KTH Computer Science
and Communication

Analysis of Drumbeats – Interaction between Drummer, Drumstick and Instrument

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Abstract

This thesis investigates the moment, when the drumstick hits the membrane of a drum. Experiments were conducted to obtain information about the interaction of drummer, drumstick and membrane during a single stroke. Using a modified drumstick, the force on the drumstick, the acceleration of the stick's tip, the duration of the contact between stick and membrane and the sound of the drum were recorded. Strokes were performed at different dynamic levels, at different positions on the drumhead and for different drumhead tensions. Furthermore, possible influences of the drumstick model and the drummer were analyzed briefly.

During the 3–6 ms of contact between drumstick and membrane, the interaction process was found to be influenced by the deflection of the drumhead, the vibration of the drumstick and a traveling wave on the drumhead. Altering the different parameters of the experiment, provided an insight into the influence of each interacting component on the process of striking the drum.

Acknowledgments

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Zu guter Letzt, herzlichen Dank an meine Eltern, die mir die Unterstützung und das Vertrauen gegeben haben, diesen Wunsch zu verwirklichen.

A human being should be able to change a diaper, plan an invasion, butcher a hog, conn a ship, design a building, write a sonnet, balance accounts, build a wall, set a bone, comfort the dying, take orders, give orders, cooperate, act alone, solve equations, analyze a new problem, pitch manure, program a computer, cook a tasty meal, fight efficiently, die gallantly. Specialization is for insects. (Robert A. Heinlein, 1973)

Contents

Chapter 1

Introduction

Trying to understand the physical details of acoustic instruments has been a major challenge to acoustics research over the last decades. A remarkable amount of knowledge about this topic has been compiled by Fletcher and Rossing in [9]. Based on the results of these works, attempts have been made to create computer models imitating the physics of the drum. Different physical modeling techniques are described by Välimäki et al. [17] and an extensive list of publications dealing with this topic has been collected by Smith [16].

Drums in particular have been subject to many publications, which in the majority of cases focus on the vibrational behavior of the drum. Examples for research on different kinds of drums can be found in [14] and [2]. Recent publications on physical modeling of drums pick up the results of these works and try to model the vibration of the drumhead, as for example [10] and [1].

With so much knowledge about the vibrations of the drum, the physical process of striking the drum has been left widely unregarded. Research on the sound of impacts in general has been performed e.g. within the Sounding Object Project [13]. Concerning musical instruments, the interaction of hammer and string in the piano has been subject to many publications (e.g. [3]) but the interaction of drumstick and drum has remained a nearly white spot on the map of music acoustics research.

It was therefore the aim of this work to take a close look at the moment when the drumstick touches the membrane of a drum and gain knowledge about the interaction between drum, drumstick and drummer.

The following Chapter 2 provides the reader with basic information on the physics of drums. Chapter 3 then describes the main experiment conducted in the context of this work. The results of this experiment are presented in chapter 4. The conclusions drawn from the analysis are discussed in chapter 5 and summarized in chapter 6. Additional experiments are described in appendices A and B.

Chapter 2

Theoretical Background

Drums have evolved along with the human race for ages and have always played an important role in virtually every musical culture. In [4], Blades gives a detailed insight into the history of drums and other percussion instruments.

As vibrating systems, Fletcher and Rossing [9] divide all drums into three categories:

- single membrane over enclosed air cavity (e.g. timpani, tabla)
- single membrane over an open shell (e.g. tom-toms, congas)
- two membranes coupled by an enclosed air cavity (e.g. snare drums)

This work focuses on the *snare drum* or *side drum* and hence the third category of drums; peeking into the second category as well by removing the second drumhead for a few experiments.

2.1 Design of the Snare Drum

Today, snare drums are mainly used in drum set playing, orchestral music and marching bands. As illustrated in Fig. 2.1, a snare drum consists of a cylindric *drum shell* with circular membranes – the *drumheads* – stretched over the openings of the shell. On top of the drumheads, metal or wooden *drum hoops* are placed over the *rim* of the drum. *Tension screws* around the drum hoops pull these towards the drum shell and tension the drumheads.

The shell of most snare drums measures 25.40 cm (10 in)[1] to 40.64 cm (16 in) in diameter, 12.70 cm (5 in) to 25.40 cm (10 in) in depth and is made of either wood, fiberglass or metal. The influence of size and material of the shell on the sound of the drum leads to a huge variety of different drum models to satisfy each drummer's preferences.

Figure 2.1: *Typical snare drums used for orchestral or drum set playing.*
Source: www.pearldrum.com

As for drumheads, they exist in as many variations as drum shells. Basic material for most drumheads is Mylar®, a strong polyester film. Based on this common material, manufacturers have come up with a large number of variations like e.g. multi-layered drumheads or drumheads with a thicker region along the edge. All those modifications influence the vibrational behavior of the drumhead as described in section 2.2.1 and hence the sound of the drum.

In addition to the shell and the drumheads, strands of wire – the *snares* – stretched across the lower, or *snare head* determine the sound of the snare drum. When the upper, or *batter head* is struck, the snares begin to vibrate against the snare head and give the drum its unique, sharp sound. Furthermore, most snare drums are equipped with a lever mechanism that allows to release the snares from the snare head. Without the characteristic buzzing of the snares, the drum then sounds more like a two-membrane tom-tom.

[1]In connection with drums, *inches* (1 in = 25.4 mm) are often used to describe a length or thickness while all measures in this work use metric units. If a measure is commonly annotated in inches, this value is therefore added in parenthesis.

Usually, the snare drum is played with a pair of *drumsticks*. Although there are sticks made of synthetic materials such as carbon fiber, most sticks are made of wood – preferably hickory, maple or oak. Most drumsticks measure between 38 cm and 42 cm in length and weigh between 40 g and 70 g. For some models, the wooden tip of the stick is replaced with a tip made of nylon that grants higher durability. When striking a cymbal with the drumstick, the shape of the tip influences the sound and therefore the tip shape has become an important parameter as well. There is however up to now no real indication that the shape of the tip has any effect on the sound of drums. In [11], Irwin examined the sound initiated by different sticks and states that "the amplitudes of the various induced partials were not significantly different when the shape/size of the tip was changed dramatically".

Fig. 2.2 shows the front end of four different drumsticks and gives an impression of the variety of available models. A more detailed description of the different designs of drumsticks is given by Zoutendijk [18].

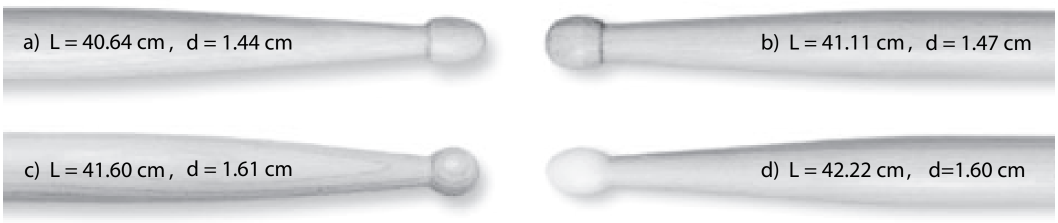

Figure 2.2: *Four drumstick examples with length L and shaft diameter d. Tip shapes: a) 'teardrop', b) 'barrel', c) 'round', d) 'oval' (nylon material). Source: www.vicfirth.com*

2.2 Physics of the Snare Drum

As for any other complex musical instrument, there is no simple generic model to explain or even simulate the physical behavior of a drum as a whole. It is therefore practical to take the drum apart and take a close look at each component with its individual physical properties.

Since the experiments conducted in the context of this work did focus on a drum with the snares removed, the vibration of the snares against the lower drumhead will be left out. The interested reader will find this *snare-action* explained in [9] and [14].

2.2.1 Vibrations of the Drumheads

The most obvious parts generating sound in drums are the drumheads. A vibrating drumhead is a complex physical system and to understand its behavior, it is useful to start with a simplified model – the ideal circular membrane.

The Ideal Circular Membrane

An ideal circular membrane is defined by its diameter d [m], surface tension T [N/m] and superficial density ρ [kg/m^2]. Attributes like bending stiffness or a non-uniform material density are neglected in this model.

The vibrational behavior of an ideal circular membrane, fixed at its circumference, is well known (see e.g. [9]): when excited, the membrane vibrates with a superposition of *modes* of order (m, n), where m defines the number of *nodal diameters* and n the number of *nodal circles* including the circular boundary. These nodal lines and circles are defined as regions, where the displacement of the membrane for the particular mode is minimal. Fig. 2.3 illustrates the first twelve modes of an ideal membrane with the nodal regions indicated by solid lines. Based on the distribution of these regions, the modes can be divided into three groups, indicated by different colors in Fig. 2.3:

- *circular* modes with $m = 0$, $n \geq 1$ (blue)
- *radial* modes with $m \geq 1$, $n = 1$ (green)
- *mixed* modes with $m \geq 1$, $n \geq 1$ (red)

The frequency of the fundamental $(0, 1)$ mode is calculated as

$$f_{0,1} = \frac{2.405}{\pi d} \sqrt{\frac{T}{\rho}} \tag{2.1}$$

The frequency ratios between the higher modes and the fundamental mode as annotated in Fig. 2.3 are in general inharmonic, leaving the sound of the sole membrane without a defined pitch.

Real Membranes

The model of the ideal membrane neglects two major influences: the properties of the membrane material and the environment, i.e. the air surrounding the vibrating membrane.

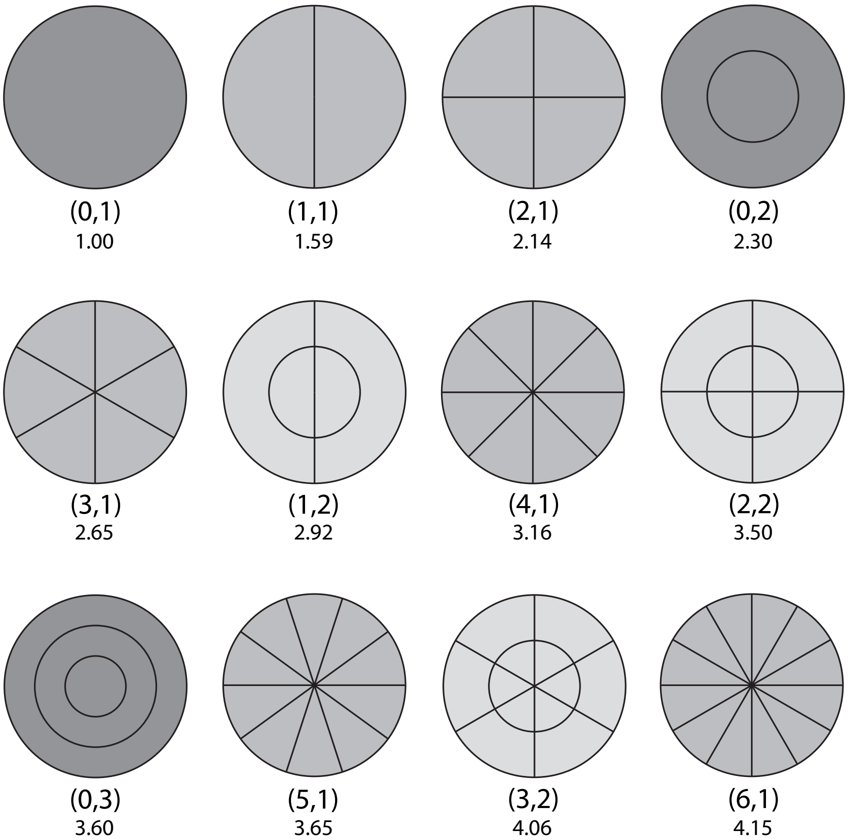

Figure 2.3: *Modes of an ideal circular membrane. Values (m, n) denote the number of nodal diameters m and nodal circles n and determine the order of the mode. The fraction below the order gives the ratio of the mode frequency $f_{m,n}$ in relation to $f_{0,1}$. The used colors divide the modes into three groups: circular (blue), radial (green) and mixed modes (red).*

The vibration of a membrane leads to a bending and shearing of the material. A real membrane will resist this elastic deformations to a certain degree due to the thickness and properties of the membrane material. This resistance is called *bending stiffness* and *stiffness to shear* and will in general raise the modal frequencies of the membrane.

Adding to this effect, any irregularity in the material can alter the modal structure. In modern drumheads for example, additional rings or layers of mylar on the membrane are used to specifically create irregularities in the membrane structure. These modifications then dampen or shift certain frequency modes and influence the sound of the drum, e.g. make it more harmonic. An extreme example for this effect is the Indian tabla as described in [9]: a thick patch applied in the middle of the membrane causes a completely different pattern of vibrational modes.

Concentrating on thin, single-layered membranes, another effect plays a more dominant role – *air loading*. This term describes the fact that the membrane interacts with the surrounding air when it is vibrating. Air that is moved by the membrane, can affect the frequency of the resonance modes in two possible directions. The mere presence of air around a membrane causes a *lowering* of the modal frequencies. In contrast, an air volume confined by a second membrane or a closed drum kettle will *raise* the frequencies, especially of the circular modes. In practice, the effect of air loading can be observed in the sound of the kettle drum or *timpani*: the enclosed air volume of the timpani's kettle is responsible for a shifting of modes which results in a nearly harmonic ratio of modes and gives the sound of the timpani a defined pitch.

Coupled Membranes

In drums with two drumheads, both membranes interact through the shell or the enclosed air volume between them. This interaction, or *coupling*, leads to the formation of mode pairs as illustrated in Fig. 2.4, especially for modes of lower order. As stated above, a single membrane shows a single mode of order $(0, 1)$ at frequency $f_{0,1}$. When two membranes are stretched over a drum shell, they *together* produce two modes of order $(0, 1)$ at frequencies $f_{0,1a}$ and $f_{0,1b}$. In the case of $f_{0,1a}$, the movements of the two drumheads are in phase while for $f_{0,1b}$, the membranes move with opposite phase. In [14], Rossing et al. describe this effect in detail and calculate the resulting modal frequencies using a two-mass model.

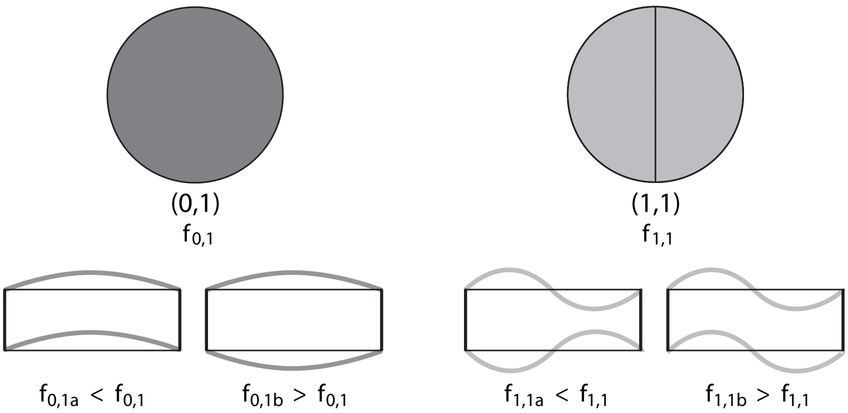

Figure 2.4: *Coupling of two membranes at the first two resonance modes. The side view of the drum illustrates the relative phase of the two vibrating membranes. $f_{0,1}$ and $f_{1,1}$ represent the modal frequency of a single membrane, the additional subscripts a and b mark the frequencies of the mode couples.*

2.2.2 Membrane Excitation

For percussion instruments, the vibrational structure of the membrane as described in section 2.2.1 is excited by striking the drumhead with either the hand or a striking tool – the drumstick in the case of the snare drum. Concerning the process of excitation, two variables are of great importance: the position of the stroke and the properties of the striking tool.

Striking Position

As illustrated in Fig. 2.3, the modes of a membrane have circular or diametric nodal regions, i.e. regions where the displacement of the membrane is minimal for this mode. When exiting a vibrational system at a certain point, all modes that have a nodal region at this point are theoretically not excited. Hence, when striking a circular membrane, modes with a nodal line or circle at the striking position will not be able to build up. A stroke at the very center of a membrane will for example excite only circular $(0, n)$ modes, since all other – radial and mixed – modes have a nodal line through the center. In contrast, a membrane excited near its circumference will theoretically contain all modal frequencies in its vibration spectrum.

The importance of this fundamental principle becomes audible when a drum is struck at the center and near the rim, respectively. The centered blow will produce a short, low pitched thud with the major part of its energy contained in the lowest circular $(0, 1)$ mode(s). Striking the drum near the rim will, on the other hand, result in a longer lasting sound with more energy in the higher modes. The normal place for striking an orchestral timpani for example, lies at about one fourth of the way from the edge to the center of the membrane. Exciting the drum at this position, transfers most energy to the radial $(m, 1)$ modes, contributing to the timpani's tonal character. When striking the timpani at the center, this character is lost since the circular $(0, n)$ modes dominate the spectrum of the drum sound.

