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vibrations - the initial course
of the piano tone**

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B. FROM TOUCH TO STRING VIBRATIONS - THE INITIAL COURSE OF THE PIANO TONE*

A. Askenfelt & E. Jansson

Abstract

This article describes an experimental study of the initial stages of sound production in the grand piano. In a first section, the timing in the piano action is studied. Important timing properties included are the relation between key-bottom contact and hammer-string contact, the "free" time of the hammer motion before striking the string, and the hammer-string contact duration. The influence of the regulation and the dynamic level on these timing properties is analyzed. In a second section, the motion of the key and hammer at different dynamics and using different types of "touch" is studied. The accelerating force on the hammer and the motion of the hammer after release ("let-off") are investigated. Resonances in the hammer are measured and their influence on key and hammer motions are discussed. In a third section, string motion is analyzed. Typical differences in string motion and spectra over the compass of the piano are presented. The influence of changing the hammer mass and adjusting the hammer compliance ("voicing") on the spectral properties is investigated, as well as the spectral differences evoked by the pianist by changing the dynamic level.

Introduction

This study is devoted to an experimental investigation of tone production in the grand piano, starting with the motion of the key and ending with the string vibrations. The aim of the study was to explore the different steps in this process, especially to describe properties which could be assumed to be of importance for playing and for the sound produced. In particular, the extent of the pianist's and the piano technician's influence on the performance was given special attention** (Askenfelt & Jansson, 1982a; 1982b; 1983; 1985; Jansson, 1978).

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** Reports from pilot studies during the period 1978-83 have previously been presented (Askenfelt & Jansson, 1982a; 1982b; 1983; 1985; Jansson, 1978).

A series of recent works, several with a theoretical approach, are related to different parts of this study (Boutillon, 1988; Dijksterhuis, 1965; Hall, 1986; 1987a; 1987b; Hall & Askenfelt, 1988; Hall & Clark, 1987).

The study is divided into three sections: (1) the timing of the action, (2) the motion of the key and hammer, and (3) the interaction between hammer-string and string vibrations. The first section maps the function of the action onto a timetable for the motion of the action parts. The possibilities for the piano technician and the pianist to change the timing patterns are explored by varying the regulation and the dynamic level, respectively. In particular, the timing relation between hammer-string contact and key bottom contact is investigated, reflecting the relation between the mechanical and the acoustic instrument response to the pianist. Further, the duration of contact between hammer-string, decisive for the high-frequency cut-off in the string spectrum, is studied.

The second section studies the motion of the key and the hammer. The influence of the dynamic level and touch on the key and hammer motions is examined. The accelerating force on the hammer during a blow is measured, illuminating the moment when the force transmission from key to hammer is interrupted. Also, oscillations in the hammer during acceleration are analyzed and their connection with different types of touch is discussed. Further, resonances in a piano hammer are studied by modal analysis.

The third section presents registrations of the string vibrations. The string waveforms on opposite sides of the hammer are analyzed during and after hammer-string contact. Typical changes in waveform and spectra over the compass of the piano are presented, together with measurements of the influence of the manufacturer's scaling of the hammer properties. These spectral changes are compared with the range of variation available to the piano technician and the pianist, respectively; the technician by voicing the hammer, and the pianist by changing the dynamic level.

Each section opens with a short survey of the particular part of the piano, followed by a description of the measurement equipment and technique used. The results of the measurements are then presented and discussed, with a focus on the implications for piano playing, regulation, and design.

The experiments were conducted on a 7-foot grand piano in concert condition*, unless otherwise stated.

*Steinway & Sons, Model B 444001, Hamburg, 1975.

PART I. TIMING IN THE GRAND PIANO ACTION

A. Properties of the piano action

1. Mechanical function of the grand action

The action in all grand pianos of today exhibits a high degree of functional similarity; the differences are essentially limited to the design of the individual parts. The construction stems from the Italian harpsichord maker Cristofori's invention of the hammer action in 1709, revised and improved by the French piano manufacturer Erard around 1820. The development stabilized before the turn of the century, after which no major changes have been made.

The action consists principally of four major parts: the key, the lever body with appurtenant parts, the hammer, and the damper (see Fig. 1). In short, the motion of the key is transferred via the lever body to the hammer. Shortly before hammer-string contact, the force transmission from key to hammer is interrupted and the hammer is left swinging free against the string. Upon return, the hammer is checked. The damper is lifted off the string by the key before the hammer-string contact, and let down when the key is released.

The successive steps in the operation of the action during a blow is illustrated in Fig. 2a-d:

- (a) The key is a two-armed lever pivoting at approximately its midpoint. In rest position, the playing end of the key is pushed in its upper position by the weight of the lever body and hammer, which presses down on the far end of the key. The hammer rests via its roller on the repetition lever, a part of the lever body. The lever body rests on the capstan screw, which is screwed into the key. The damper is resting on the string, pulled down by lead weights.
- (b) As the key is depressed, its inner part moves upwards, which in turn causes the lever body to rotate upwards and the repetition lever to push on the hammer. Immediately after the motion has started, the support of the hammer switches from the lightly spring-supported repetition lever to the jack. As the hammer has traveled half of its distance to the string, the inner end of the key starts to lift the damper. When the hammer is close to the string, the upper end of the repetition lever touches the drop screw and the lever is stopped at this level.
- (c) The lever body and the hammer continue to move upwards until the tail end of the jack is stopped by the escapement dolly which makes

the jack turn backwards ("let-off"). The top of the jack is rapidly withdrawn from the roller and the action loses contact with the hammer. The free hammer continues upwards, strikes the string, and bounces back.

- (d) Upon return, the hammer roller falls on the repetition lever in front of the tripped jack, and the hammer is captured by the back check at the far end of the key. The stroke may now be repeated, either by releasing the key as normal, or by using the double-repetition feature. If the key is let up entirely, the hammer, the damper, and the other parts of the action return to their initial positions, and the whole process described is repeated upon making the note sound again.

When the double-repetition mechanism is used, the key is let up only about a third of its travel. At this stage, the hammer has been released from the check and lifted slightly by the spring-supported repetition lever. This allows the spring-loaded jack to slip back into its initial position under the roller, and the action is set for a second blow. This feature of the grand action enables very fast repetitions on the same key, without the damper touching the string between the notes.

A correct function of the action, as outlined above, requires a careful regulation. Of crucial importance is the distance between the top of the hammer at rest and the string in the following hammer-string distance ("blow level"). This distance is adjusted with the capstan screw. Of equal importance is the setting of the release of the jack ("let-off"). This is adjusted with the escapement dolly. The adjustment is made by observing the distance between the string and the top of the hammer at the highest point of its travel (let-off distance), when the key is depressed slowly.

Nominal values for the grand piano in the experiments as given by the manufacturer were 47 mm for the hammer-string distance, and 1-3 mm for the let-off distance* (Dietz, 1968a). A proper adjustment of the repetition lever should position its upper surface above the top of the jack by "the thickness of a paper". A correct regulation gives the pianist a precise feeling of the hammer release at let-off ("pressure point"), and a proper amount of remaining key travel after let-off ("after-touch").

*In the bass, the let-off distance is set to half the string diameter, in the middle and in the treble between 1 and 2 mm.

