary for the effect. Periodicity pitch may be present even when the individual partials cannot be resolved (Nordmark, 1978). It is, for instance, difficult to see how the harmonics of complex tones making up a minor second can be resolved. All harmonics of one tone are at a distance from the other of a fraction of a critical band - a concept used to define the limits of frequency resolution.

It is therefore tempting to assume that all the partials of a tone can act together collectively both to contribute to the sensation of pitch and, in the presence of other complex tones, to the sensation of consonance and dissonance. The roughness we associate with dissonance can be understood to be mainly the result of interference on the basilar membrane between the collective waveforms of the constituent tones, with only a minor contribution from the individual beats. A consequence of this view is that we should expect complex tones made up of inharmonic partials to form less distinct dissonant intervals.

There are problems, however, with any form of the additivity hypothesis. A comparison between the dyad C-B, the major seventh, and the tetrad C-E-G-B (7) shows that the tetrad has a lower rating (3.02) than the dyad (5.35), whereas we would, of course, expect the opposite, as any addition of tones should only increase dissonance. Furthermore, the third inversion of the chord, where B is the tone just below C (14), was judged much less dissonant (4.15) than the minor second without the third (E) and the fifth (G), which at 5.95 was the most dissonant dyad of all. A similar observation has been made by Kunitz (1960), who pointed out that any addition of consonant intervals, such as E and G to the C-B major seventh, decreases dissonance.

Would the same be true if we add a tone which forms a dissonant interval with one or more of the constituents of a chord? Unfortunately the experiments did not include examples of simple additions of dissonances. However, a comparison of chords 9 and 19, which are identical except for one tone, goes some way towards answering the question. In this case the chord with one dissonant interval is perceived as less than half as dissonant as the one with three dissonances. Can we therefore conclude that the addition of a tone or tones to other tones with which they form consonant intervals reduces dissonance? We would then expect a major chord to sound more consonant than the major third alone, and the minor chord more consonant than the minor third. The results give some small support for this conclusion. The rated dissonance value of the minor chord, for instance, was 2.28 compared with 2.7 for the minor third. There is a similar difference for the major chords.

With dissonance added to dissonance there is, as we have seen, sometimes an additive effect. At other times the effect appears to be the opposite. One such example would be the addition of a D and an F sharp to the major seventh interval C-B. Our impression is that the resulting chord is dissonant, but less sharp and perhaps more pleasing and musically meaningful.

What then is the reason for these unexpected variations in consonance and pleasantness?

There must obviously be some other factor involved, one for want of an explanatory term we may simply call the musical factor, and which is responsible for the reduced unpleasantness of dissonant intervals when consonant ones are added. In some ways we have come close to Terhardt's (1984) separation of musical consonance into two components, sensory consonance and harmony. The difficulty for us with his particular definition is that sensory consonance, as the psychophysical basis of pleasantness for all kinds of sounds, is considered to be the mere absence of roughness. This definition is too narrow for our purpose. To make sense of our results we have to include musical factors even in isolated, static chord, factors which Terhardt would consider to be part of the harmony component and to be of less importance for single chords. The difference in our respective approach, however, is not only one of emphasis. It also reflects our differing views on the origin of roughness and harmony. We think the periodicity concept is relevant to both roughness, as described earlier, and to harmonic effects, such as the naturalness and pleasantness of certain intervals or chords. No "explanation" can be given at present for the phenomena of octave similarity or consonance of intervals with simple or close to simple numerical ratios. But it is at least more natural from this point of view to link these effects to relations between the time intervals giving rise to the pitch of the constituent tones in a chord than to assume that they are entirely due to experience or cultural habits.

In spite of the attractiveness of periodicity as a unifying concept for a number of related phenomena, we must be aware of its limitations. Consonance in music has many meanings, some of which are only distantly related to the phenomena considered here. Dissonant chords, for example, are often the musically most pleasing, and consonant chords sometimes are dissonant from a theoretical point of view.

The widely held belief that consonance exists as a separate entity to be defined, measured, and explained is almost certainly based on an illusion.

Acknowledgements

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