Striking Tool

The properties of the striking tool or *mallet* also play an important role in the excitation of the modes. When striking a drum or any other percussion instrument, the force transmission between mallet and instrument is responsible for the energy transferred to the instrument. This interaction reflects a convolution in the time domain – or a multiplication in the frequency domain – of the applied force pulse $f(t)$ and the pulse response of the instrument as vibrational system $h(t)$.

This results in the vibration $s(t)$ of the instrument:

$$s(t) = f(t) * h(t) \tag{2.2}$$

or in the frequency domain:

$$S(j\omega) = F(j\omega) \cdot H(j\omega) \tag{2.3}$$

Eq. 2.3 clearly shows that the spectrum $F(j\omega)$ of the force pulse determines the highest modal frequency excited in the instrument. This behavior has been investigated by Bork for the xylophone [5] and by Asano et al. for a Japanese drum [2].

2.2.3 Vibrations of the Drum Shell

Like the drumheads, the shell of the drum vibrates in modes. Although the motion of the shell is very small compared to the displacement of the drumhead, a strong coupling between drumhead and shell motion leads to a significant contribution of shell modes to the sound spectrum of the drum. Since the vibration of the shell has no direct influence on the interaction between drumhead and drumstick, it will be left out at this point. The interested reader may find a detailed description of the shell motion in [14].

Chapter 3

Conducted Experiments

3.1 Experimental Setup

Spatial Setup

To analyze the interaction of drummer, drumstick and instrument in a realistic context, an experimental setup as illustrated by Fig. 3.1 was created. As in drum set playing, the drum was supported on a drum stand with the drummer sitting in front of it. This positioning allowed the drummer to play in a relaxed and natural way.

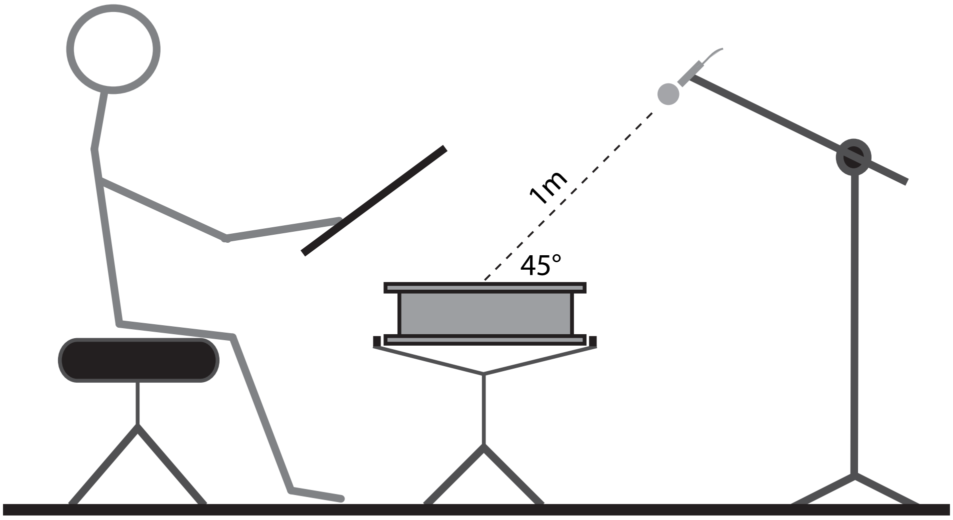

Figure 3.1: *Spatial setup of the conducted experiments. The drum is supported on a drum stand and the drummer is seated in front. This is the normal playing position for drum set playing. A microphone at one meter distance captured the sound of the drum.*

Drum and Drumheads

Subject to all conducted measurements was a Ludwig snare drum with a chrome-metal shell, 35.56 cm (14 in) in diameter and 12.70 cm (5 in) in depth. With these dimensions, it represents a popular type of drum used in both drum set and orchestral playing.

The drum was equipped with *Remo Ambassador* drumheads – *coated* for the batter head, *clear* for the snare head. This 0.254 mm (0.01 in) thick, single layered batter head model is a very popular choice among drum set players of all genres and was chosen as a typical drumhead for the snare drum.

To obtain a clean audio signal of the vibrating drumheads, the snare and its lever mechanism were removed from the shell. Whilst recording only one drumhead, the lower hoop and the snare head were removed as well.

Drumsticks

A pair of *Vic Firth American Classic 5B* drumsticks (referred to below as *Vic Firth*) was selected as a good example of the average drumstick used for drum set playing. This stick model, made of hickory, is 40.64 cm long, has a shaft diameter of 1.50 cm and weighs 53 g. The shape of its wooden tip falls into the category 'teardrop'.

During the experiments, a pair of *Premier Jim Kilpatrick KP2* drumsticks (referred to below as *Premier*) was additionally chosen to contrast the properties of the *Vic Firth*. This maple stick weighs 56 g, is 40.40 cm long, has a wooden 'barrel'-shaped tip and a slightly conical shaft which reaches a diameter of 2.00 cm at the end of the stick. Fig. 3.2 compares the dimensions of the two selected models.

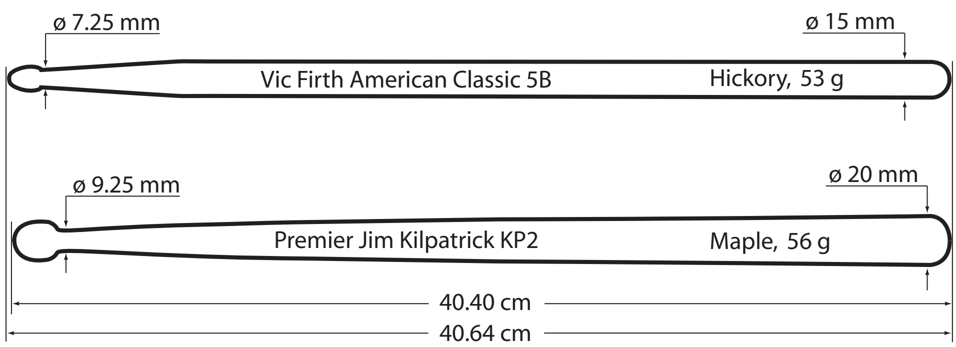

Figure 3.2: *Illustration of the two drumstick models used in the experiment.*

Drummer

Due to the large number of possible parameter permutations, the number of subjects was kept to a minimum. Most of the recordings were therefore performed by the author of this work, having basic experience in drum set playing.

In order to validate that the recorded strokes were not significantly different from the strokes a professional drummer would perform, a professional studio drummer with more than 20 years of drumming experience was invited to take part in the experiments. During the analysis, it was found that the key features of the signals were present in both the amateur and the professional recordings.

Grip Technique

Both drummers involved in the experiments held the stick using the *matched grip* and the *German position*. For this grip technique, the drumstick is clamped between thumb and forefinger at the so-called *fulcrum point* (see Fig. 3.3a). To balance the stick properly, the fulcrum point is normally chosen at approximately one third of the stick length measured from the rear end.

During the stroke, the stick rotates around the fulcrum point and is loosely guided by the three remaining fingers. For the German position, the back of the hand faces upwards during the movement as illustrated in Fig. 3.3b.

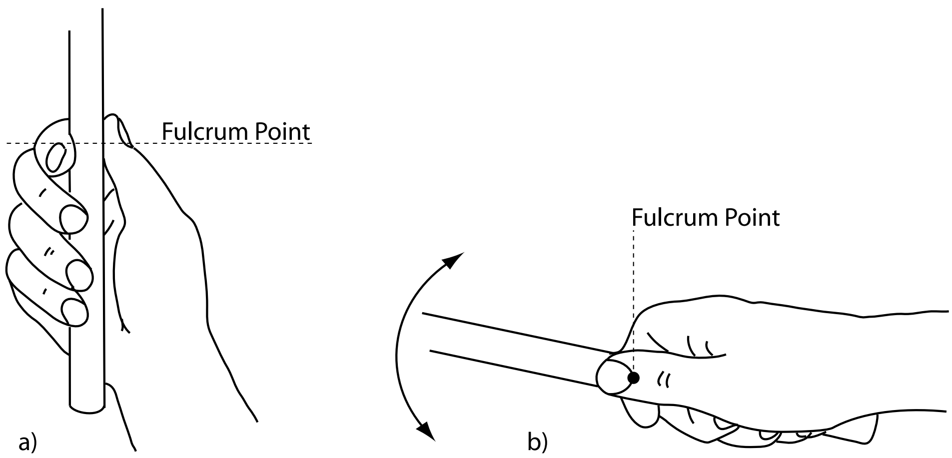

Figure 3.3: *a) Illustration of the matched grip technique. The stick is held between thumb and forefinger and rotates around the fulcrum point at approximately one third of the stick length from the rear end. b) For the German position, the stick rotates around the fulcrum point and the back of the hand faces upwards. Source: [8]*

3.2 Measurements and Measuring Equipment

Different methods to obtain knowledge of the interaction process between drumstick and drumhead were explored. This evaluation process resulted in the final four measured quantities *contact force*, *acceleration of the drumstick*, *contact time* and *sound.*

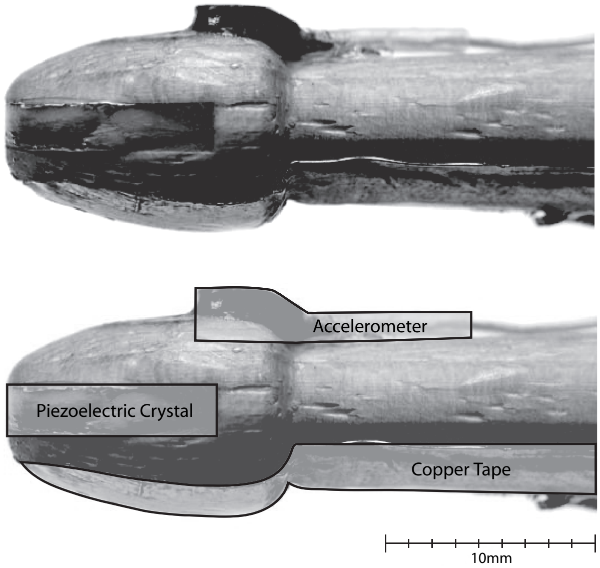

Figure 3.4: *Side view of the modified tip of the Vic Firth drumstick used in the experiment. The positions of the measuring equipment shown in the upper picture are outlined in the lower illustration: accelerometer on the top, piezoelectric crystal glued into a cut in the tip and conductive copper tape on the contact area of the tip.*

Contact Force

To obtain the force on the drumstick during its contact with the drumhead, a small, circular piezoelectric crystal was glued into a cut in the drumstick's tip (see Fig. 3.4). Applying mechanical stress on the tip and therefore on the crystal, resulted in a change in electric charge on the crystal. Via a thin coaxial cable soldered to the two contact surfaces of the crystal, this small signal was fed into a *Brüel & Kjær Type 2635* charge amplifier for signal conditioning before recording.

The crystal was calibrated by comparing the created force signal with the signal of a calibrated force transducer (*Brüel & Kjær Impedance Head 8001*). While striking the calibrated transducer with the drumstick, the input amplification of the crystal's charge amplifier was adjusted until both the signal from the drumstick and the transducer signal matched.

Stick Acceleration

Perpendicular to the crystal's contact surfaces, a small piezoelectric accelerometer (*Endevco Model 22*, weight: 0.4 g) was glued on top of the drumstick's tip to obtain the acceleration of the tip during contact with the drumhead (see Fig. 3.4). The signal of the accelerometer was amplified by a second charge amplifier (*Brüel & Kjær Type 2635*) and then recorded.

Contact Time

To measure the exact contact time of stick and drumhead, a piece of thin (0.2 mm) copper tape was attached to the tip of the drumstick (see Fig. 3.4). During contact with the drumhead, this tape closed an electric circuit through an area of conductive paint on the drumhead. The change in voltage, measured over a serial resistance in the circuit, marked the contact between drumstick and drumhead.

The graphite based paint was sprayed on the drumhead to form a 2 cm wide conductive strip from the center of the drumhead to the rim as illustrated in Fig. 3.6. This thin layer of paint was assumed not to alter the vibrational behavior of the membrane.

Adding the measuring equipment – including the attached cables – to the drumstick, slightly altered its mass and its center of gravity. However, both drummers involved in the experiment stated that it was still possible to hold the drumstick in a comfortable position and play normally.

Sound

As a fourth measurement, the sound of the drum was recorded. A cardioid microphone *(Brüel & Kjær 4021)* was placed one meter from the center of the batter head, pointing at the drum at an angle of 45° as illustrated in Fig. 3.1.

Recording Equipment

Force and acceleration signals were recorded using a DC-coupled digital audio interface working at a sampling frequency of 32 kHz with a resolution of 16 bit. Sound and contact time were recorded at 96 kHz/16 bit using a *M-Audio Delta 1010LT* digital audio interface. Although the two recording systems were not synchronized, the transient nature of the recorded signals allowed a precise synchronization during the analysis of the measured data.

3.3 Varied Parameters

Dynamic Level

Figure 3.5: *Drumming exercise used in the experiment. A crescendo from piano to forte is followed by single strokes at three dynamic levels. The pause between the last four strokes at each dynamic level allowed the drum sound to decay completely.*

Fig. 3.5 shows the exercise that was presented to the drummer. The first part of the exercise was a crescendo from *piano* to *forte* to gather data over a certain dynamic range. To obtain more reliable data, a row of single strokes was then recorded at the three dynamic levels *piano*, *mezzo forte* and *forte*. The last four measures of these single stroke patterns included pauses between the strokes to ensure a complete decay of the drum sound.

For each dynamic level, four strokes of similar force amplitude were later selected for analysis. In this way, strokes could be matched according to amplitude and artifacts, affecting single strokes, could be excluded.

Position of the Stroke on the Drumhead

The exercise was played at four positions between the center and the rim of the drum as illustrated in Fig. 3.6. These four positions will be referred to as *striking positions* and were named *Center*, *Off-Center*, *Two Third* and *Rim*. A small colored circle on the drumhead marked each position and it was up to the drummer to control his stroke so he hit the drum at the desired position.

Although the exact position of the stroke was not recorded, some information about it could be extracted from the measured force and contact time. The narrow strip of conductive paint on the drumhead forced a certain positioning accuracy to obtain a proper contact signal. Changing the striking position along the conductive strip lead to a slightly different shape of the measured force pulse. With these two indicators, variations of the striking position could be minimized for the strokes selected for analysis.

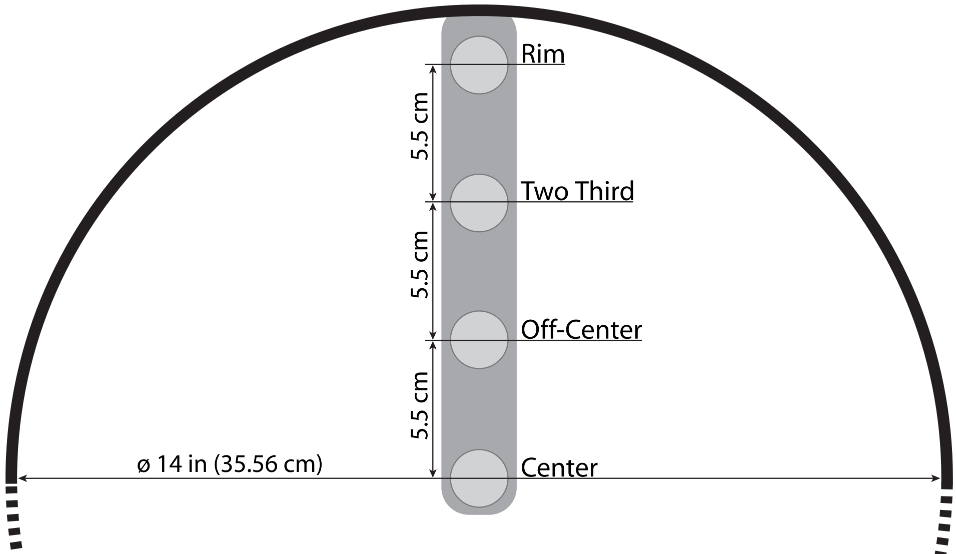

Figure 3.6: *Illustration of the four marked striking positions on the drumhead. With an equal distance of 5.5 cm between the spots, the outermost position Rim was 1.28 cm close to the boundary of the drum. The grey strip running along the striking positions indicates the position of the area of conductive paint applied on the drumhead.*

Tension and Number of Drumheads

The exercise was performed at three different tensions of the drumheads. In addition, recordings were made with only the batter head present and the snare head removed.