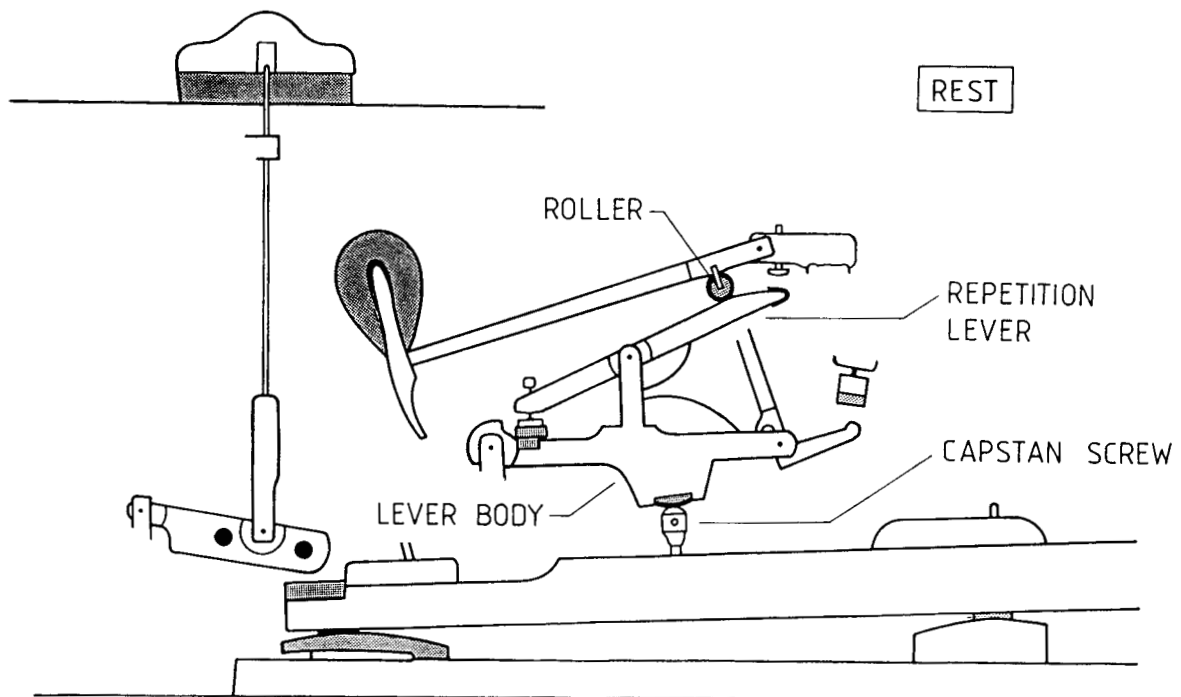
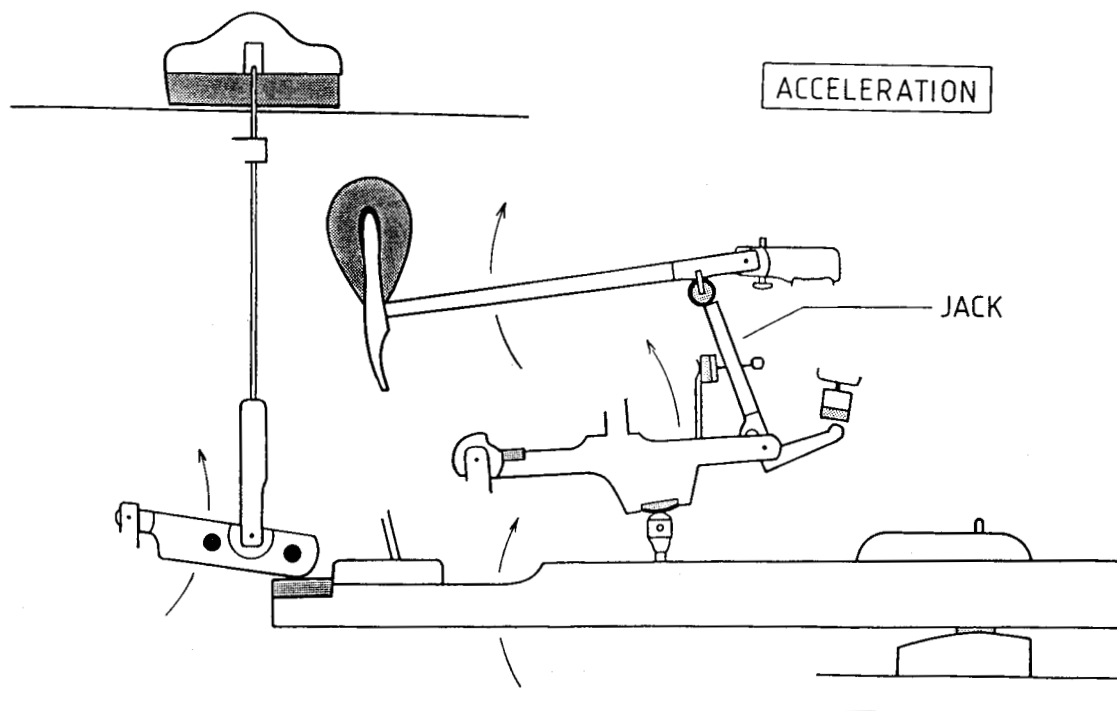
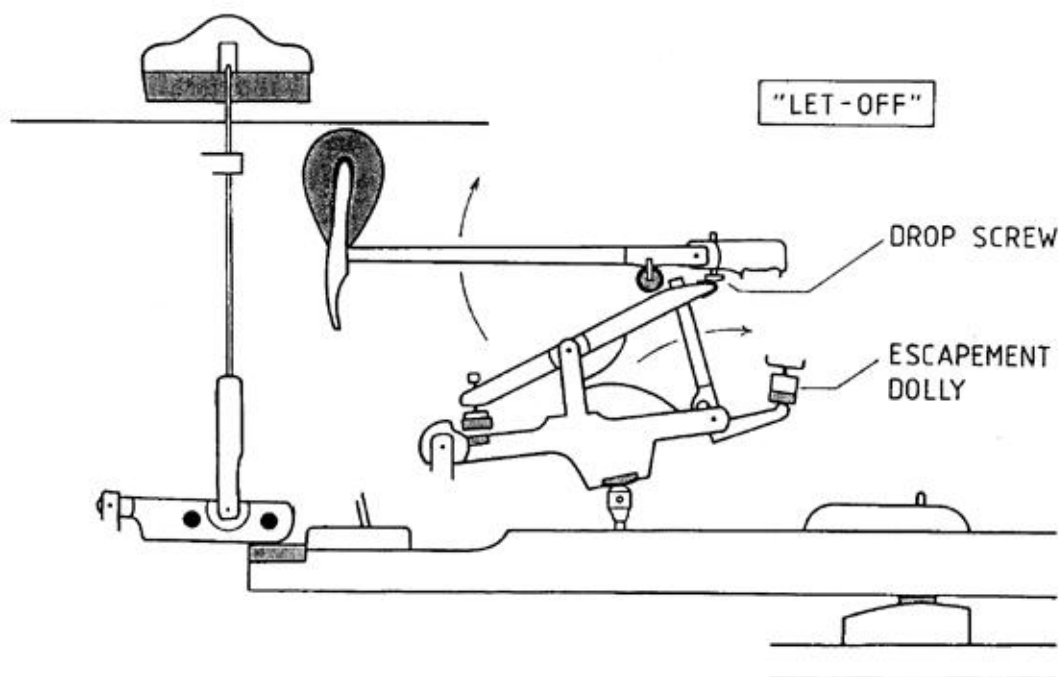


Fig. 2. Illustration of the function of the action at successive stages during a blow.

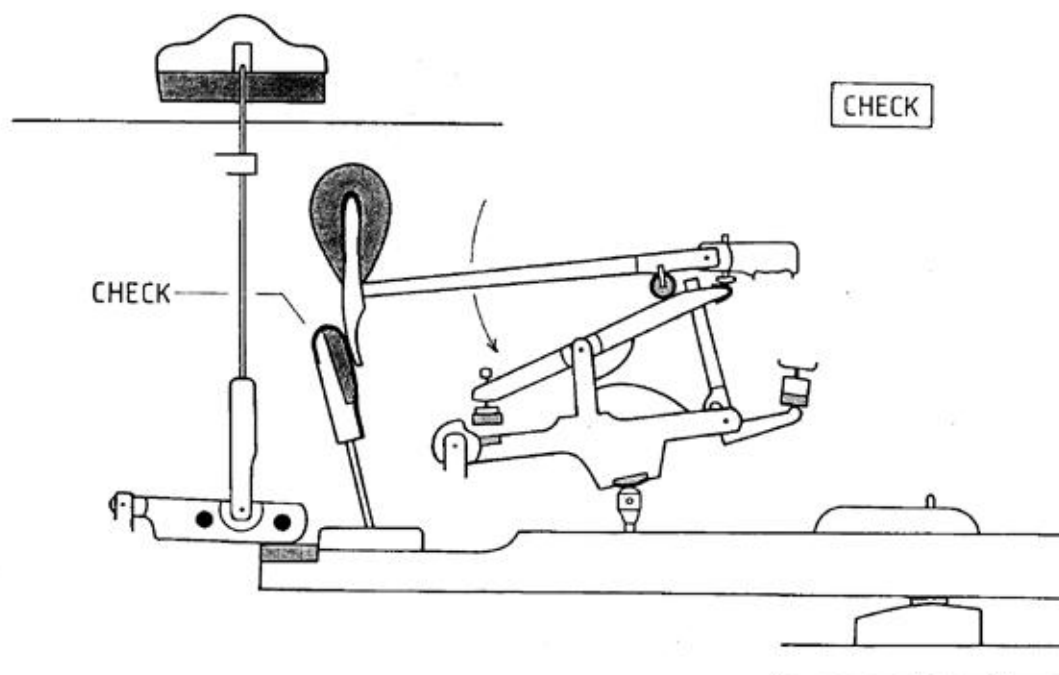
a. Rest position. The hammer rests via the hammer roller on the repetition lever, a part of the lever body. The lever body stands on the key, supported by the capstan screw. The weight of the hammer and lever body holds the playing end of the key in its upper position. The damper is resting on the string.



b. Acceleration. When the pianist's finger depresses the key, the lever body is rotated upwards. The jack, mounted on the lever body, pushes on the roller and accelerates the hammer. The damper is lifted off the string by the far end of the key.



c. "Let-off". The tail end of the jack is stopped by the escapement dolly, and the top of the jack is rotated away from the hammer roller. The free hammer continues towards the string. The repetition lever is stopped in waiting position by the drop screw.



d. Check. The rebounding hammer falls with the hammer roller on the repetition lever in front of the tripped jack. The hammer is captured at the hammer head by the check at the far end of the key.

In all contact points between the moving parts, one of the surfaces is covered with felt or leather in order to ensure a smooth and silent motion, free from backlash. Even thin shafts, for example, the shaft for the jack in the lever body, are supported in felt bushings. This makes the action change condition according to wear and changes in temperature and humidity. Periodic regulation is thus necessary in order to keep the action in optimum condition.

2. Hammer-string contact duration

The hammer-string contact duration and the mechanisms of hammer release from the string have been discussed in depth by Hall (1986; 1987a; 1987b). Effects included in these theories are: the restoring force from the deflected string ("bow and arrow"), the repeated impulses on the short string segment between hammer and agraffe during string contact, and the oscillation of the hammer mass against the string caused by the compliance of the hammer felt. Properties determining the contact duration are the mass ratio of the hammer and the string, the point of excitation, the fundamental period of the string, and the effective compliance of the hammer. The distinction effective compliance is necessary as the piano hammer has been shown to exhibit a nonlinear compression characteristic (Boutillon, 1988; Hall & Askenfelt, 1988; Suzuki, 1985; Yanagisawa & Nakamura, 1934).

The dependence of hammer and string masses, excitation point, and fundamental period of the string implies that the string contact duration varies over the compass of the piano. Further, the variation in hammer compliance implies that the contact duration for a given note will change with dynamic level.

B. Scope of measurements

The aim of the measurements was to provide a view of the timing process in the action at different dynamic levels and by using different types of "touch" (ways of depressing the key). Characteristic properties of the timing patterns which could be assumed to be of particular importance in playing include the relation between the onset of the note, i.e., the hammer-string contact, and the mechanical response to the player, i.e., the key bottom contact. The relation between the release of the hammer ("let-off") and the moment of the string contact is another interesting timing property, which answers the question whether the contact between the key and the hammer is broken at string contact or not. Further, the magnitude of the shifts in timing patterns introduced by differing regulations is of interest in order to judge the implications for the pianist's timing of the notes. Also the hammer-string contact duration is an important timing parameter, influencing the content of the string spectra.

C. Measurement methods

1. Detectors for timing

The timing in the action was measured by applying thin copper wires (diameter 0.10 mm), copper foil adhesive* or graphite dispersion** on the contact surfaces between selected parts of the action. The integrated "micro-switches" thus obtained were fed from a DC-supply. The contact signals were added in a resistor network, which condensed several contact signals into a step-wise signal on one trace on an oscilloscope. The individual contact signals were identified by using summation resistors of different values, which gave each contact point its characteristic step height.

The contact signals displayed the moments of mechanical on/off contact between the parts. In some cases, as between jack and roller, the contact signals could also give information on the contact force. The force estimation was made possible by the variation in resistance with pressure between two graphite covered surfaces.

All contact points measured had one of the two joining parts covered by felt or leather. This was at first expected to give a poor definition of the on/off contact positions, as the felt and the leather pieces would compress and expand due to the contact forces during a blow. The compression of the material could also be expected to introduce some hysteresis, making the on/off contact positions somewhat dependent on previous movements. However, these expected difficulties were of secondary importance. After some vivid playing which always followed each preparation, the method gave a high reproducibility in the measurements, possibly with the exception of very low dynamic levels (ppp).

The following eight contact points were prepared (see Fig. 3).

Key bottom - the underside of the key and the felt washer on the keybed.

Key-lever body - the capstan screw and the felt-covered cushion in the bottom of the lever body.

Repetition lever-roller - the upper surface of the repetition lever and the leather on the hammer roller.

* 3M, 1181 "Copper foil with conductive acrylic pressure sensitive adhesive", thickness 0.08 mm.

**DAG, dispersion 568-E, CRT external wall coating.

Jack-dolly - the tail end of the jack and the felt-covered underside of the escapement dolly.

Jack-roller - the top of the jack and the leather on the hammer roller.

Hammer-string - the top of the hammer head. The metal string itself served as the other part of the switch.

Hammer-check - the curved backside of the hammer head tail and the check leather.

Damper-string - the underside felt of the damper. The metal string itself served as the other part of the switch.

It turned out to be advantageous to use the copper foil for most of the contact points. The foil could simply be attached on top of a felt piece, and it followed the movements of the felt. On the hammer head, however, a copper wire was used. The thin copper wire was soon embedded in the hammer felt and caused only a minor deviance in tone quality. With the foil on the hammer, the tone quality was affected noticeably.

For the crucial contact between the jack and the roller, graphite dispersion was used. The upper part of the wooden jack was painted with graphite dispersion and connected with pieces of copper foil with conductive adhesive*. The leather roller was also treated with graphite. In addition, two copper wires (diameter 0.10 mm) were sewn to the leather, close to the edges of the path of the jack. As the jack and the roller already are lubricated with graphite at the manufacturing, the response was not seriously affected. A professional piano technician and a professional pianist, respectively, considered the deviation from normal response as marginal, with only a minor loss of a clear pressure point.

2. Mechanical playing

A special tool was used for producing repeated strikes with high reproducibility. The pianist was substituted by a pendulum (length 0.8 m), supplied with a weight (mass 0.4 kg) which could be secured at any desired position along the rod. The pendulum tip was covered with a folded rubber tube in order to make a rough simulation of the finger tip. The pendulum was supported in an initial position above the key by a wooden stick resting on the key bed (see Fig. 3).

Although the pendulum touch gave very reproducible blows as regards the dynamic level, the way of depressing of the key was a little different compared to a human player. When the pendulum was not used, the strength of the excitation was measured by the amplitude of the first velocity pulse on the string.

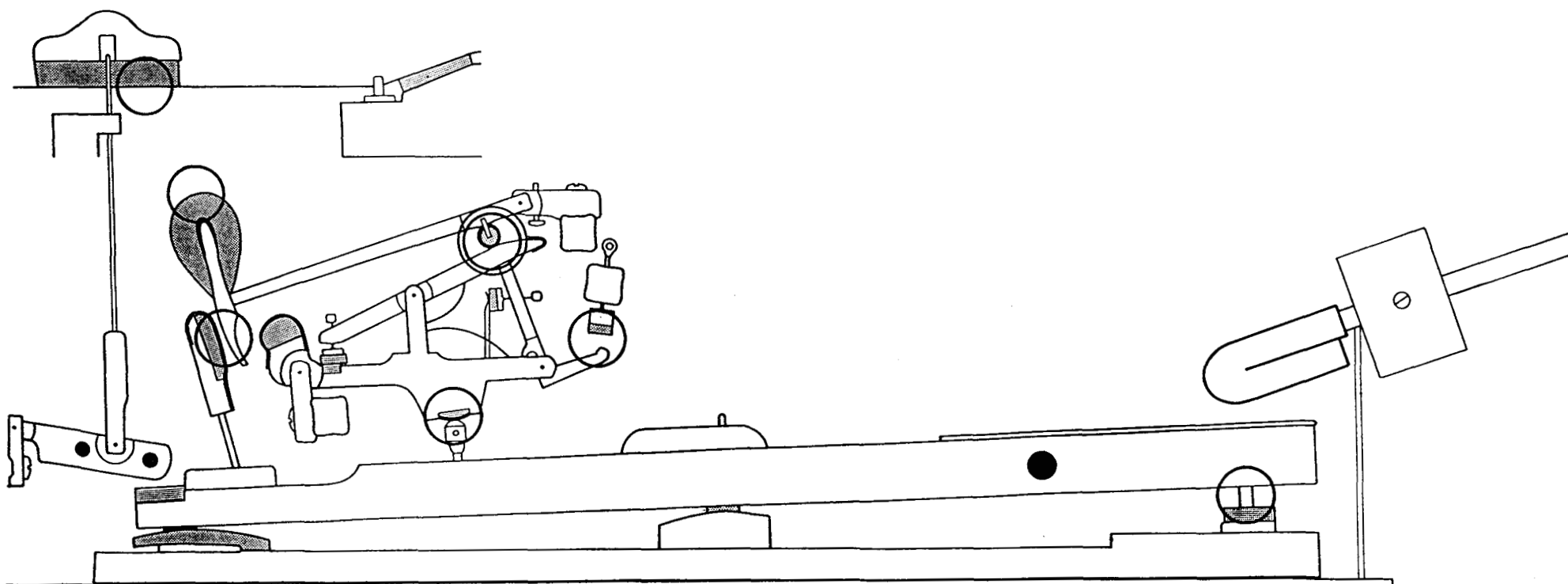


Fig. 3. View of the grand action indicating the contact points prepared with copper foil or graphite (circles). An arrangement of a pendulum for mechanical playing is also shown.