As described by Rossing et al. in [14], measuring the tension of a drumhead is a time-consuming and complex process. In this work, the actual value of the tension was of lesser interest than the ability to reproduce a certain tension. For this purpose, a tool called *Tension Watch* (see Fig. 3.7) was used to tune the drumheads. The Tension Watch does not measure the actual tension of the drumhead as its name suggests, but rather a local stiffness, which can be used to reproduce different drumhead tensions.

To tune the drumhead, the Tension Watch was successively placed near each screw of the drum hoop which was then tensioned or loosened until the dial of the Tension Watch indicated the same value for all measured positions. After tuning the drumhead to the desired value on the Tension Watch, the drumhead was fine-tuned by ear and finally controlled using the Tension Watch again.

Following this process, the batter head was tensioned to values of 75, 80 and 85 on the Tension Watch. Those values match the tension examples published by Tama for the snare drum and will be referred to as *low* (75), *mid* (80) and *high* (85). For all experiments using both drumheads, the snare head was tuned five scale divisions higher than the batter head.

Figure 3.7: *Tama Tension Watch used for tuning the drumheads to create reconstructible tensions. Source: www.tama.com*

Drumstick

As described in section 3.1, two different drumstick models were used during the experiments. Most recordings were performed using the *Vic Firth* drumstick. To learn more about the influence of the drumstick vibration, an additional recording was performed using the *Premier* drumstick.

Chapter 4

Analysis and Results

The analysis of the measured signals is divided into two parts. Section 4.1 gives the reader a description of the obtained signals by analyzing a single stroke. Section 4.2 then deals with the influence of the parameters on the measured quantities.

4.1 Signal Analysis

To understand the features of the measured signals, a single stroke will be analyzed in this section. The chosen example is a *forte* stroke with the *Vic Firth* drumstick at the *Center* position. The batter head was tuned to a *low* tension and the snare head was removed.

The following sections will look at force, acceleration, contact time and sound for this particular stroke and describe the characteristics of the measured signals.

4.1.1 Force on the Drumstick

The top panel in Fig. 4.1 shows the force on the tip of the drumstick during the stroke. At time $t_0 = 0\,\mathrm{ms}$ the drumstick touches the drumhead and the force on the stick begins to rise due to the deflection of the membrane. The deformation of the drumhead alone would cause the force to increase up to a maximal value and then decrease as the stick is rejected by the membrane. While this would result in a force pulse with one absolute maximum, the measured signal features a local minimum at the top of the pulse marked A in Fig. 4.1.

An explanation for this shape of the force pulse was found in the vibration of the drumstick that can be observed as a periodic component in the acceleration signal in the lower panel of Fig. 4.1. The contact force between drumhead and drumstick not only sets the membrane in motion but also initiates a vibration in the drumstick.

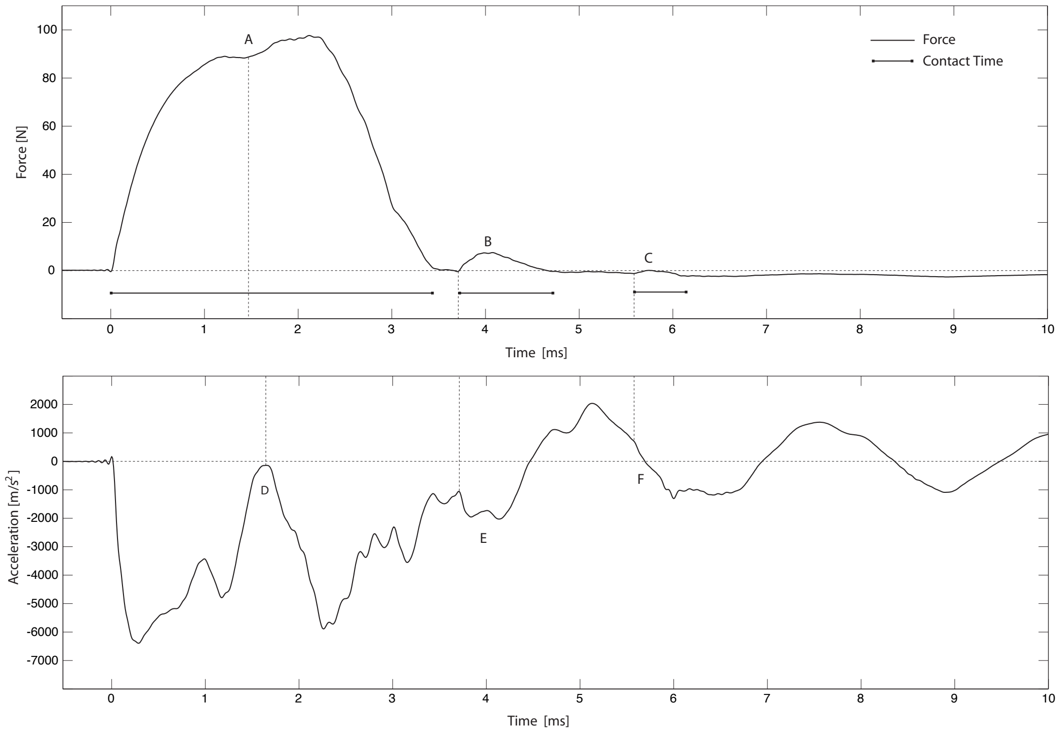

Figure 4.1: *Measured force and acceleration of a forte stroke at the center of the drumhead. The horizontal lines below the force signal indicate contact between drumstick and drumhead. Letters A and D mark an influence of the drumstick vibration on force and acceleration. Letters B,C,E and F mark the interaction of the drumstick with a traveling wave on the drumhead.*

It is this vibration – especially the peak marked D in the acceleration signal – that affects the interaction between stick and drumhead and leads to the characteristic local minimum at marker A. A more detailed analysis of the drumstick vibration and its influence on the force signal can be found in appendix A.

At $t \approx t_0 + 3.5\,\text{ms}$ the stick leaves the membrane and the force on the drumstick ceases. After the drumstick has left the membrane, the force signal shows two weaker impacts starting at $t = 3.75\,\text{ms}$ and $t = 5.6\,\text{ms}$ marked B and C in Fig. 4.1. As the contact signal indicates, stick and membrane touch again during these shorter pulses and it was concluded that an interaction of the stick with the reflections of a traveling wave on the drumhead is responsible for this second and third impact.

This circular, traveling wave is initiated by the impact of the drumstick on the drumhead – similar to the wave caused by a stone dropped into a pool of water. Starting at the point of contact, it spreads across the drumhead and is reflected at the rim. In this particular example with the stroke placed at the center of the drumhead, the first reflections from the rim reach the striking position at the same time from all directions. The time delay between the impact of the drumstick at the center of the drumhead and the arrival of the traveling wave at the rim was measured with an accelerometer mounted on the membrane near the rim of the drum. The measured delay of approximately 1.7 ms indicated that it is indeed the returning wavefront that touches the drumstick on its way upwards and generates the two traces in the force signal.

In the chosen example, the drumstick loses its contact with the membrane before the reflected wave arrives. The deflection of the membrane due to the arriving reflections then leads to a second and third contact. Such *multiple contacts* can also be observed in the interaction between hammer and string in the piano as described for example by Askenfelt and Jansson in [3].

To summarize, three parts were identified in the recorded force signal:

- **Deformation of the drumhead**, causing a positive force pulse
- **Vibration of the drumstick**, altering the shape of the force pulse (marker A in Fig. 4.1)
- **Reflections of a traveling wave on the drumhead**, interacting with the contact process (markers B and C in Fig. 4.1)

It may be noted that after the stick is completely released from the membrane, a small alternating, negative force can be observed. This signal was concluded to be caused by the vibration of the drumstick. Due to the stiff fixation of the piezoelectric crystal in the wood of the stick, a bending of the stick and hence a shearing of the wood caused a measurable force signal. The magnitude of this force was, however, small compared to the force caused by actual contacts and was therefore neglected.

4.1.2 Contact Time

The contact time is defined as the time during which drumstick and drumhead are in physical contact. For multiple contacts, the contact time was measured from the beginning of the first to end of the last contact. Fig. 4.2 shows the measured contact signal for the example stroke. Since the amplitude does not carry any information, the contact signal was abstracted to a horizontal line, indicating the contact time.

In theory, the contact time can also be determined from the force signal as the time span during which the force is greater than zero. As Fig. 4.2 shows, this is roughly true for the chosen example. For force signals with less defined onsets and offsets, however, the measured contact time proved to be more accurate than the value estimated from the force signal.

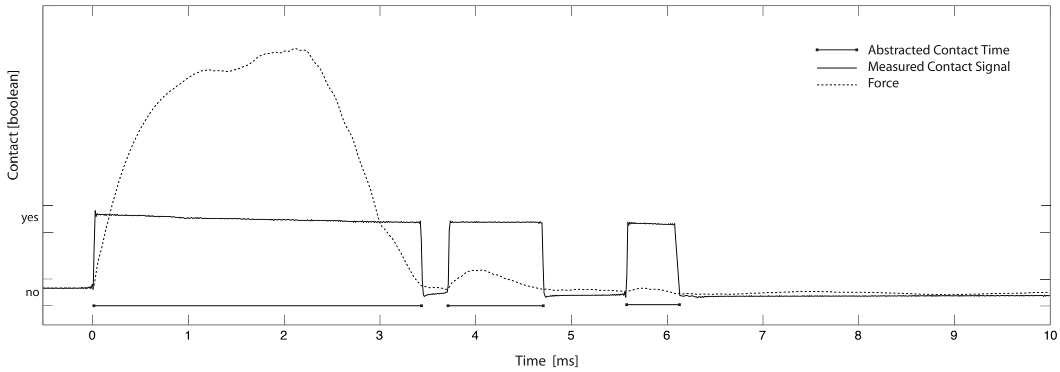

Figure 4.2: *Contact signal measured for the stroke. The alternating curve reflects the actual measured contact voltage. The horizontal line below is an abstraction of this signal. Comparing the force signal with the contact signal indicates the relation of force and contact time.*

4.1.3 Acceleration of the Drumstick

Although the acceleration of the drumstick in Fig. 4.1 has a more periodic character than the force signal, it was found to reflect the same components – deformation of the drumhead, vibration of the drumstick and reflections of a traveling wave on the drumhead.

When the drumstick makes contact with the drumhead, the deformation of the drumhead first decelerates the drumstick and then accelerates it in the opposite direction when rejecting it. This leads to a negative pulse in the acceleration signal corresponding to the positive force pulse. In Fig. 4.1, this acceleration pulse becomes visible as a negative DC-component during the contact time.

Superposed on this DC-component is a strong periodic signal containing modal frequencies of the drumstick at approximately 400, 1.0k, 1.7k and 2.7k Hz. This vibration continues after the drumstick has left the drumhead and lasts for about 100 ms in the selected example. Because of its effect on the contact force, this drumstick vibration was analyzed in more detail. Appendices A and B hold information about these additional experiments.

As on the force signal, the interaction of the stick with the reflections of the traveling wave on the membrane has an effect on the measured acceleration. When the first reflection arrives 3.75 ms after the impact, the signal shows a negative peak marked E in Fig. 4.1. The weaker second impact has a slight effect on a falling slope of the stick vibration marked F. With increasing tension, the reflections arrived earlier and had a more prominent effect on both force and acceleration (see section 4.2.3).

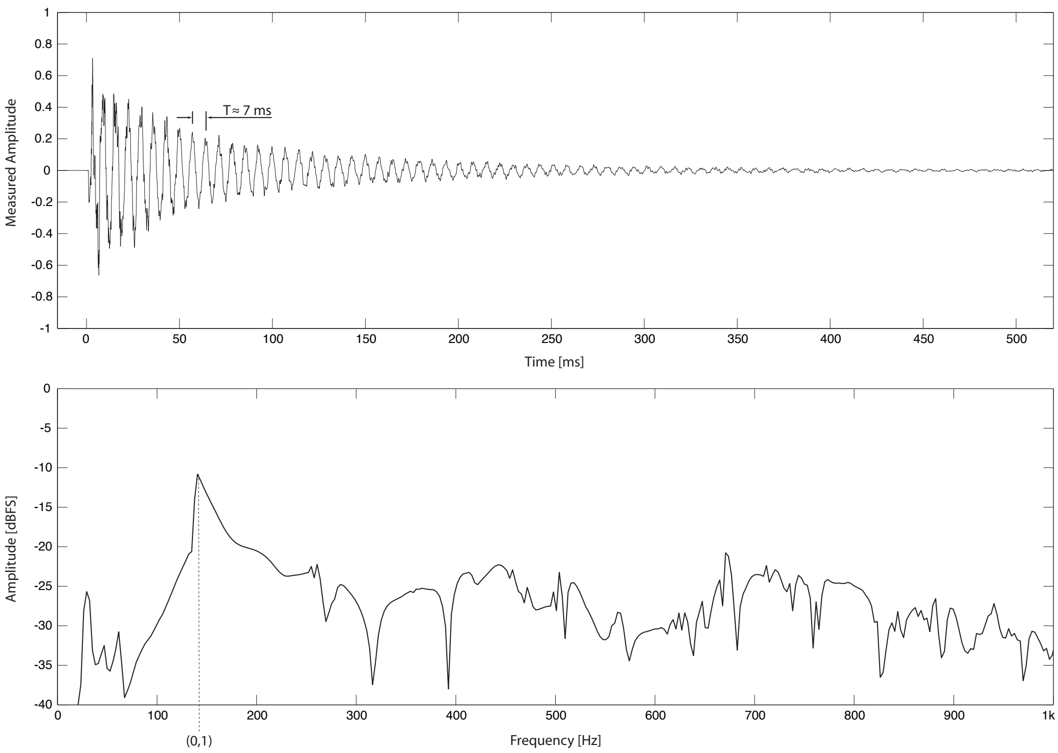

Figure 4.3: *Recorded sound of the example stroke in time and frequency representation. The circular (0,1) mode at* $f_{0,1} = 140\,\mathrm{Hz}$ *is marked in the spectrum.*

4.1.4 Sound of the Drum

Fig. 4.3 shows time and frequency representations of the recorded sound created by the *forte* stroke. The time signal illustrates the transient character of the drumbeat and reveals a dominant vibration with a period of $T \approx 7\,\mathrm{ms}$. The frequency spectrum identifies this vibration as the fundamental $(0, 1)$ mode at $f_{0,1} = 140\,\mathrm{Hz}$. The fundamental mode was present in all recorded signals and was therefore a good indicator of a possible shift of modal frequencies due to a variation of parameters.

All displayed sound spectra were created using a Fast Fourier Transformation (FFT) with a window length of 340 ms. Due to the transient nature of the signal, no windowing function was applied. Since the modes were not always distinguishable in this representation, a three dimensional representation of a Short Time Fourier Transformation (STFT) was used to identify single modes. Here, the window length was 170 ms and a Hanning window was applied to the signal slices.

4.2 Parameter Analysis

Based on the signal overview in the previous section, this section will analyze the influence of changes in parameters on the measured quantities. For each parameter, a set of representative strokes was selected to illustrate the influence of the parameter.