3. Measurement equipment

The signals were recorded by means of a digital storage oscilloscope (Gould, Digital Storage Oscilloscope 1425) and a plotter.

D. Measurements and results

This section opens with an overview of the timing in a well-regulated action. Then the effect of changes in the regulation on the timing are examined. The pianist's role in determining the timing by changing the dynamic level and touch is also analyzed. In particular, the timing relation between the key bottom and the hammer-string contact is described. Finally, the duration of the hammer-string contact is examined.

1. Overview of the timing in the action

An overview of the time events in a well-regulated action during two different types of touch are shown in Fig. 4a and 4b. The example in Fig. 4a is representative of a "staccato-touch" in forte with the finger striking from some distance above the key, and releasing the key immediately after the blow.

This timing diagram includes about 15 important events. Immediately after the process was started by the player's finger on the key, the support of the hammer was switched from the repetition lever to the jack (not shown in figure). Next, the damper was lifted off the string. At about 5 ms before the hammer-string contact, the tail end of the jack made contact with the escapement dolly, and the top of the jack was withdrawn from the roller. Shortly before the impact on the string, typically less than 1 ms, the jack lost contact with the hammer roller, and the hammer swung freely towards the string. The key reached its bottom position a few milliseconds before the hammer struck the string.

The hammer-string contact time occupied approximately 2 ms. Almost immediately after the hammer-string contact had ceased, the hammer returned, first making contact between the repetition lever and the roller. After additional 5 ms, the hammer was captured by the check. If the key were held down after the stroke, the action would remain in this state while the note decayed undisturbed.

As the key was released, the parts of the action reverted to their initial positions. The tone was terminated when the damper came down on the string approximately 80 ms after the hammer-string contact and brought the string to rest after some bouncing. The falling damper and damper lever, which landed on the far end of the key, caused the key to occasionally lose contact with the lever body.

In this example, the duration of the note was approximately 140 ms, corresponding to a sixteenth note in andante tempo (MM = 107). The

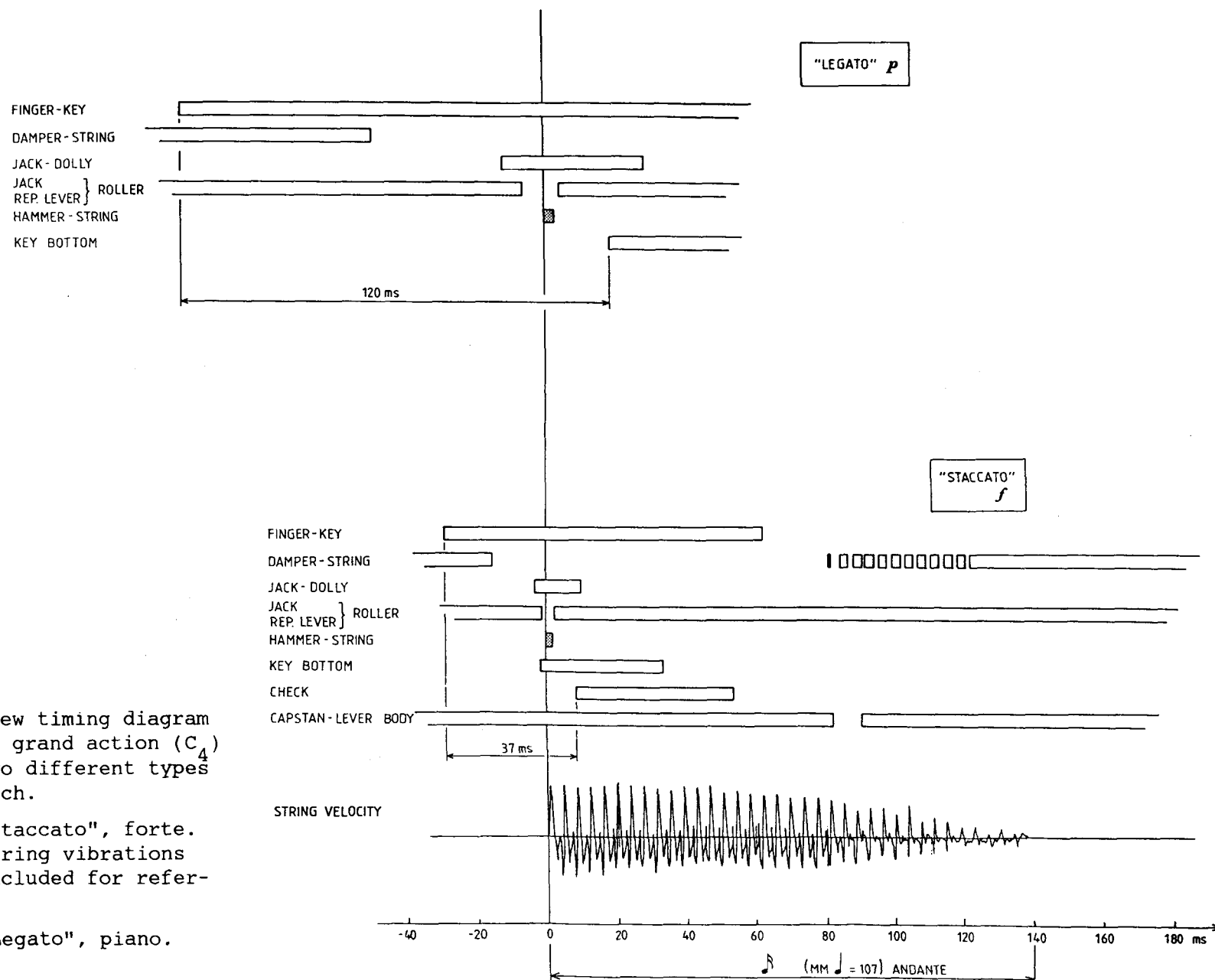


Fig. 4. Overview timing diagram of the grand action (C₄) for two different types of touch.

a. "Staccato", forte. The string vibrations are included for reference.

b. "Legato", piano.

process from the moment the finger touched the key until the hammer was captured by the check, lasted about 40 ms.

In a soft "legato-touch", with the finger resting on the key from the beginning, the entire process was slowed down (see Fig. 4b). The time from the start of the key motion to the check of the hammer was roughly quadrupled compared to the "staccato-touch". Interestingly, the key bottom contact was much delayed relative to the hammer-string contact as compared to the forte touch. Further, the hammer-string contact time was extended.

This example suggests that the dynamic level, controlled by the strength of the player's touch, causes large changes in the timing patterns in the action. Further experiments showed that the timing patterns also could be changed by the piano technician's regulation of the action. The influence of these factors will be the topics of the following paragraphs.

The aspect of the repetition rate was studied briefly. It was confirmed by measurements that the double repetition mechanism allows fast repetitions. A professional player was easily able to reach a repetition rate of about 8 notes/s in the middle register by using two fingers repeatedly on the same key (120 ms between the note onsets). By using three fingers, the repetition rate was increased to about 16 tones/s. A semitone trill was performed with the same repetition rate.

2. Influence of regulation

a. Hammer-string distance

The hammer-string distance was varied in three steps from long, via normal, to short setting. The distances were $47 \text{ mm} \pm 3 \text{ mm}$ ($\pm 6\%$), respectively. The changes from the normal setting could be characterized as large, but with the instrument still in a playable condition. The longer distance represents considerable wear caused by a long period of playing, while normally the shorter hammer-string distance is not observed in practice, except when due to a blunder in the regulation.

The effect of the changes in the hammer-string distance on the timing pattern is shown in Fig. 5. The let-off distance was adjusted after each change of the hammer-string distance to a normal value (1.8 mm). The dynamic level was held constant at a mezzo-forte level.