4.2.1 Dynamic Level of the Stroke

Force, Acceleration and Contact Time

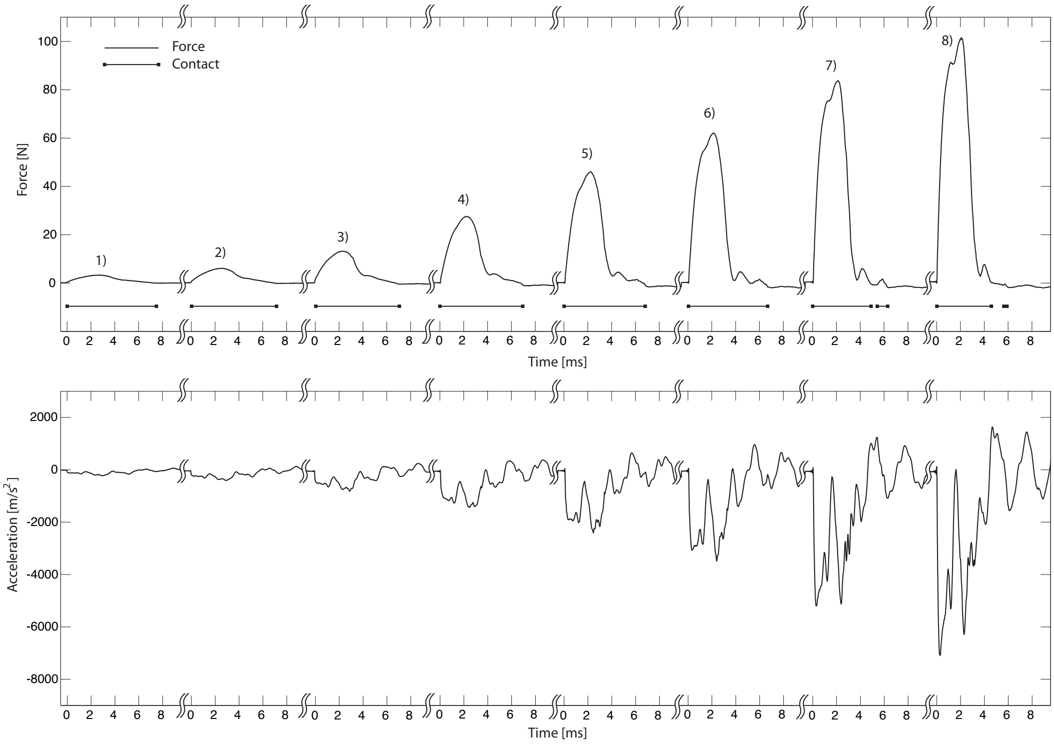

Figure 4.4: *Force and acceleration for a crescendo at the center of the drum. Vic Firth drumstick, one drumhead at low tension. The crescendo begins at piano (1) and ends with a forte stroke (8). The force rises during the crescendo by a factor of about 33 from piano to forte.*

To increase the dynamic level of a stroke, the drummer increases the velocity of the drumstick as reported by Dahl [7]. During a crescendo on the drum, this increasing striking velocity leads to harder impacts and hence rising impact forces. Fig. 4.4 shows force and acceleration for a crescendo played at the center of the drum and clearly illustrates the increase in force on the drumstick during a crescendo. For this particular recording, the peak amplitude of the measured force pulses rises from 3 N (*piano*) up to slightly over 100 N (*forte*) – an increase in force by a factor of 33.

The shape of the force pulse changes with dynamic level as well. As stated in section 4.1.1, the vibration of the drumstick is responsible for the characteristic shape of the force pulse. The acceleration signal in Fig. 4.4 shows that this vibration becomes stronger with increasing impact force. For the last four strokes, the local minimum at the top of the force pulse therefore becomes more and more pronounced.

Similar to the influence of the drumstick vibration, the influence of the traveling wave on the drumhead also becomes stronger with increasing force. For the first three strokes, the arriving reflections of this wave appear as a flattening in the falling slope of the force pulse, beginning about 4 ms after impact. From the fourth stroke on, the arrival of the reflections can be distinguished as small peaks around 4 ms and close to 6 ms. In the acceleration signal, the first contact with the reflections becomes visible as a negative acceleration peak at 4 ms.

While the force peak at 4 ms slightly gains amplitude with increasing dynamic level, the second one at 6 ms seems to lose effect. This observation can be explained by the increase in velocity for the rebounding stick. For the eighth stroke, the stick is already so far away from the membrane that the second contact with the traveling wave is too weak to create a real peak in the force signal.

With increasing amplitude, the interaction between drumstick and the reflected wave on the drumhead also seems to occur earlier. For the *forte* stroke (stroke (8) in Fig. 4.4), the first contact with the traveling wave can be observed approximately 160 µs earlier than for the previous stroke. This suggests that with increasing force on the drumhead, the propagation velocity of the traveling wave increases as well. The measured 160 µs correspond to an increase in velocity by approximately 4% between the seventh and eighth stroke of the crescendo. A possible explanation for this behavior is an increase in tension of the drumhead caused by the force, which the drumstick applies on the membrane. As described in section 4.2.3, an increase in tension was found to cause such a higher propagation velocity of the traveling wave.

For the combination of high dynamic level, low drumhead tension and central striking position, the drumstick temporarily lost contact with the membrane either before or during its interaction with the reflected traveling wave. This caused multiple contacts as described in section 4.1.1. For the crescendo in Fig. 4.4, double contacts can be observed for the seventh and eighth stroke when the stick loses contact during the interaction with the traveling wave. The low speed of the traveling wave caused by the low tension of the membrane and the maximal distance to the rim caused by the central striking position, result in a rather late arrival of the reflections. This late arrival gives the drumstick a chance to part from the membrane before interacting with the reflected wavefront. At higher head tensions or at different striking positions, the reflections of the wave arrive earlier and interact with the main force pulse (see sections 4.2.2 and 4.2.3).

The striking velocity together with the reflections on the membrane determines the contact time of the stroke. At higher dynamic levels, it becomes obvious from the shape of the force pulse that the drumstick would already leave the drumhead after approximately 3.5 ms. Due to the arriving reflections, however, the stick remains in contact until it has gained enough distance from the drumhead and is no longer affected by the membrane deflection. With increasing dynamic level, i.e. increasing velocity of the drumstick, the contact time decreases (see Fig. 4.7).

Sound

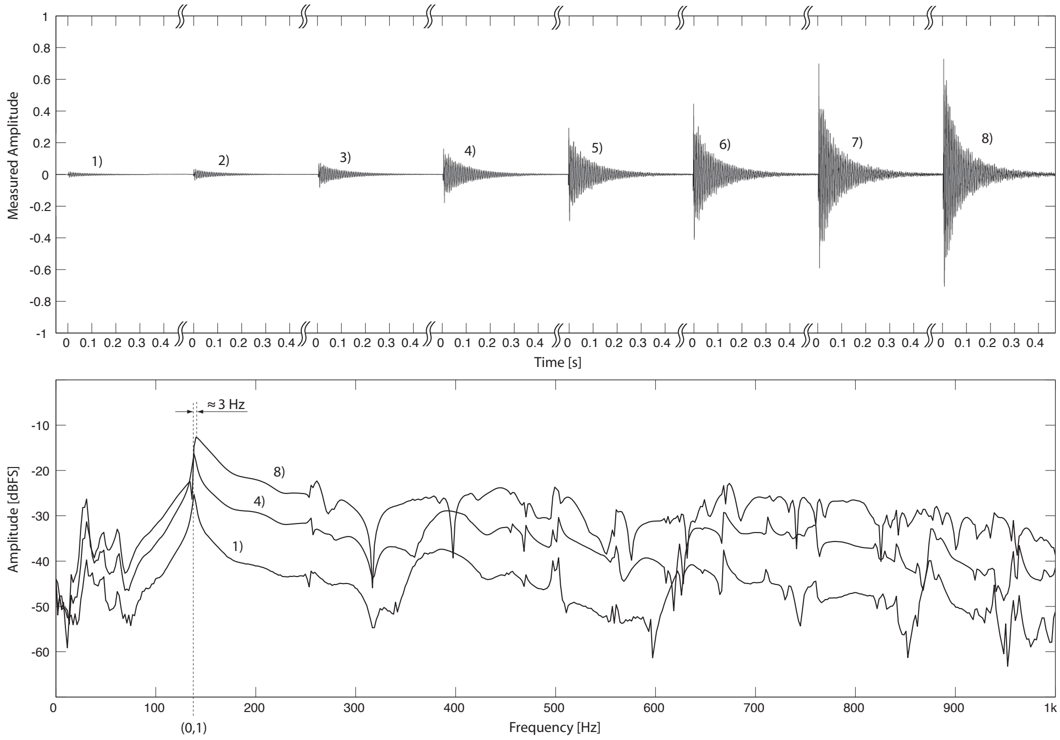

Figure 4.5: *Time and frequency representation of the recorded sound for a crescendo played at the center of the drum. Vic Firth drumstick, one drumhead at low tension. With increasing dynamic level, the sound level increases while the modal structure remains widely stable. For a high dynamic level, the frequency of the (0,1) mode increases slightly by 3 Hz.*

Fig. 4.5 shows a time and frequency representation of the recorded sound for the crescendo. Comparing the increasing attack amplitudes of the strokes, their rise was found to be proportional to the measured increase in force on the drumstick.

The frequency spectra of the strokes show that the fundamental mode remains dominant for all strokes of the crescendo. While the modal structure of the sound does not change significantly for higher dynamic levels, the frequency of some modes was found to increase slightly for the seventh and eighth stroke. This shift of approximately 3 Hz for the fundamental mode was concluded to be caused by an increase in drumhead tension due to the striking force applied on the drumhead. A similar behavior was analyzed for the tom-tom by Dahl [6]. The observed frequency shift also supports the theory for the increase in propagation velocity observed for the traveling wave at higher dynamic levels.

Summary

Increasing the dynamic level of the stroke had the following effects:

- Increase in force on the drumstick
- Stronger excitation of the drumstick vibration
- Decrease in contact time
- Slight increase in propagation velocity of the traveling wave on the drumhead
- Increase in sound level
- Stronger excitation of the same membrane modes
- Slight shift of modal frequencies at higher dynamic levels

4.2.2 Striking Position

Force, Acceleration and Contact Time

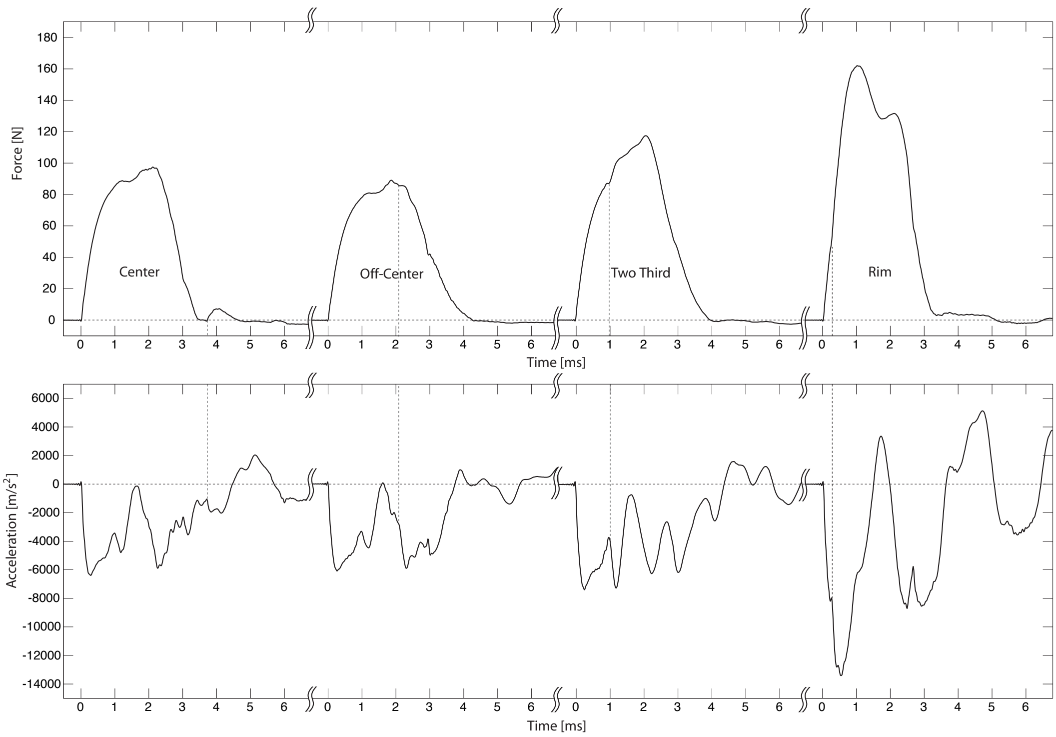

Figure 4.6: *Force and acceleration of a forte stroke at four different positions on the drumhead: Center (0 cm), Off-Center (5.5 cm), Two Third (11 cm) and Rim (16.5 cm from the center of the drumhead). Vic Firth drumstick, one drumhead at low tension. The change of position leads to a change in shape for both the force and acceleration signal mainly due to the interaction with the traveling wave on the drumhead. The estimated beginning of this interaction is marked with a vertical line for each position.*

Moving the striking position from the center of the drum towards the rim affected again all measured quantities. Fig. 4.6 shows the measured force and acceleration for a forte stroke at the four selected positions on the drum – Center, Off-Center, Two Third and Rim (see section 3.3).

As seen in the figure, the force pulse is more narrow and has a higher amplitude when striking the drum at the Rim position, compared to the other three positions. This can be explained by an increase in local stiffness of the membrane close to the rim. Together with the increase in force, the amplitude of the acceleration signal grows as well.

Besides the differences in amplitude, the shape of the force pulse changes for each position. For the Off-Center and Two Third position, the differences in shape are mainly caused by the interaction of the stick with the traveling wave on the drumhead. Leaving the central position on the drum, the arrival of the reflected wave is spread in time since the distance from its origin to the rim is no longer equal in all directions. The first arrival of the reflection at the different positions is indicated with vertical lines in Fig. 4.6. Due to the earlier arrival, compared to the central position on the drum, the reflections interact with the contact process between stick and membrane directly and alter the shape of the force pulse. At the Rim position, the first reflection arrives immediately and interacts with the rising slope of the force pulse. Its arrival is indicated by a small peak in the descending slope of the acceleration signal and has only a minute effect on the force pulse.

Although the force pulses from the Center and Rim positions are free from the influence of interfering reflections, they show different shapes at peak level. For the central stroke, a small local maximum is followed by a stronger one. For the Rim position, this order is reversed. A possible explanation for the difference in shape could be a change in frequency of the drumstick vibration superposing the force pulse and altering its shape (see appendix A).

While the frequencies of the stick modes are unlikely to change, their ratios in the stick vibration could be altered. Indeed, the frequency spectra of the four acceleration signals revealed a different excitation of the stick modes for each position. Tab. 4.1 lists the amplitudes of the first three stick modes extracted from the frequency spectra of the acceleration signals in Fig. 4.6. The values show that the 400 Hz mode is excited much stronger at the Rim position compared to the other positions. The dominance of this frequency can also be observed in Fig. 4.6 as a change in the shape of the acceleration signal. This alteration of the acceleration signal is probably enough to cause the difference in shape for the Center and Rim force pulses.

[dB]	**Stick Modes**		
Striking Position	400 Hz	1 kHz	1.7 kHz
Center	0	-7	-10
Off-Center	0	-4	-11
Two Third	0	-2	-6
Rim	+6	-2	-8

Table 4.1: *Amplitudes of the drumstick modes excited at different striking positions. The values are normalized to the level of the 400 Hz mode at the Center position. For the Rim position, the 400 Hz mode clearly dominates the vibration of the stick and alters the shape of the acceleration signal.*

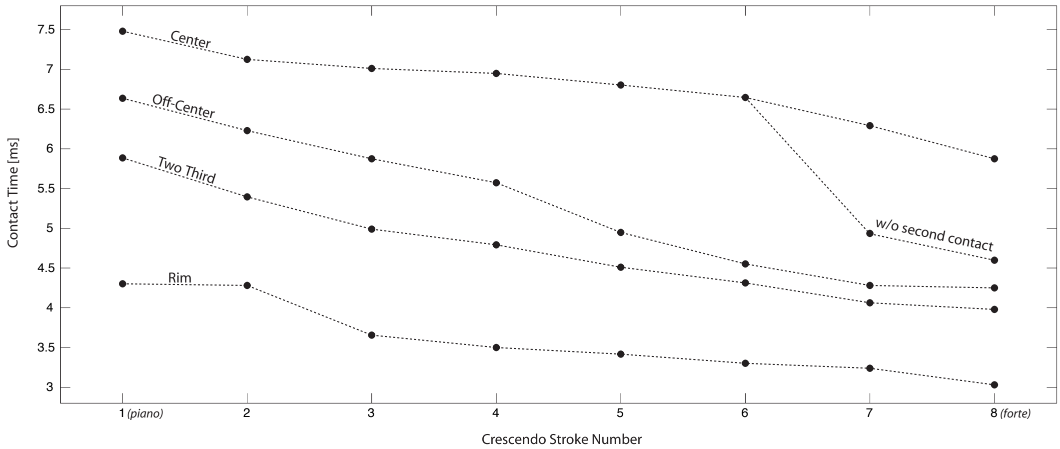

Figure 4.7: Measured contact times for the crescendo at different positions on the drumhead. For the last two strokes at the center of the drum, the contact time is denoted with and without the multiple contacts taken into account.

The earlier arrival of the traveling wave, when moving the striking position towards the rim, also influences the measured contact time. Fig. 4.7 compares the contact times of a crescendo played at the four different striking positions. Moving towards the rim, the contact time decreases for all strokes – indicating the decreasing influence of the traveling wave. The reduced width of the force pulse at the Rim position, due to the higher stiffness of the membrane, lowers the contact time additionally.

Sound

The striking position mainly determines which modes of the membrane are excited by the stroke (see section 2.2.2). In theory, a stroke at the center of the membrane can only excite circular modes because all other modes have a nodal line through the center. Due to the properties of the real membrane, in combination with the variability in striking position, the drumstick usually touches the membrane outside the nodal region and excites radial and mixed modes as well.

Fig. 4.8 shows time and frequency representations of the recorded sound for the four striking positions on the drumhead. In addition to a strong $(0, 1)$ mode, the spectrum of the centered stroke contains a weak trace of the $(1, 1)$ mode and other radial and mixed modes. As expected, these radial and mixed modes become more prominent towards the rim and the $(0, 1)$ mode loses its dominance. This effect can be observed in the spectral illustration as well as in the time signal. Comparing the time signal for the Center and the Rim stroke, the sound loses amplitude and the signal becomes more dense. In the spectrum, the peak of the fundamental $(0, 1)$ mode decreases by 5 dB and the $(1, 1)$ mode gains about 5 dB in amplitude.

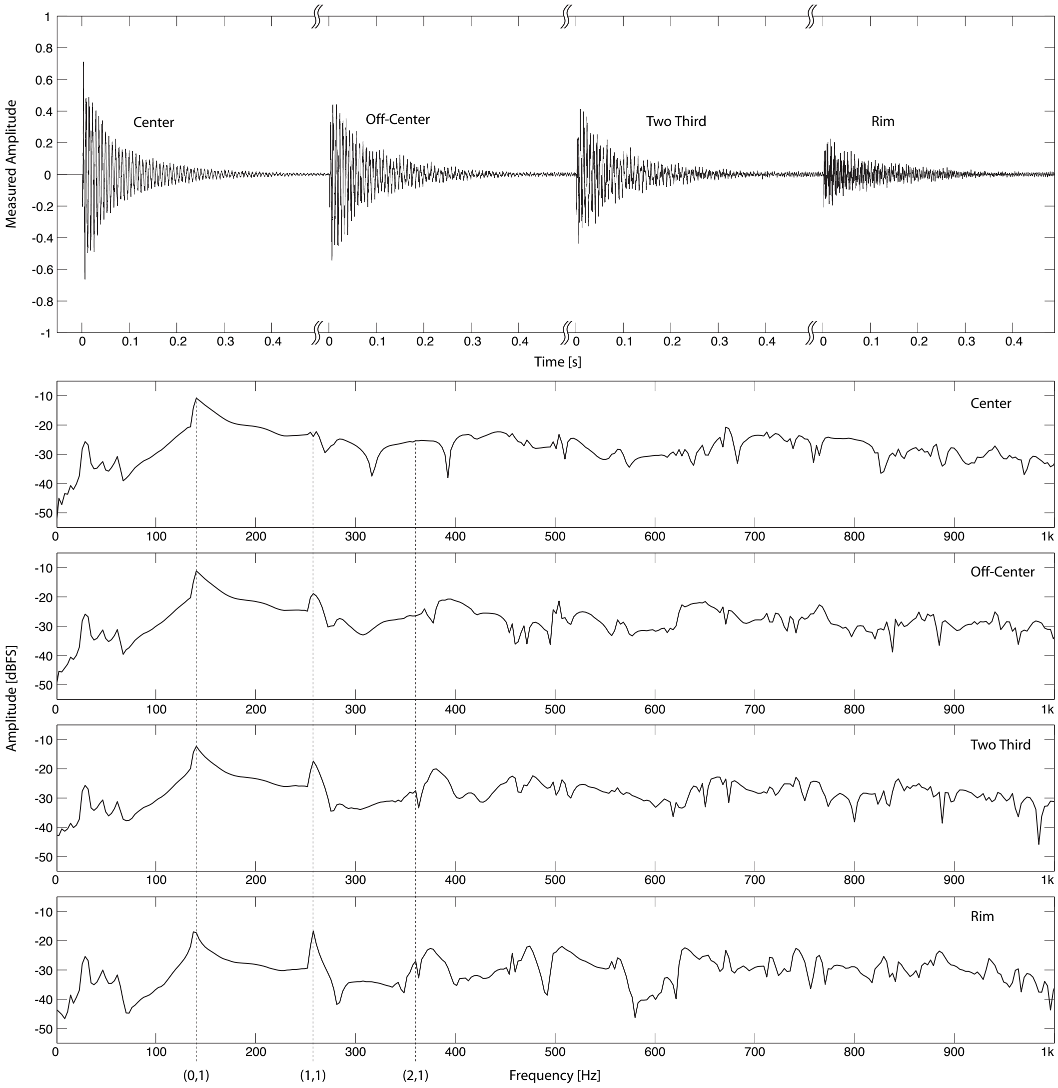

Figure 4.8: *Time signal and spectra of a forte stroke at different positions on the low-tensioned, single drumhead. Moving towards the rim, the marked radial (1,1) and (2,1) modes gain, and the fundamental (0,1) mode loses power.*

Summary

Moving the striking position from the center towards the rim of the drum had the following effects:

- Increase in force on the drumstick, particularly at the Rim position
- Reduced width of the force pulse at the Rim position due to a higher stiffness of the membrane
- Changes in shape of the force pulses
- Changes in the excitation of the drumstick vibration
- Earlier arrival of the traveling wave on the drumhead, resulting in changes in the interaction between drumstick and drumhead
- Decrease in contact time
- Decrease in sound level
- Stronger excitation of higher membrane modes

4.2.3 Tension of the Drumhead

Force, Acceleration and Contact Time

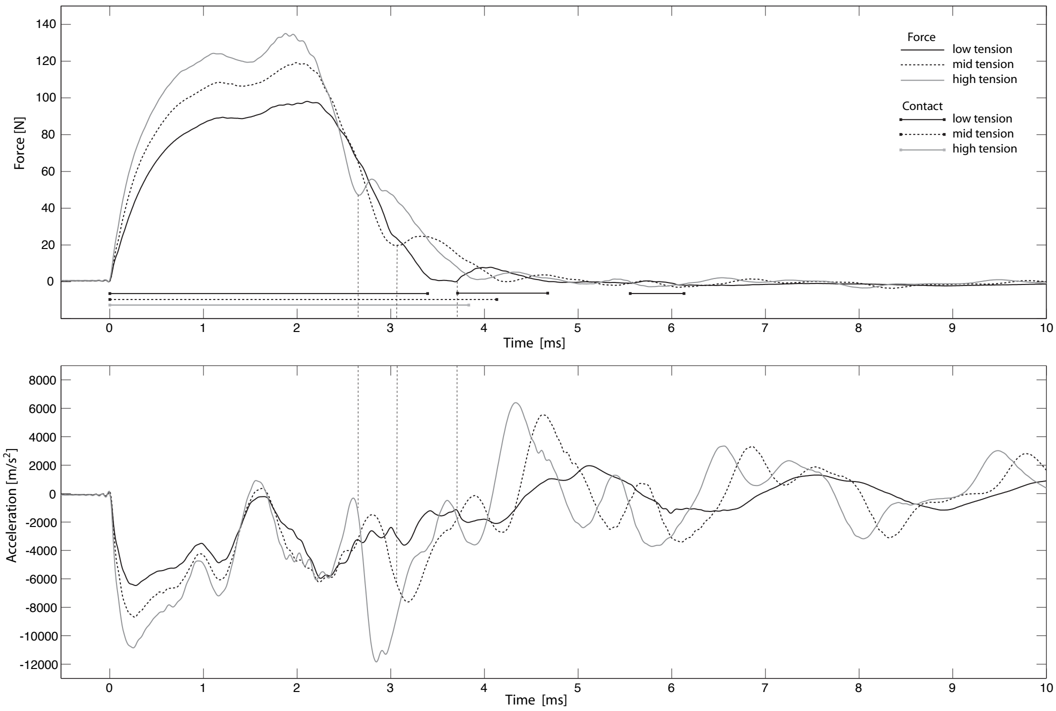

Figure 4.9: *Force, contact time and acceleration of a forte stroke at the center at three different tensions of a single drumhead. The first arrival of the reflected traveling wave on the drumhead is indicated by a vertical line for each tension.*

Fig. 4.9 compares force and acceleration of a centered stroke for three different drumhead tensions. Increasing the tension of the drumhead affected most notably the propagation velocity of the traveling wave on the membrane. Vertical lines in the figure mark the first contact of the reflected wave with the drumstick. At a low tension, the reflections arrive too late to have an effect on the main force pulse. With increasing tension, they arrive earlier and clearly influence both force and acceleration during the contact time. This earlier arrival indicates a higher wave speed for higher membrane tensions.

A higher tension also increases the stiffness of the membrane, leading to a slightly narrower force pulse. In combination with the earlier arrival of the traveling wave, this reduces the contact time. Fig. 4.10 compares the contact times of *forte* strokes at different striking positions and for different tensions of the batter head. With increasing tension, the contact time decreases for all striking positions on the drumhead. At the center of the drum, where the local stiffness changes the most, the contact time also shows the largest change.

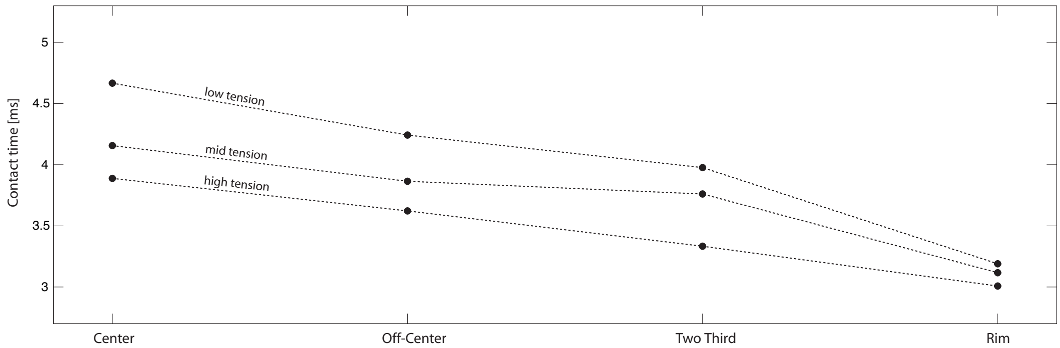

Figure 4.10: *Contact times of forte strokes at different positions and different tensions of the single batter head. With increasing tension, shorter force pulses and an earlier arrival of the reflected wave cause decreasing contact times.*

As illustrated in the example in Fig. 4.9, a higher tension caused higher force amplitudes in most recordings. This increase was, however, not constant and dependent on the performer. In section 4.2.2, an increase in force has been noticed due to a higher stiffness of the membrane at the Rim position. An increase in contact force due to a higher tension and therefore higher stiffness of the whole drumhead would be plausible as well.

Sound

The tension is a variable in the equation for the modal frequencies of a membrane (Eq. 2.1). As expected, all modes were affected and raised by an increasing tension. Fig. 4.11 illustrates this frequency shift by comparing the spectra of strokes at three different drumhead tensions. The values for the identified modal frequencies and their percentage increase compared to the low tension are listed in Tab. 4.2.

[Hz]	**Tension**		
Mode	low	mid	high
$(0, 1)$	138	170 (+23%)	188 (+36%)
$(1, 1)$	255	311 (+22%)	352 (+38%)
$(2, 1)$	363	439 (+21%)	504 (+39%)

Table 4.2: *Identified modal frequencies for three different tensions of a single drumhead. The percentage in parenthesis gives the increase of each modal frequency compared to the low tension.*

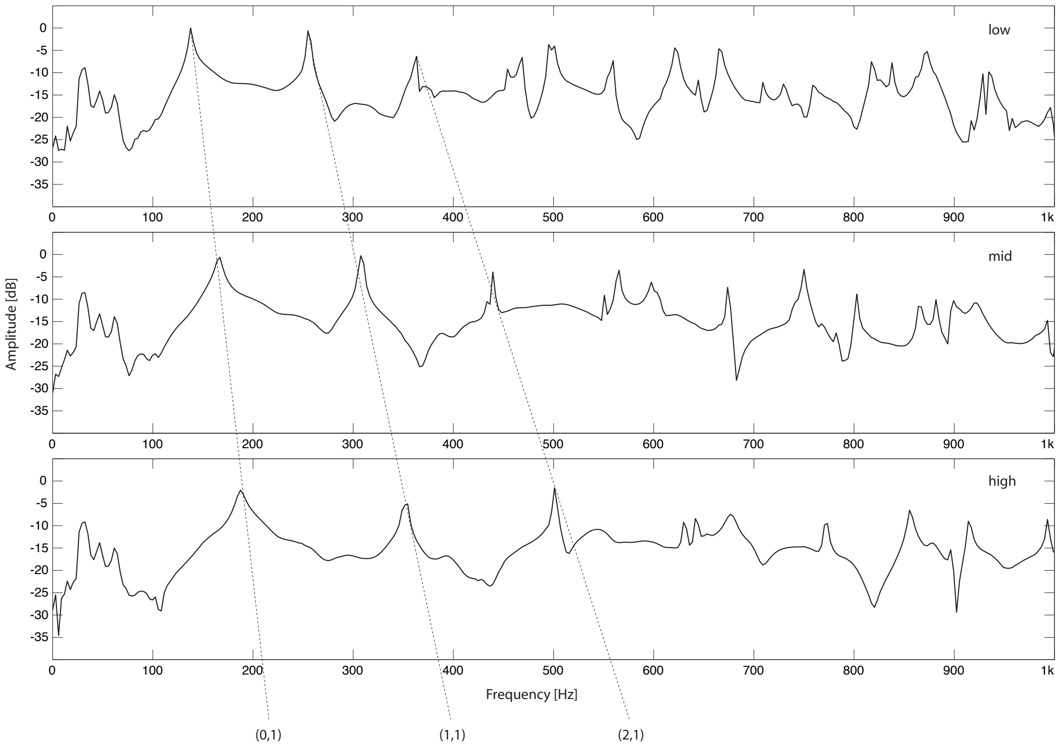

Figure 4.11: *Sound spectra of strokes at the Rim position for three different tensions – low, mid, high. The values are normalized to the level of the (0,1) mode at a low tension. The frequencies of the marked (0,1), (1,1) and (2,1) modes rise with increasing drumhead tension.*

Summary

Increasing the tension of the drumhead had the following effects:

- Reduced width of the force pulse
- Increase in propagation velocity for the traveling wave on the drumhead
- Earlier contact between drumstick and the reflections of the traveling wave
- Decrease in contact time
- Upward shift of the membrane modal frequencies

4.2.4 Number of Drumheads

Force, Acceleration and Contact Time

Adding the snare head at the bottom of the drum produced a confined air volume between the two drumheads. The effect of the second drumhead on force, acceleration and contact time was very small – the properties of the head in contact with the drumstick were clearly dominant.

Sound

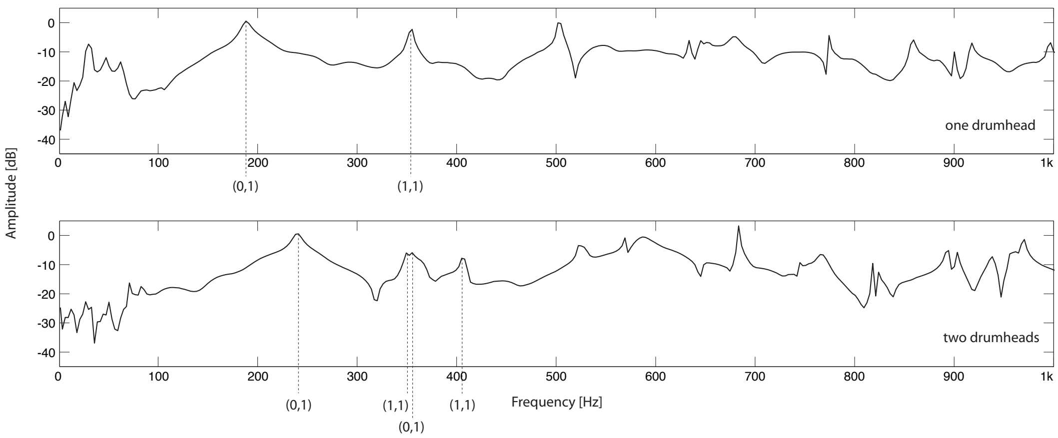

Figure 4.12: *Sound spectra of strokes at the Rim position with one and two drumheads present. The single (0,1) and (1,1) modes for the one-headed drum are split into coupled modes when adding the second drumhead.*

The greatest influence of the second drumhead was found in the sound of the drum. Fig. 4.12 shows the frequency spectra of a *forte* stroke at the Rim position with and without the second drumhead. The most evident change was the creation of coupled modes as described in section 2.2.1. For the single-headed drum, the $(0,1)$ and $(1,1)$ mode each appear once. When adding the second drumhead, these modes are split and appear twice in the spectrum – one with the two membranes moving in-phase and one with opposite phase. The measured frequencies for the single and coupled modes are listed in Tab. 4.3

Mode	$(0,1)$	$(0,1)$	$(1,1)$	$(1,1)$
Batter head only	188	–	352	–
Batter head and snare head	240	366	349	407

Table 4.3: *Frequencies of the (0,1) and (1,1) modes in Hz. The coupling effect introduced with the second membrane splits each mode into two modes of opposite phase.*

4.2.5 Drumstick

Using the *Premier* drumstick (see section 3.1) instead of the *Vic Firth* model had little influence on the shape of the recorded signals.