The changes in the hammer-string distance affected mainly the timing relation between the key bottom contact and the hammer-string contact. As the hammer-string distance was made longer, the moment of the key bottom contact occurred earlier, as compared to the hammer-string contact. With the normal setting of the hammer-string distance, the key reached its bottom position approximately 2 ms after the hammer string contact. This delay was approximately doubled in the short setting, while it shortened to almost no delay at all in the long setting.

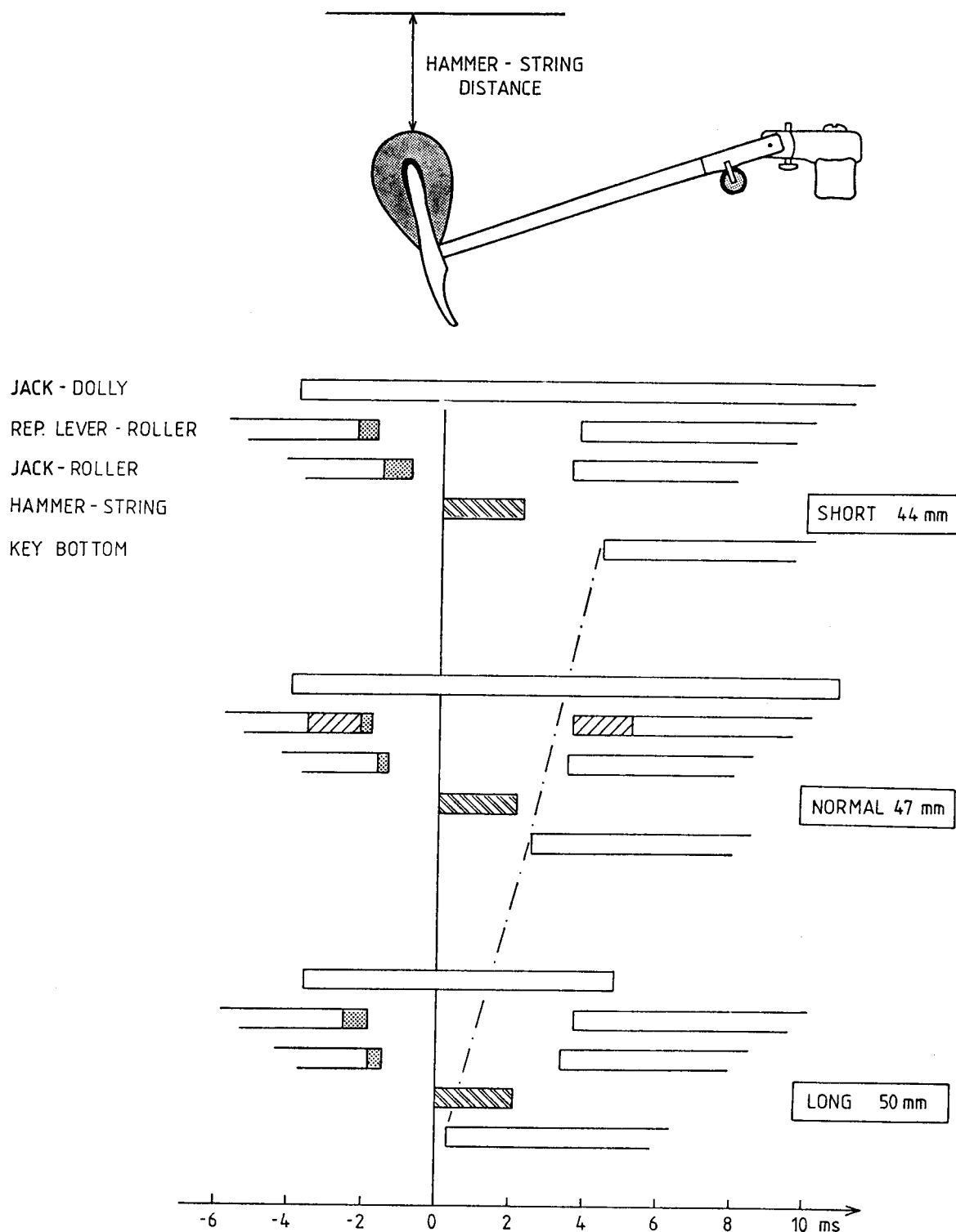


Fig. 5. Influence of the regulation of the hammer-string distance on the timing in the action (C_4), mezzo-forte. The three regulation conditions correspond to short hammer-string distance (top), normal (middle), and long (bottom). The dashed line indicates the shift in key bottom contact. The dotted parts indicate segments with low contact force. The hatched portions in the bars for the repetition lever-roller indicate the changes caused by lowering the drop screw by 0.8 mm (one turn of the drop screw).

With a short hammer-string distance, the hammer release and the hammer-string contact occur at a high level in the key travel, which results in a long "after-touch". This means that the key still has a long way to go after the hammer release, and, consequently, it will reach its bottom position late after the string contact. A long hammer-string distance, on the other hand, gives hammer release and string contact with the key far down and almost no "after-touch", which means that the key reaches its bottom position close to, or even after, the string contact.

The contact between the repetition lever and hammer roller is interrupted as the lever is stopped by the drop screw. This moment is not influenced by a change in the hammer-string distance but occurs at a certain level in the hammer motion, set by the drop screw. The repetition lever is not involved in the force transmission from key to hammer. The lever is held in continuous contact with the roller by its spring until it is stopped and held in waiting position for the returning hammer.

b. Let-off distance

The let-off distance was varied in three steps from long, via normal, to very close setting. The corresponding values were 3.8 mm (+110 %), 1.8 mm (+ 0%), and 0 mm (-100%). The changes from the normal setting could be characterized as large, but with the action still in a playable condition. The let-off distance is normally set within the range 1.0-2.0 mm in the middle register. The distance changes with time, mainly because of the normal increase in hammer-string distance, but also due to deformation of the roller and compression of the felt on the escapement dolly. The direction of the change depends on the initial condition of the action, but in most cases the let-off distance increases with time*.

The effect of changes in the let-off distance on the timing pattern is illustrated in Fig. 6. The hammer-string distance was adjusted to a normal value prior to the experiment. The dynamic level was kept the same at a piano-level in all three cases.

The adjustment of the let-off distance affected mainly the time during which the hammer swung freely before and after striking the string. At this dynamic level, a normal setting of the let-off distance gave a free interval for the hammer of approximately 2.5 ms, before the string contact. The roller reached contact with the jack again about 0.5 ms after the string contact had ceased.

* Personal communication, principal piano technician Hans Norén, The Swedish Radio Company, and Conny Carlsson, The Keyboard Committee, Royal Swedish Academy of Music, Stockholm.