One clear difference in the recording with the *Premier* stick was found in the amplitude of the recorded signals. Without intention to play louder, the amplitude of force, acceleration and sound was nearly doubled. Fig. 4.13 shows force, contact time and acceleration for a crescendo identical to the one in Fig. 3.5 and illustrates the higher amplitudes.

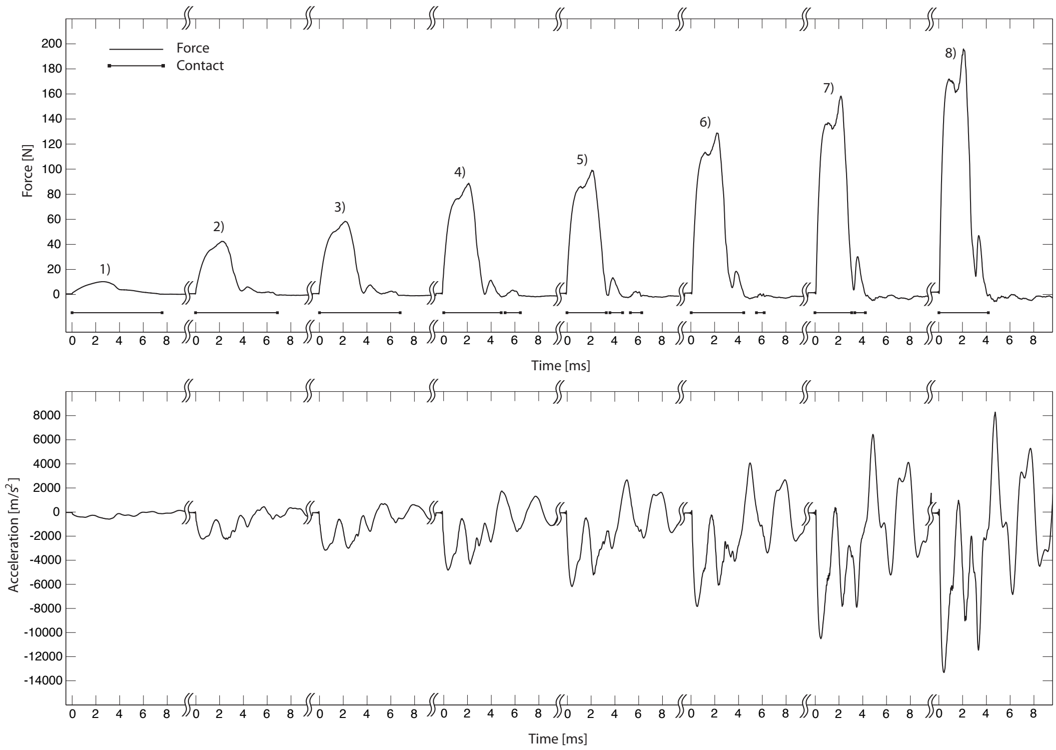

Figure 4.13: *Force, contact time and acceleration of a crescendo played with the Premier drumstick at the center of the drum. One drumhead at low tension.*

A brief analysis of the recordings made with the *Premier* stick, supported all conclusions drawn for the signals obtained with the *Vic Firth* model. Due to time limitations, no additional recordings were performed with the *Premier* model.

Chapter 5

Discussion

The process of striking a drum appears simple and yet is a rather complex physical process. The presented measurements allowed an insight into the interaction of performer, instrument and striking tool. This chapter will summarize the findings and comment on some conclusions and choices.

5.1 Influence of the Drummer

The drummer prepares the stroke and makes a decision for striking velocity and positioning of the stroke. The striking velocity determines the force transmitted to the drumhead and hence the dynamic level of the stroke. The analyzed crescendi illustrated an increase of force, acceleration and sound amplitude with increasing dynamic level.

The positioning of the stroke on the drumhead mainly determines, which modes are excited in the membrane. The greatest difference in sound was observed between a stroke at the center of the drumhead and one close to the rim. These observations were consistent with both the theoretical model of the membrane and the experimental results published by Fletcher and Rossing in [9].

One important decision in this work was to use a human drummer rather than a mechanical device to perform the strokes. Benefits of this decision were a natural grip and motion of the drumstick – possibly leading to more representative signals. On the other hand, a human drummer also introduces minor inconsistencies in dynamic level and positioning of the stroke. While the positioning of the stroke was quite constant, the measured amplitudes for strokes at constant dynamic level showed some variations. Furthermore, a human performer is likely to adapt the stroke to different playing conditions, e.g. different tensions of the drumhead. Drum and drumstick give a feedback to the drummer through the rebound of the stick and the sound of the drum. When this feedback changes, the drummer might unconsciously choose to alter his stroke. Some observations based on the magnitude of the force pulses, e.g. an increase of force for higher tensions of the drumhead, could be biased by the drummer.

To learn about a possible influence of the drummer's individual grip on the interaction process, a professional studio drummer was invited to take part in the experiment. Comparing his force and acceleration signals with the signals produced by the author, did not reveal any significant differences. For two subjects, this may be pure coincidence but the nature of the grip and the measured force signals suggest one possible explanation:

For the grip variant used in this work, the drummer holds the drumstick clamped between thumb and forefinger and the other three fingers control the movement of the stick (see Fig. 3.3). During the stroke, the force on the tip of the drumstick can reach 200 N according to the measurements, causing the drumstick to pivot around the fulcrum point. To control these high forces, a more firm grip would be necessary. A stronger locking of the drumstick though, would reduce the rebound of the stick. To learn how to use this rebound is considered essential for a drummer and facilitates a relaxed and fluent drumming. It may therefore be to the advantage of the drummer, to keep the dampening of the rebound to a minimum and let the stick bounce back freely.

5.2 Influence of the Drum

For the drum, the interaction of the drumhead with the drumstick was investigated. The response of the batter head at different positions and tensions was analyzed as well as effects caused by the presence of the snare head.

Moving the striking position from the center of the drum towards the rim clearly altered the interaction process, affecting the shape of both force and acceleration. A change in the tension of the drumhead affected force and acceleration as well. Two factors were found important for these changes: the local stiffness of the drumhead and the reflections of a traveling wave on the drumhead caused by the impact of the drumstick. These factors are mainly determined by the batter head and adding the snare head showed little or no effect on the interaction.

Close to the rim, the stiffness of the membrane was found to be higher than at the center. It was presumably this higher stiffness that caused stronger and more narrow force pulses at the Rim position.

When the stick hits the drumhead, a traveling wave is initiated that propagates across the membrane. This wave is reflected at the rim and interacts with the drumstick if it returns during the contact time. With increasing tension, the propagation velocity of the wave was found to increase and the reflections interacted earlier with the drumstick.

The interaction between the reflections and the drumstick was most clearly visible for strokes at the center of the drumhead. At the central position, the distance to the rim is equal for all directions and the reflections therefore arrive simultaneously. This synchronized arrival was visible in both the force and acceleration signal. Striking the drum out of its center, spread the arrival of the reflections in time. For these strokes, only the beginning of the interaction could be determined from the measured force or acceleration – the effects of the interaction were continuous and therefore difficult to identify.

One observed effect that could possibly be caused by the continuous interaction with the traveling wave, was a different excitation of the drumstick modes at different positions on the drumhead. The vibration of the drumhead could dampen the excitation of specific stick modes. Another plausible explanation for the observed change in the drumstick vibration could be the variation of the local stiffness across the membrane.

Both the stiffness of the membrane and the reflections of the traveling wave are responsible for the different contact times measured at the different positions on the membrane. This difference in contact time contradicts the results of Sánchez and Irwin [15], who measured contact times of a steel ball dropped on a drum. In the abstract of their work, they mention that initial results indicated no dependency between impact location and contact time. Unfortunately, only the abstract of the complete work was available.

Besides the effects on force, acceleration and contact time, the membrane mainly determines the sound of the drum. Increasing the tension of the membrane, all identified modal frequencies were shifted upwards, as expected from the theory.

The presence of the second drumhead was found to have little influence on force, acceleration and contact time, but a considerable effect on the sound of the drum. Adding the snare head, lead to an observable coupling of the membranes and confirmed the observations by Rossing et al. in [9].

5.3 Influence of the Drumstick

One important result of the work was the insight that the drumstick does not act as a simple mass in the interaction process – when hitting the drumhead, the drumstick begins to vibrate as well. The vibration of the drumstick was found to dominate the acceleration signal, alter the shape of the transduced force pulse and hence play an important role in the interaction process. A modal analysis of the drumstick, conducted at the Division of Experimental Mechanics, Luleå Technical University, Sweden, visualized the resonance modes of the two used drumstick models (see appendix B).

To learn more about the effects of the drumstick vibration, an additional drumstick was selected. Although the investigated *Premier* model differed in wood type, shape and diameter, it developed vibrations very similar to the *Vic Firth* model. The *Premier* model was chosen because its vibration felt differently and it was therefore surprising to find a similar vibration characteristic in the recorded signals and in the modal analysis. A possible explanation for this can be found in the equation for the modal frequencies of a bending bar. In this equation, the length of the vibrating bar is squared and hence has a strong influence. With a similar length for both drumsticks, the calculated modal frequencies become very similar (see Tab. B.1).

The however different 'feel' of the drumstick might explain the higher amplitudes found in the recorded signals. In the case of the *Premier* stick, a different feedback from the stick appears to let the drummer strike harder. As the rebound of the stick was not measured in this work, the reason for the more powerful performance remains unclear. A possible explanation might be found in the shape of the *Premier* drumstick – its conical shaft distributes the weight differently compared to the *Vic Firth*. This might result in a different rebound and hence a different feedback for the drummer.

As stated in section 2.2.2, the properties of the striking tool can influence the sound spectrum of the drum. Rigid and soft mallets have been analyzed e.g. by Bork [5], who found differences mainly in the shape of the force pulse transduced by the mallet. A different shape, and hence a different spectrum of this pulse, had an effect on the vibrations excited in the instrument. In this work, the vibration of the drumstick was found to have an influence on the shape of the transduced force pulse. This implies that the vibration of the stick could slightly influence the sound of the drum. In this work, however, no measurable evidence was found in support of this hypothesis.

Chapter 6

Conclusions and Future Work

In the process of striking the drum, all three analyzed factors – drummer, striking tool and instrument – were found to influence the measured quantities force, acceleration, contact time and sound.

With striking velocity and positioning of the stroke on the drumhead, the drummer decides where and how strong the membranes of the drum are excited. With these two decisions, the drummer controls level and timbre of the drum's sound during the performance. An additional possible influence of the drummer's grip technique could not be analyzed in detail. An analysis of the grip in connection with a measurement of the rebound of the stick, will probably be of interest for future research.

During the contact with the drumhead, the drumstick transfers energy in the form of a force pulse to the membrane. The impact not only sets the drumhead in motion but also initiates a vibration in the drumstick. A modal analysis was used to visualize the individual resonance modes contained in this drumstick vibration.

Analyzing the measured force, the drumstick vibration was found to influence the shape of the force pulse transmitted to the membrane. Based on this information, a simple model was defined that allows to simulate the effect, the drumstick vibration has on the force pulse. For the research on physical models of drums, this simulation could be of use to create more accurate hammer models, i.e. functions used to excite a physical model of a membrane.

Comparing a hammer model including the drumstick vibration with a conventional model, could furthermore give information about a possible influence of the drumstick vibration on the sound of the drum. This hypothesis arose during the work but could neither be disproved nor confirmed by the performed experiments.

For the drum, the tension of the batter head mainly determines, how the membrane reacts to the force transmitted by the drumstick. The impact of the stick initiates a traveling wave on the drumhead, which spreads across the membrane and is reflected at the rim. The tension of the drumhead determines the propagation velocity of this traveling wave. Depending on this velocity and the striking position, the reflections of the traveling wave can arrive at the point of impact fast enough to interact with the contacting drumstick and affect the measured force and acceleration.

To summarize, this work allowed an insight into the short moment, when the drumstick touches the membrane of a drum. The interaction process between stick and membrane could be captured and three different components of this interaction were identified – the deflection of the membrane, the vibration of the drumstick and a traveling wave on the membrane initiated by the impact.

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Appendix A

Further Analysis of Force and Acceleration

In order to identify the various components of the measured force and acceleration, the drum as influencing factor was removed and substituted by a 11 mm thick *rubber pad*. This substitution allowed to eliminate the complex vibrational behavior of the drumhead while imitating the tension of the drumhead with the elasticity of the rubber.

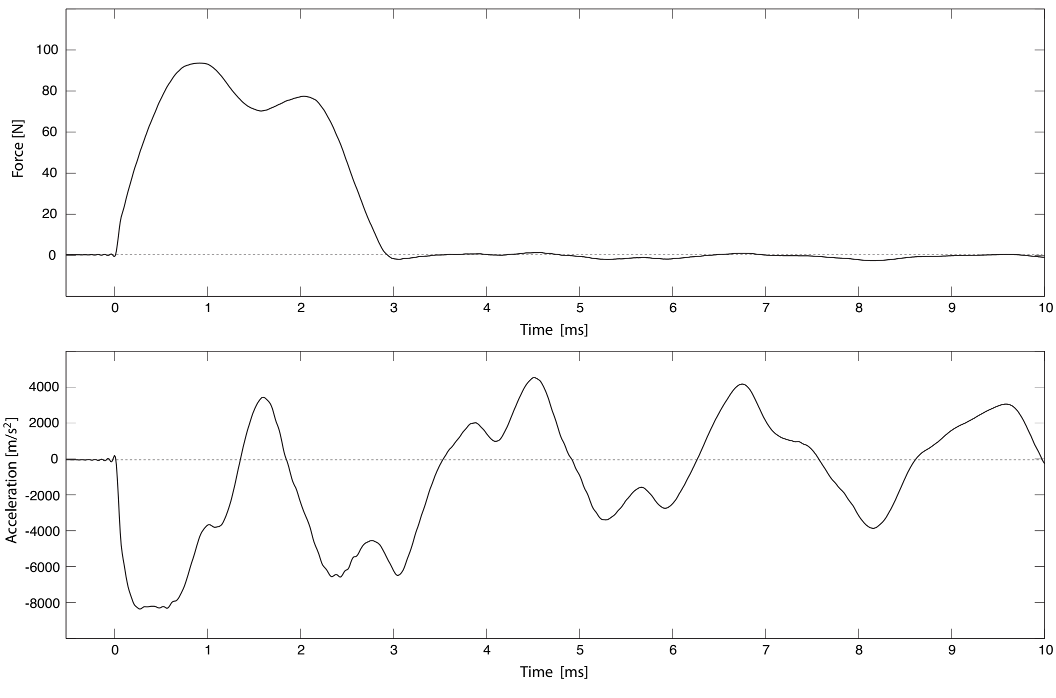

Figure A.1: *Force and acceleration measured for a stroke with the Vic Firth drumstick on an 11 mm thick rubber pad. The shape of force and acceleration was found to be similar to the signals obtained from striking a drum.*

Striking the pad with the *Vic Firth* drumstick produced force and acceleration signals as shown in Fig. A.1. Like the signal obtained from striking the drum (see e.g. Fig. 4.1), the stroke on the rubber pad showed a local minimum at the top of the force pulse. By excluding the drum as influencing factor, it was shown that the vibration of the drumstick caused the particular shape of the force pulse.

A.1 Acceleration Analysis and Synthesis

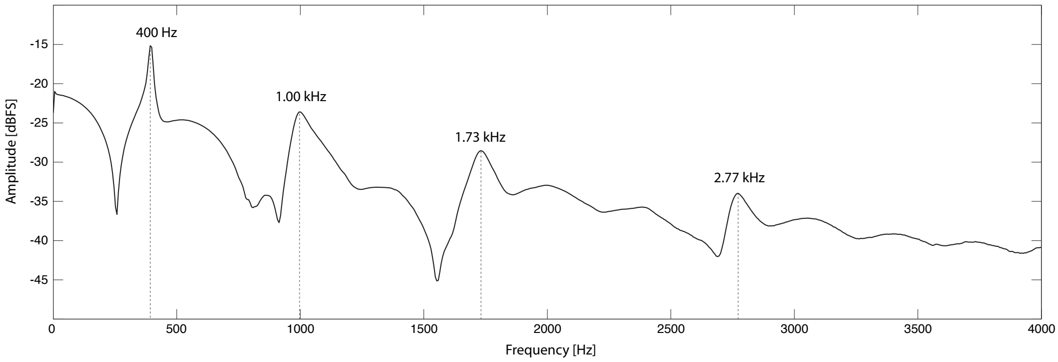

Figure A.2: *Frequency spectrum of the acceleration signal obtained from striking the rubber pad. The interaction with the pad introduces a DC-component. Drumstick modes were identified at the marked frequencies.*

Most information about the drumstick vibration could be obtained from the acceleration of the drumstick. Fig. A.2 shows a frequency analysis of the acceleration signal from Fig. A.1 and reveals two major components:

- A frequency structure similar to a sinc function beginning at 0 Hz (DC)
- Four prominent single frequencies at approximately 400, 1k, 1.7k and 2.8k Hz

As reported by Bork [5], a collision between a soft and a rigid object produces a $\sin^2$ shaped force pulse due to the compression/decompression of the soft object. Measuring force and acceleration of a simple, round mass dropped on the rubber pad, confirmed this observation and revealed a similar, negative pulse in the acceleration signal. The frequency representation of such a $\sin^2$ pulse corresponds to the first component found in the spectrum of the acceleration.

The four prominent frequencies were assumed to reflect the frequency modes and hence the vibration of the drumstick. An optical modal analysis of the drumstick at the Division of Experimental Mechanics, Luleå Technical University, Sweden confirmed this assumption. The results of the modal analysis are included in appendix B.

With this information from the spectrum, the acceleration signal was concluded to be a composition of a negative $\sin^2$ pulse and the vibration of the drumstick. To separate the drumstick vibration from the pulse, a curve fitting algorithm was used to resynthesize the acceleration signal from the identified components. The algorithm tried to fit the following function to the first 4 ms of the measured signal by successively altering the values of the input parameters amplitude A, damping factor $b^{-\delta}$, frequency f and time t:

$$\begin{aligned} a_{syn}(\underline{A}, \underline{b}, \underline{\delta}, \underline{f}, t, \tau) \quad = \quad & A_0 \cdot b_0^{-\delta_0 t} \cdot \sin^2(2\pi f_0 \tau) + & \text{(A.1)} \\ & A_1 \cdot b_1^{-\delta_1 t} \cdot \sin(2\pi f_1 t) + & \text{(A.2)} \\ & A_2 \cdot b_2^{-\delta_2 t} \cdot \sin(2\pi f_2 t) + & \text{(A.3)} \\ & A_3 \cdot b_3^{-\delta_3 t} \cdot \sin(2\pi f_3 t) & \text{(A.4)} \end{aligned}$$

The function consists of four different terms. Term (A.1) models the compression/decompression of the rubber pad with a $\sin^2$ pulse. The use of the variable τ instead of the continuously increasing time t guarantees that the $\sin^2$ part forms a pulse and not a continuous signal:

$$\tau = \begin{cases} t & : \quad t \leq \frac{1}{2f_0} \\ 0 & : \quad else \end{cases}$$

Terms (A.2) to (A.4) model the first three modal vibrations of the drumstick with simple sinusoids. For each term, frequency f, amplitude A and damping factor $b^{-\delta}$ were modified by the fitting algorithm.

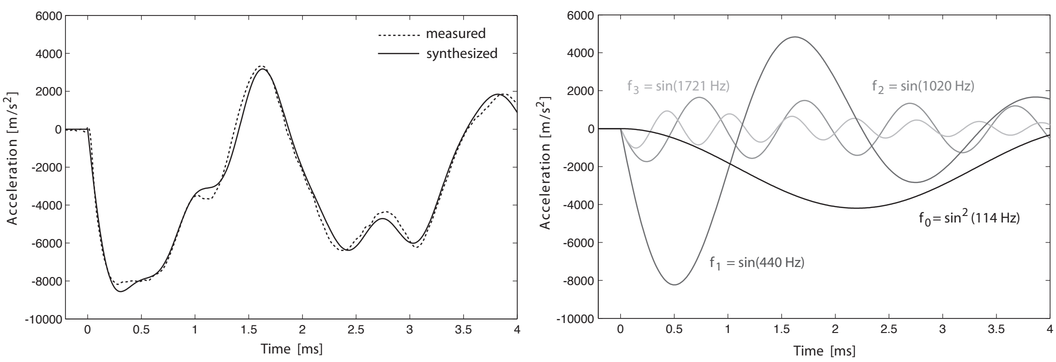

Figure A.3: *Result of the curve fitting algorithm for the acceleration signal. Components found in the signal (right) and comparison of the synthesized signal with the original (left).*

Using only the four components defined above, the result of the curve fitting was close to the measured original. Fig. A.3 compares the synthesized and the measured signal and illustrates the components used for the synthesis. This result confirmed the assumed composition of the measured acceleration. The next step was to analyze the force signal and look for a possible influence of the drumstick vibration.

A.2 Force Synthesis

The hypothesis was to find the same components in the force signal as in the acceleration signal: a $\sin^2$ pulse caused by the rubber pad and a trace of the drumstick vibration. To support this hypothesis, different combinations of these elements were simulated.

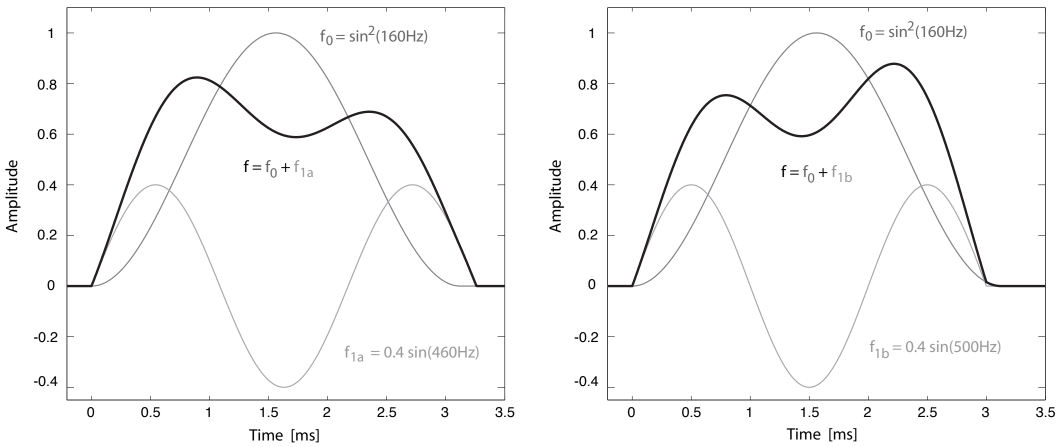

Figure A.4: *Model to imitate the shape of the force pulse by adding a* $\sin^2$ *pulse and a single sinusoid. Their frequency relation determines the shape of the resulting pulse.*

Fig. A.4 shows that a pulse similar to the measured force pulse could be synthesized by adding a $\sin^2$ pulse and a single sinusoid. The shape and intensity of the two local maxima at the top of the pulse is determined by the amplitude and frequency ratios of f_0 and f_1. A similar change of shape was observed on the drum when moving the striking point from the Center to the Rim position (see section 4.2.2).

A curve fitting algorithm similar to the previous one was then used to separate the $\sin^2$ pulse in the force signal from the superposing vibration. Fig. A.5 shows the result of the fitting process. Other than in the model in Fig. A.4, the resulting vibration f_x is not a single sinusoid but a composition of several frequencies.

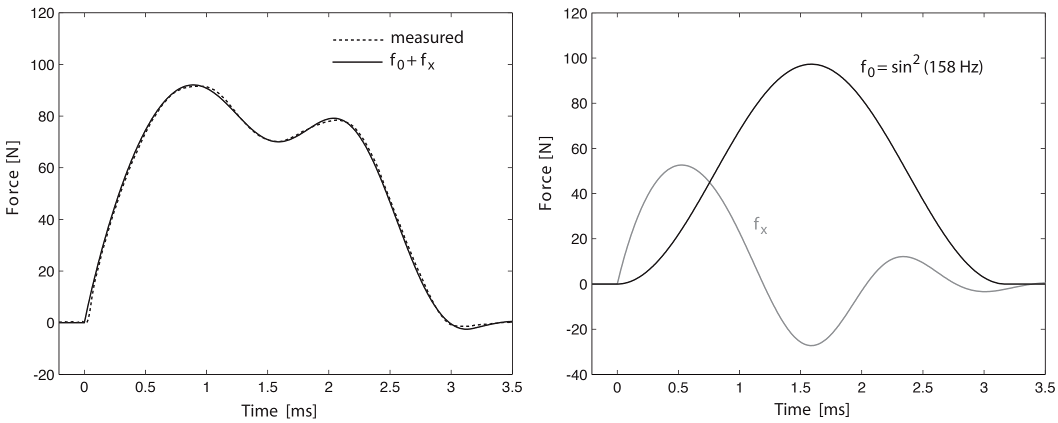

Figure A.5: *Result of the component analysis (right). Comparison between reconstructed and measured signal (left). Similar to the acceleration signal, the force pulse could be split into a sin*^{2} *and a vibrational part.*

Finally, the vibration f_x was compared to the stick modes extracted from the acceleration signal. As Fig. A.6 illustrates, the unknown vibration from the force signal corresponds well to the strong 440 Hz mode of the drumstick for the first 2 ms. The close match suggests that it is indeed the vibration of the drumstick that is responsible for the particular shape of the force pulse.

This relatively simple synthesis of the force pulse could also be of interest in connection with a physical model of the drum to model a more realistic excitation of the membrane.

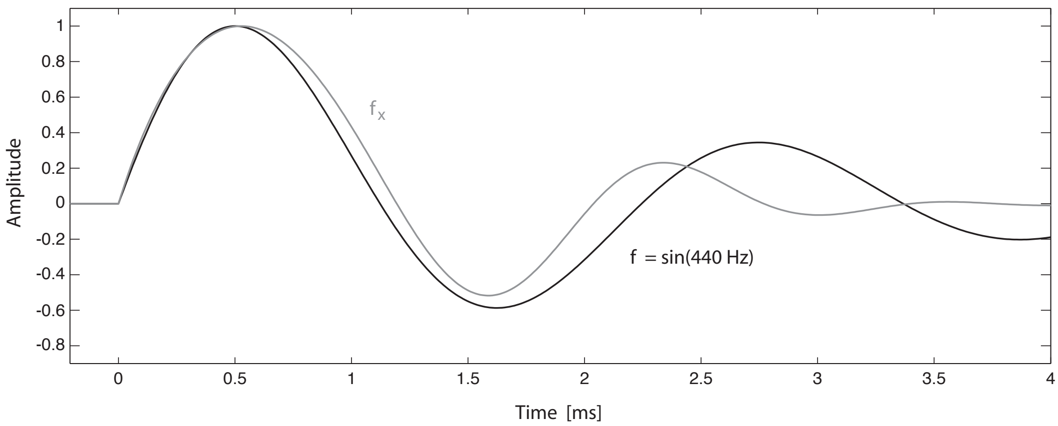

Figure A.6: *Comparison of the stick mode at* $f = 440\,\mathrm{Hz}$ *with the vibration* f_x *extracted from the force signal.*

A.3 Double Integral of Acceleration

Interpreting the measured acceleration directly was found to be nontrivial. A second attempt to determine, if the vibration of the drumstick had an influence on the force pulse was therefore to integrate the acceleration $a(t)$ twice and hence obtain first the velocity $v(t)$ and then the deflection $x(t)$ of the drumstick:

$$\int_0^t a(\tau)\ d\tau = v(t) + v_0 \tag{A.5}$$

$$\int_0^t (v(\tau)\ + v_0)\ d\tau = x(t) + x_0 \tag{A.6}$$

Fig. A.7 shows the results of a numerical integration of the acceleration signal obtained from striking the rubber pad. After the first integration, the initial velocity v_0 of the drumstick in Eq. (A.5) had to be estimated. It was assumed that the velocity of the drumstick reached zero at approximately 1.5 ms after the impact – the half width of the force pulse. v_0 was then chosen accordingly, so that the result of the first integration was zero at this point. Integrating the obtained velocity curve lead to a measure of the deflection of the drumstick. For the second integration, the constant x_0 in Eq. (A.6) was chosen so that the deflection was zero at $t_0 = 0$ ms when the drumstick touched the rubber pad.

Since the initial velocity had to be estimated, the integration process was repeated for different values of v_0. Fig. A.7 shows three velocity curves with slightly different initial velocities and illustrates the effect on the resulting deflection. The calculated values around 6.5 m/s for the initial velocity correspond well to measurements presented by Dahl [7].

The resulting deflection shows a flat top, or – depending on the value chosen for v_0 – even a local minimum at the top. This shape corresponds to the local minimum observed in the force pulse and is clearly caused by the vibration of the drumstick. The results of the integration hence support the assumption that it is the drumstick vibration that alters the shape of the force pulse.

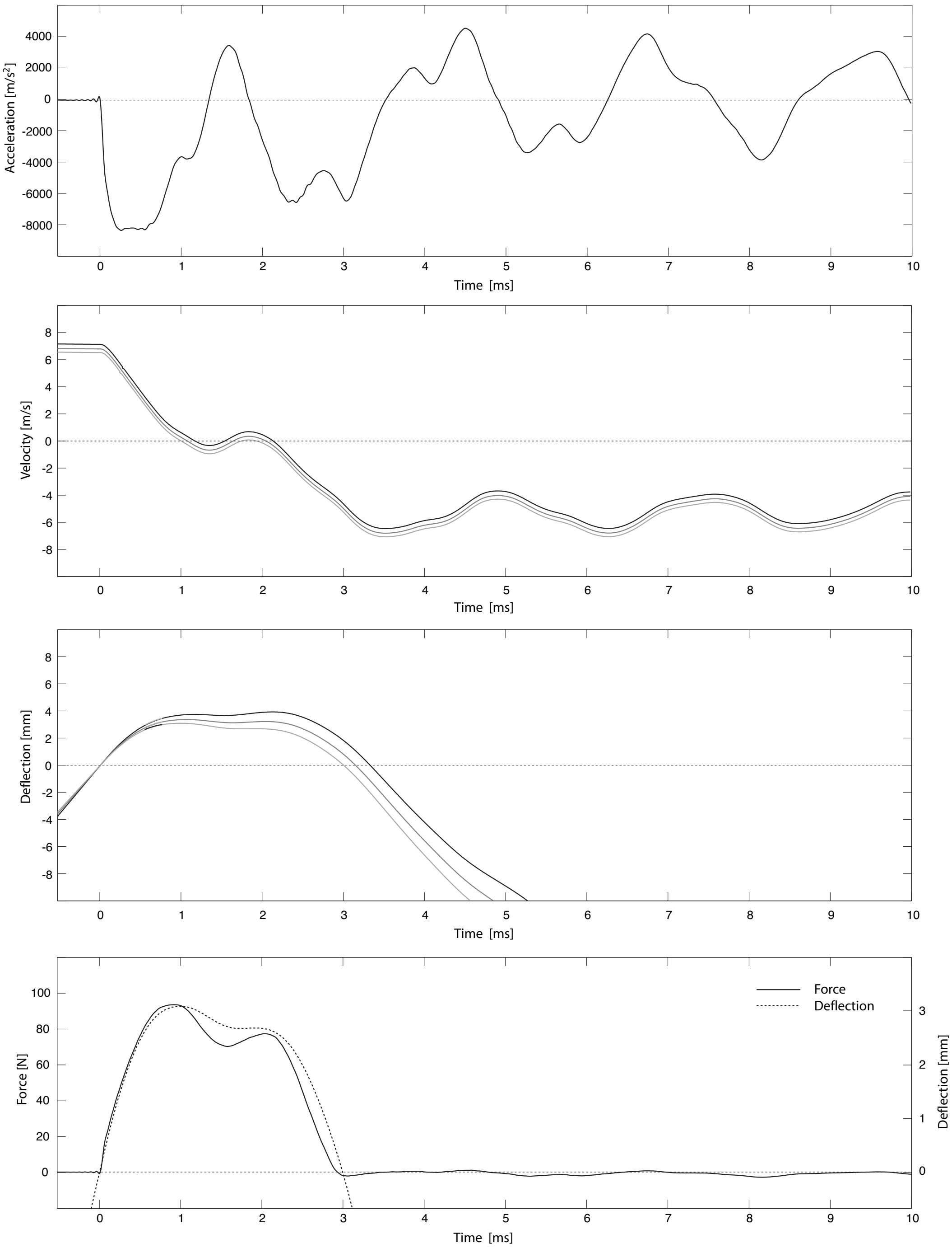

Figure A.7: *Integration of the acceleration signal obtained from striking the rubber pad. The panels show the original acceleration signal (top panel), velocity after the first integration (second panel), deflection after second integration (third panel) and a comparison of the calculated deflection with the force signal (bottom panel). The velocity graph shows three different velocity curves – each based on a slightly different estimation of the initial velocity* v_0. *The deflection graph shows the three corresponding deflections. The comparison of one magnified deflection curve with the force pulse illustrates, how the vibration of the drumstick affects the shape of the pulse.*

Appendix B

Modal Analysis of the Drumstick Vibration

The observed prominent frequencies in the spectrum of the acceleration signal (see Fig. A.2) were assumed to be modes of the drumstick. To confirm this assumption, an optical modal analysis of the two used drumstick models was performed at the Division of Experimental Mechanics, Luleå Technical University, Sweden.