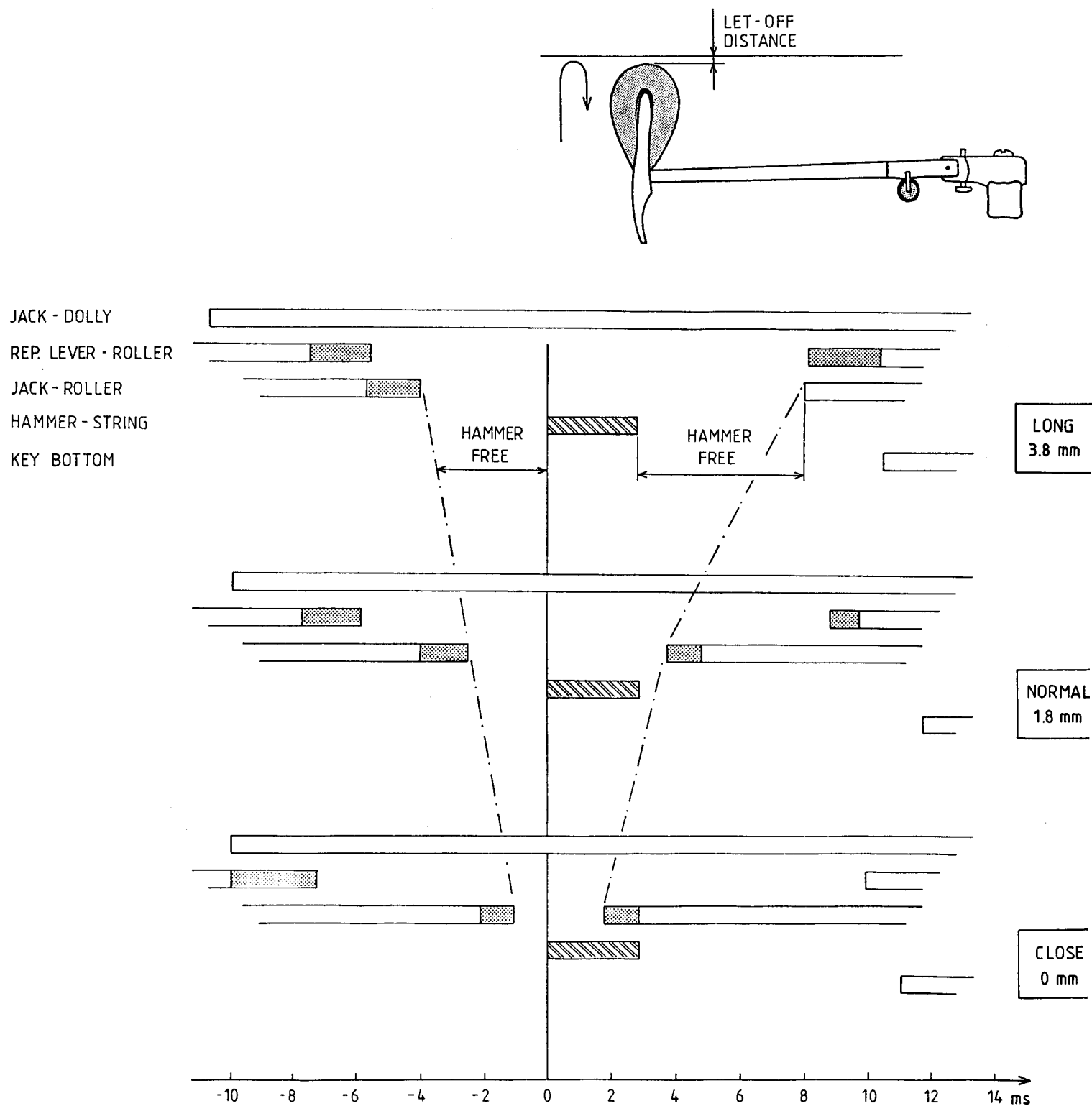


Fig. 6. Influence of regulation of the let-off distance on the timing in the action (C₄), piano. The three cases correspond to long (top), normal (middle), and close let-off distance (bottom). The dashed lines indicate the shifts in the free time for the hammer. The dotted portions indicate segments with low contact force.

The longer the setting of the let-off distance was, the longer became the free interval before the string contact. This was a consequence of the fact that the tail end of the jack reached the escapement dolly at an earlier moment. With the long setting of the let-off distance, the free interval was almost twice as long as normal, and in the very short setting it was approximately halved.

For the pianist, a close adjustment of the let-off distance means that the key can be depressed further down before the "let-off"-point is reached, which facilitates pianissimo playing under strict control. A close setting also means that the hammer is accelerated closer to the moment of the string contact, which means that more power could be delivered to the string (Norén & Carlsson, personal communication; Edwards, 1984; see also Part II). While a pianist will be able to break a string with a close setting of the let-off distance, the piano technician may easily eliminate this danger by increasing the let-off distance.

As the hammer returned from the string, the roller first touched the side of the tripped jack before it landed on the repetition lever. In the close adjustment, this contact had already begun before the hammer had left the string. This was probably caused by a forward motion of the jack, as the entire action settled after the acceleration process*.

The timing relation between key bottom contact and hammer-string contact was only marginally influenced by the adjustment of the let-off distance. This was a somewhat unexpected result. For example, with the let-off distance adjusted to the close condition, the hammer was accelerated closer to the string resulting in a shorter free time for the hammer. This could be assumed to cause an earlier string contact compared to the key bottom contact. However, the free time for the hammer seemed to have marginal influence only on the timing between the string contact and the key bottom contact. It was only the level of the key at "let-off", set by the hammer-string distance, which determined the relation between the hammer-string contact and the key bottom contact for a given dynamic level.

At higher dynamic levels, the motions of the parts of the action were faster and the entire timing pattern compressed.

* High-speed film made by H. Suzuki, included in "How to regulate a Steinway", Steinway & Sons.

3. Influence of dynamic level

a. General observations

Some of the events in the timing patterns showed a strong dependence on the dynamic level. In general, the motions of the parts of the action were faster and the time scale compressed at louder dynamics. Examples of registrations, typical for three dynamic levels (p, mf, f) with the action in a normally regulated condition, are given in Fig. 7. More specifically, an increase in the dynamic level resulted in the following four major changes:

- (1) The moment of the contact between the jack and the escapement dolly was reached closer to string contact.
- (2) The free time for the hammer before the string contact became shorter.
- (3) The key bottom contact was reached earlier compared to the hammer-string contact.
- (4) The hammer-string contact duration decreased (discussed later in this section).

The changes in the timing patterns according to (1) and (2) could be attributed to the higher key and hammer velocities for the louder dynamics. In combination with (3), these changes ought to give the pianist a more precise control over the moment of the tone onset at a higher dynamic level.

As regards the free time of the hammer before and after the string contact, the following was observed. At forte level, the jack remained in contact with the roller for a short moment after that the hammer had reached string contact, but with low contact force. Presumably, the jack was almost tripped, but the side remained in light contact with the roller. The moment of "let-off" could not be experienced in the touch. With a close setting of the escapement (cf. Fig. 6), the jack remained in effective contact with the roller during the initial 0.5 ms of the hammer-string contact. In this case, the "let-off" of the hammer could be perceived as a snap in the touch.

b. Key bottom and hammer-string contact

The observed variation in the timing relation between the moments of key bottom contact and hammer-string contact, (3), was studied in further detail. This timing relation was found to be strongly dependent on the dynamic level (see Fig. 8a).

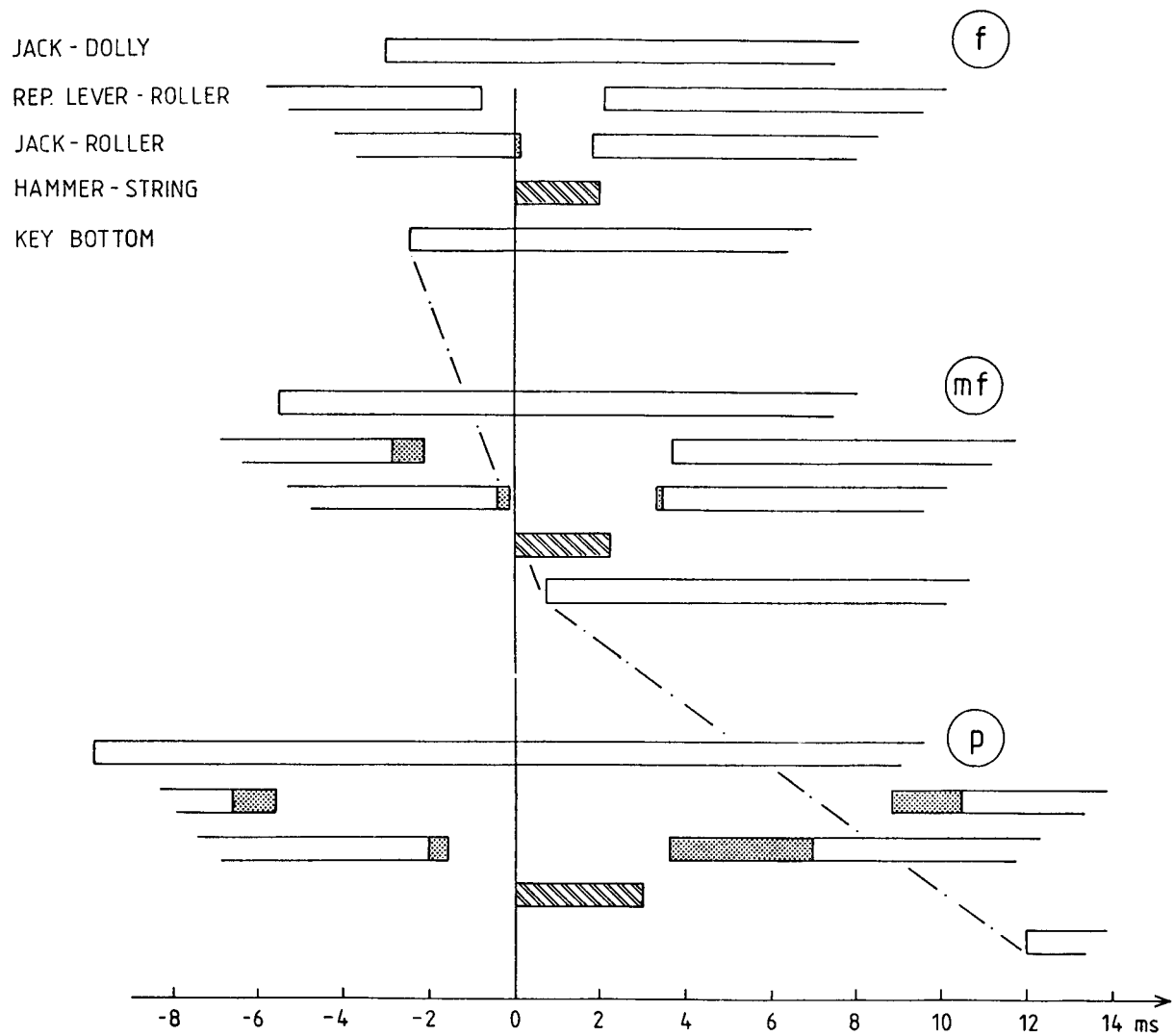


Fig. 7. Influence of dynamic level (p - mf - f) on the timing in the action (C₄). Normal regulation condition. The dashed line indicates the shift in key bottom position.

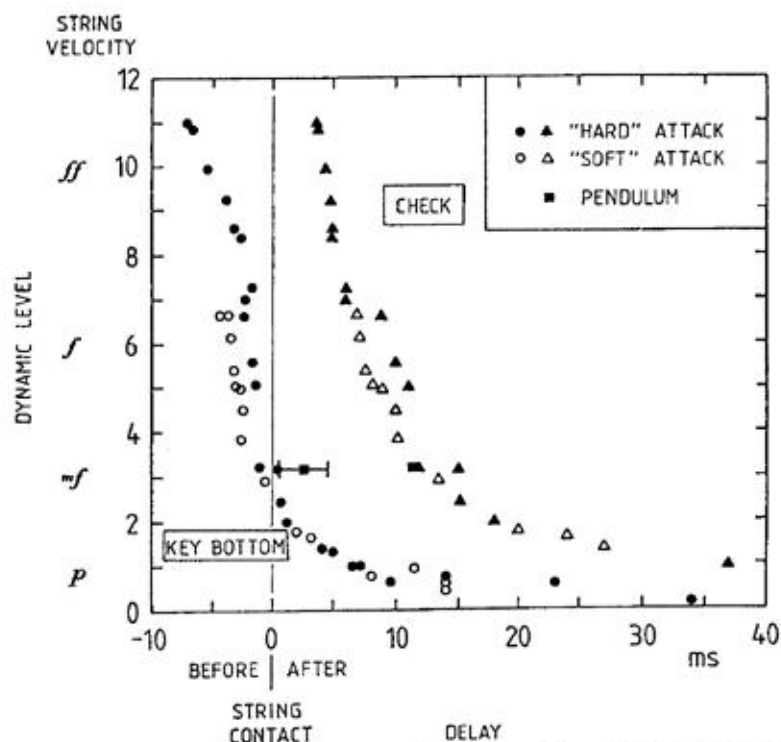
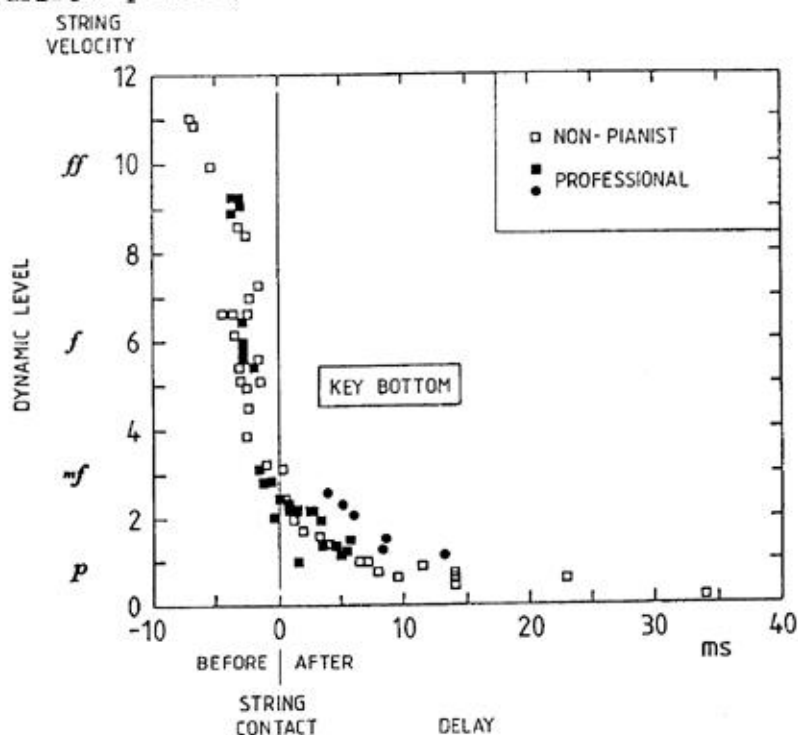


Fig. 8. a. Influence of the dynamic level on the timing relation between key bottom contact and hammer-string contact (untrained subject). Also included is the moment of hammer capture by the check. The unfilled symbols represent a "soft" attack with the finger resting on the key from the beginning. The filled symbols represent a "hard" attack with the finger striking the key from above. An artificial touch by a pendulum is represented by squares. The horizontal bar line indicates the changes caused by an adjustment of the hammer-string distance according to Fig. 5 (short - normal - long). The dynamic span ranges from *pp* to *ff*, corresponding to 33 dB. The dynamic level was estimated as the velocity amplitude of the first pulse on the string (arbitrary units).



b. Comparison of the timing relation between key bottom contact and hammer-string contact in the performances of an untrained subject (unfilled symbols) and a professional pianist (filled symbols). The dots mark notes played with very unusual types of touch.

In fortissimo, the key reached its bottom position approximately 5 ms before the hammer struck the string, in mezzo-forte at approximately the same moment, and in pianissimo up to 35 ms after string contact. Different principal types of touch with the finger resting on the key initially ("soft touch"), vs. with the finger striking the key from above ("hard attack"), showed almost no differences at all in the relation between key bottom and hammer-string contact. However, the loudest dynamics could not be reached with the "soft touch". The dynamic range covered in this experiment was large, approximately 33 dB (Burghauser & Spelda, 1971).

The moment of hammer capture by the check followed a trend similar to the key bottom contact, but delayed. The delay between string contact and check increased from approximately 8 ms in forte to about 20 ms in piano, reflecting the differences in hammer velocity.

A comparison of the performances of an untrained subject and a professional pianist showed small differences, and only at soft dynamic levels (see Fig. 8b). The skilled player managed to make only small deviations from the general trend, even though he was encouraged to apply different unusual types of touch*.

The continuous shift in the timing relation between the hammer-string contact and the key bottom contact is probably partly caused by a compression of the felt and leather parts in the higher dynamics. This gives hammer "let-off" at a key level closer to the bottom position. As a result, the key reaches its bottom position closer to, or even before, the string contact.

Further, general differences in touch between soft and loud dynamics also contribute to the shift in timing relation. In order to achieve a soft dynamic level, the final velocity of the hammer must be low, in the very soft levels the hammer should almost turn spontaneously at the string level**. This requires a careful control of the force applied by the pianist, which must barely overcome the static force needed to depress the key and still give the hammer a well-balanced impulse (Dijksterhuis, 1965). This is feasible due to the sensitive response of the action, which lets the player feel the pressure point at which the impulse is given. After this point, the key need not be

* Interestingly, the untrained subject revealed a wider dynamic span than did the professional pianist. This difference could probably be attributed to a lack of understanding of the musical usefulness of the notes at the extremes of the dynamic range, as regards the non-pianist.

**The task bears a certain resemblance to trying to bounce a tennis ball off a racket, making the ball turn at the level of the ceiling.

further accelerated and is stopped relatively gently at the bottom position. A touch at a soft dynamic level could thus be assumed to give a late key bottom contact compared to the string contact, in concordance with the observations.

At a high dynamic level, on the other hand, the pianist aims at a high acceleration of the hammer, which makes the touch resemble a stroke rather than a push. In this case, the hammer is accelerated heavily also close to the bottom contact, and the stop of the key is abrupt*. In combination with the compression of the parts in the action, this suggests a key bottom contact close to the string contact.

The key bottom contact can be assumed to be an important mechanical response from the instrument, decisive for the player's ability to perform the desired timing of the notes. For this reason, a well-established correspondence between the key bottom contact and the onset of the note defined by the hammer-string contact seems fundamental for a successful performance on a particular instrument.

The changes in the timing relation between the hammer-string contact and the key bottom contact, introduced by an adjustment of the hammer-string distance (Fig. 5), are included in Fig. 8a. The magnitude of these changes suggests that they could offset normal timing conditions. Expressed as an equivalent change in the dynamic level, the change from normal to short vs. long hammer-string distance corresponded to a shift from mezzo-forte to forte and mezzo-forte to mezzo-piano, respectively. Presumably, such deviations from a normal regulation must be considered by pianists in their performance, especially if the regulation happens to vary between the keys. Even by applying unusual types of touch, the professional player in the experiments could not induce a deviation from normal timing of the same magnitude as the deviation introduced by changes in the hammer-string distance.