Fig. B.1 shows the experimental setup for two supporting conditions of the drumstick. In the left picture, the *Vic Firth* drumstick is clamped at its end. The right picture shows the *Premier* stick clamped at the *fulcrum point*, i.e. the position where the drummer normally holds the stick (see Fig. 3.3). For both positions, the sticks were clamped between two pieces of rubber foam to imitate the holding between thumb and forefinger of the drummer. During the analysis, the drumstick was set in motion by a shaker and its deflection was measured by a laser beam. The measuring points of the laser are marked with colored squares along the drumstick in Fig. B.1.

To investigate any possible influence of the wood structure on the vibration, the measurements were performed parallel and perpendicular to the grain structure of the wood. With the annual rings of the tree visible at the end of the stick, it was rotated to vibrate parallel and perpendicular to the direction of the grain, respectively.

Fig. B.2 illustrates the modal frequencies identified for both sticks clamped at the end and vibrating perpendicular to the grain. In [9], Fletcher and Rossing illustrate the modes of a bar clamped at one end and list their frequency ratios. With exception of the third mode at 991 Hz and 948 Hz respectively, the shapes and frequency ratios of the measured modes correspond quite well with the theoretical results for a bar with uniform cross section.

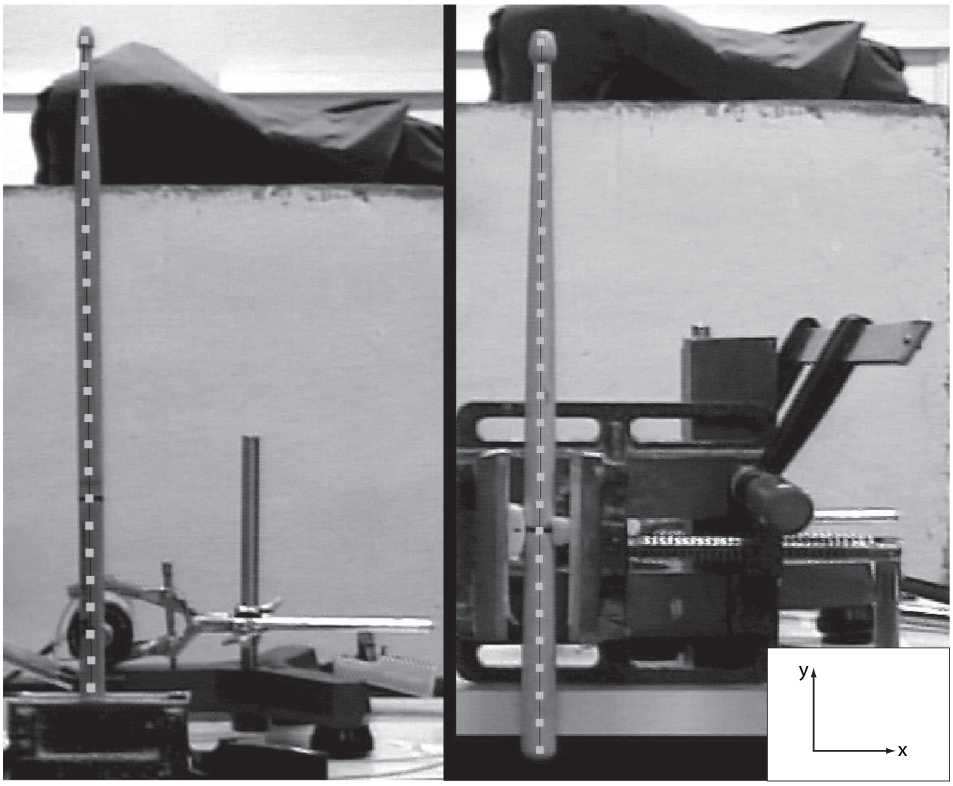

Figure B.1: *Optical modal analysis of the drumstick. For the analysis, the sticks were lightly clamped at the end (left) and at the fulcrum point (right). The colored squares along the stick mark the measuring points of the used laser beam.*

Tab. B.1 gives the theoretical modal base frequencies calculated for both drumsticks using the bending bar model in [9]. The non-uniform cross section of the sticks is neglected in this calculation. Based on these results, Tab. B.2 then compares the calculated frequencies with the actually measured mode frequencies for the drumsticks clamped at the end.

Clamping the drumstick at the fulcrum point leads to a different modal structure. Fig. B.3 illustrates the modes of both sticks vibrating perpendicular to the grain and Tab. B.3 lists all measured frequencies. It can be observed that even at the position where the stick is clamped, it shows a deflection in the measurements. Since the stick is clamped from the sides (see Fig. B.1), it maintains a certain freedom in the direction of the vibration, i.e. along the z-axis.

The four frequencies obtained from the FFT-analysis of the measured drumstick acceleration are reasonably close to the four highest modes of the optical analysis (see Tab. B.3). It was therefore concluded that the frequency components in the acceleration signal represent modes of the drumstick.

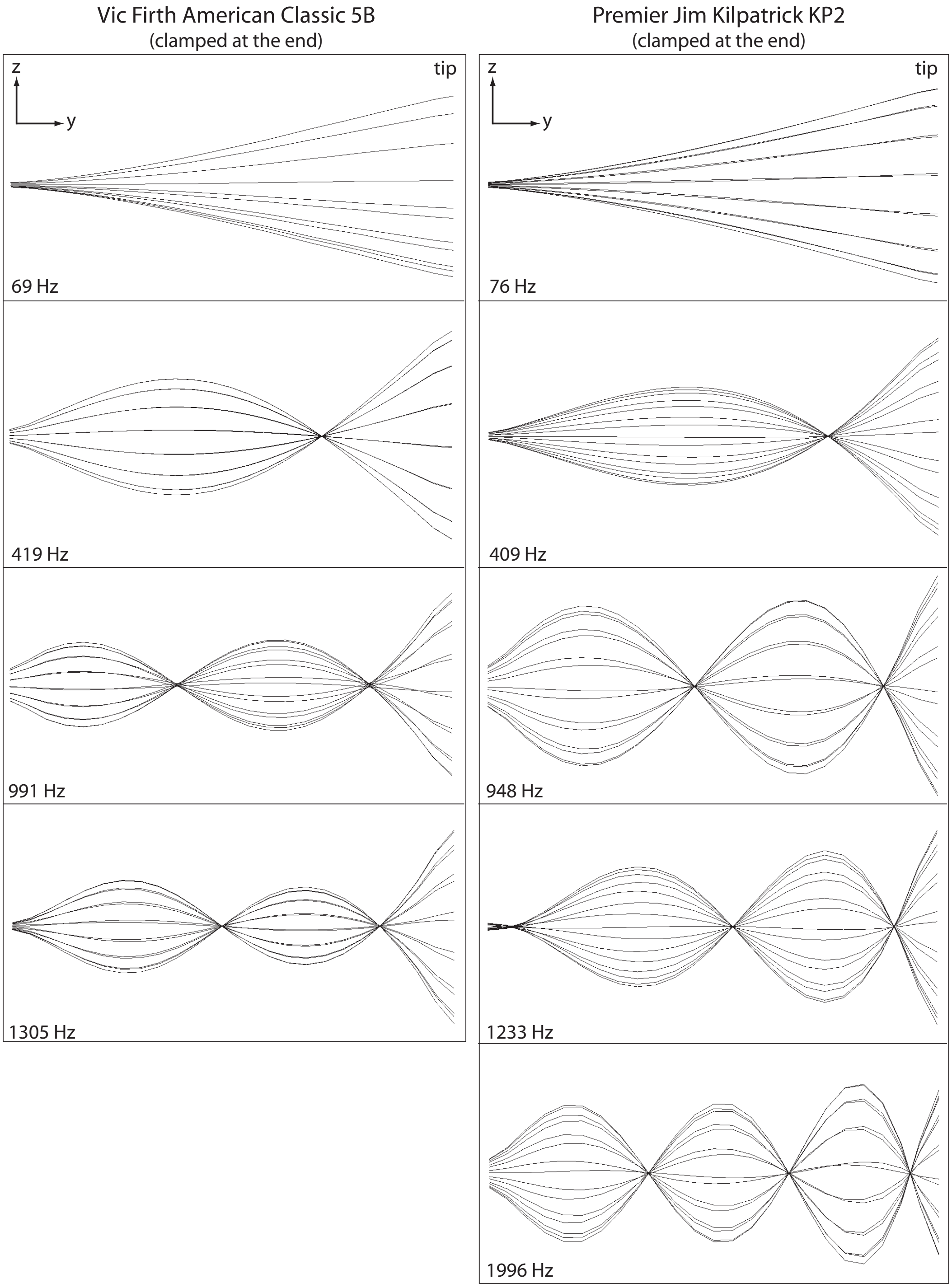

Figure B.2: *Illustration of the mode shapes for both drumsticks. The drumstick was clamped at the thick end between two pieces of rubber foam and adjusted to vibrate perpendicular to the annual rings.*

Stick model	d [mm]	L [mm]	E [GPa]	ρ [kg/m^3]	f_{base} [Hz]
Vic Firth (Hickory)	15	406.4	9.45	600	**50**
Premier (Maple)	20	404	9.79	490	**61**
d: diameter of the stick, L: length of the stick, E: Young's module, ρ: density of the wood, $f_{base} = 0.1782 \cdot (\pi d/4L^2)\sqrt{E/\rho}$					

Table B.1: *Calculated fundamental frequencies for the drumsticks modeled as simple rods clamped at the end. The calculation is based on the bending bar vibration in [9]. Material properties for the different wood types were taken from [12].*

Clamped at the End				
Vic Firth Drumstick			**Premier Drumstick**	
CF	PF	Bending Bar	CF	PF
69 (1.00)	69 (1.00)	**50** (1.00) **61**	76 (1.00)	77 (1.00)
419 (6.07)	405 (5.87)	313 (6.26) 382	409 (5.38)	403 (5.23)
991 (14.36)	913 (13.23)	–	948 (12.47)	902 (11.71)
1305 (18.91)	1220 (17.68)	878 (17.56) 1071	1233 (16.22)	1138 (14.78)
–	–	1718 (34.37) 2097	1996 (26.26)	1616 (20.99)

Table B.2: *Modal frequencies in Hz measured for both drumsticks clamped at the end. The value in parenthesis shows the ratio of each frequency to the frequency of the lowest mode. CF and PF indicate a vibration perpendicular (CF) or parallel (PF) to the grain. The column headed 'Bending Bar' holds the modal frequencies calculated from the bending bar model in [9].*

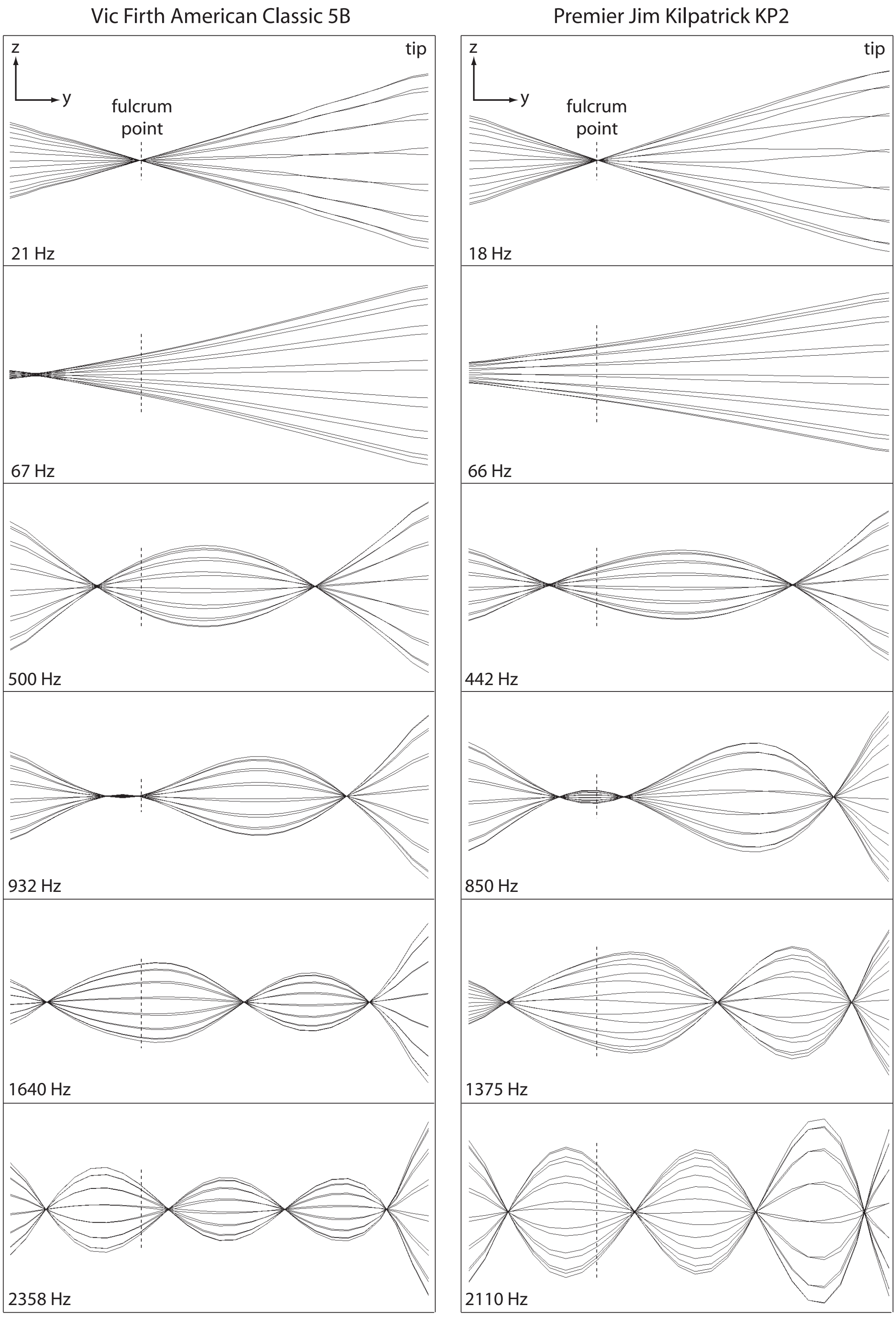

Figure B.3: *Illustration of the modal frequencies measured for both drumsticks. The drumstick was lightly clamped at the fulcrum point, i.e. the position where the drummer holds the stick. This position is indicated by a vertical line. The illustrated vibrations represent the vibrations perpendicular to the annual rings.*

Clamped at the Fulcrum Point				
Vic Firth Drumstick			**Premier Drumstick**	
FFT	CF	PF	CF	PF
–	21 (0.31)	21 (0.31)	18 (0.27)	19 (0.29)
–	67 (1.00)	67 (1.00)	66 (1.00)	66 (1.00)
400	500 (7.46)	490 (7.31)	442 (6.70)	444 (6.73)
1000	932 (13.91)	915 (13.66)	850 (12.88)	860 (13.03)
1730	1640 (24.48)	1606 (23.97)	1375 (20.83)	1392 (21.10)
2770	2358 (35.19)	2306 (34.42)	2110 (31.97)	2117 (32.08)

Table B.3: *Modal frequencies in Hz measured for both drumstick models clamped at the fulcrum point. The value in parenthesis shows the ratio of each frequency to the frequency of the second lowest mode. CF and PF indicate a vibration perpendicular (CF) or parallel (PF) to the grain. The column 'FFT' gives the frequencies obtained from the FFT-analysis of measured acceleration of the drumstick (see Fig. A.2)*