Summarizing this particular aspect of the timing, the variation in the timing relation between the hammer-string contact and the key bottom contact according to the dynamic level seems to be an inherent property of the system action-pianist. The overall variation between ff and pp was large, on the order of 30 ms. It could be assumed that the skilled pianist is familiar with this characteristic change in the timing relation with level, and takes this into account in playing. The changes in the timing relation introduced by offsets in the regulation of the hammer-string distance were much smaller than the overall variation according to the dynamic level, but they were probably large enough to have an influence on the pianist's way of playing.

* In this case, the analogy with the tennis racket would correspond to a smash.

4. Hammer-string contact duration

a. Variation over the compass of the piano

Measured hammer contact durations over the compass of the piano, sampled by one key per octave, are shown in Fig. 9a and 9b. For each note, the contact durations were measured at the loudest and softest possible playing, respectively.

The measurements in the louder dynamics showed very stable durations, while the registrations in the softest dynamics (ppp) showed a larger variation between repeated blows. This variation presumably reflected a difficulty in obtaining identical final hammer velocities upon repeating a blow, when very low touch forces are used (Dijksterhuis, 1965). These small variations in the hammer velocity give pronounced variations in the contact duration due to the nonlinear hammer compliance.

The string contact duration was found to vary from about 0.5 ms in the treble to 4 ms in the bass, referring to a mezzo-forte level. The results were in reasonable agreement with contact durations reported earlier (Quitter, 1958). In the lowest bass, multiple contacts occurred above the pianissimo level (see Part III) (Hall, 1987b; Suzuki, 1987a).

Predictions for the note C_4 , when influence of hammer compliance is excluded, give a contact duration of 1.6 ms if the hammer is considered light compared to the string, and 2.1 ms if the hammer is considered heavy (Hall, 1987a, Eq. 40; Hall, 1987b, Eq. 31). In reality, none of the cases is applicable, as the hammer mass for C_4 is approximately equal to the string mass. A better prediction, which takes the actual mass ratio into account, could be read as 1.7 ms from computer generated graphs (Hall, 1987a). An inclusion of the hammer compliance would tend to increase the contact durations slightly (Hall, 1987b). The measured contact durations for C_4 covered a range from 2.0 to 2.8 ms depending on the dynamic level, in reasonable agreement with the predicted values. The observed values are a little longer than the predicted, which probably could be attributed to the omitted hammer compliance.

The contact durations did not change over the compass in proportion to the fundamental period of the string (T_0) (see Fig. 9b). In the bass, the string contact was only a fraction of a period, in the middle register about a half period, and in the treble several periods. This change in relative string contact duration over the compass has implications for the number of prominent partials in the string spectra, as string modes with a period essentially shorter than the string contact duration will be weakly excited (Benade, 1976, Ch. 8; Hall, 1987a).

On the other hand, a string mode for which the period is much longer than the hammer contact duration will be inefficiently excited by the short hammer impulse. This implies that the present design of the

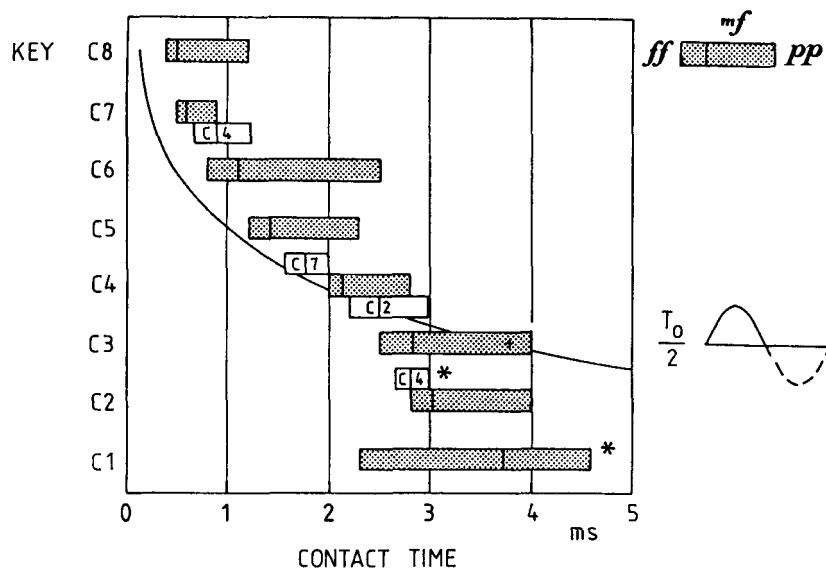
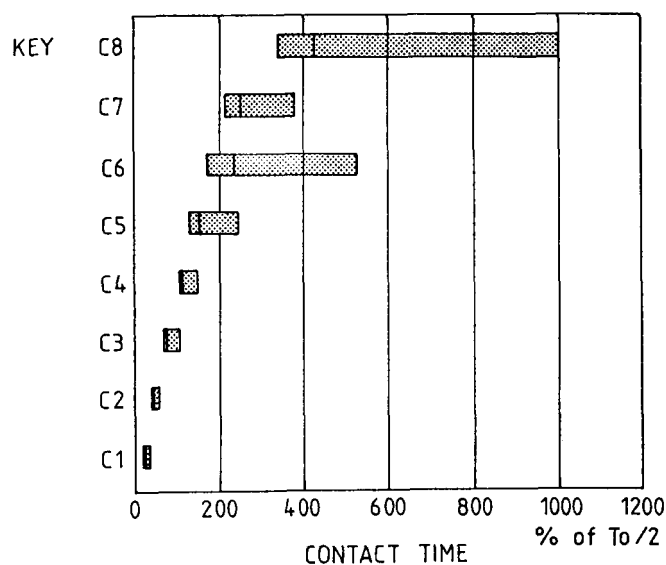


Fig. 9. Hammer-string contact durations. The bars indicate the range in contact duration between a blow in ff (left end) and pp (right end). The vertical line in each bar represents a blow by a pendulum in mezzo-forte level. Notes giving multiple contacts are marked with an asterisk. The contact durations for these notes are given as the time from the first onset to the last offset. Observe that for multiple contacts, the left end of the bar corresponds to pp and the right end to ff.

a. The contact durations expressed in absolute time. The solid curve ($T_0/2$) represents half a period of the fundamental for the corresponding notes. Symbols in unfilled bars indicate exchanged hammers.



b. The contact durations expressed as per cent of half a period of the fundamental ($T_0/2$).

piano exhibits nonuniform spectral characteristics over its compass due to the differences in excitation. The bass would exhibit prominent higher partials, comparable in strength with the fundamental, while the higher partials in the treble would be relatively weak. Typical differences in measured spectra in different ranges will be discussed in Part III.

A couple of hammers were exchanged ($C_4 \rightarrow C_2$, $C_4 \rightarrow C_7$, $C_2 \rightarrow C_4$, $C_7 \rightarrow C_4$), in order to further study the influence of the hammer mass and the compliance on the string contact duration. The contact durations were found to increase somewhat with the heavier bass hammer and to decrease with the lighter treble hammer. The predicted changes in contact durations, according to the changes in string and hammer masses, were a little smaller than the observed values. This could probably be attributed to differences in the hammer compliance. In the bass (C_2), multiple contacts developed when the original hammer was replaced by the C_4 -hammer, in line with predictions (Hall, 1987b).

b. Dependence on dynamic level

The hammer-string contact durations decreased with rising dynamic level, due to the nonlinear compliance of the hammer (Boutillon, 1988; Hall & Askenfelt, 1988; Suzuki, 1987a). The observed ratios of the contact duration for a note in ff (short) and pp (long), respectively, varied between 1.5 and 3 for the notes sampled. This pronounced shortening of the contact durations from pp to ff implies a considerable increase in the high frequency of the spectra with rising level (further discussed in Part III). In the lowest bass (C_1), multiple contacts gave a longer total contact duration in forte than in piano (see also Part III).

The influence of the dynamic level on the string contact duration was further examined in the middle register (C_4), by observing the relation between the final hammer velocity before the string contact and the hammer contact duration (see Fig. 10). The final hammer velocity was estimated by measuring the amplitude of the first velocity pulse on the string (further discussed in Part III) (Hall, 1987b). The figure shows that the contact durations increased rapidly below mezzo-forte level. The variation in contact duration over a comfortably accessible dynamic span from piano to fortissimo was roughly ± 0.4 ms ($\pm 20\%$), as compared to mezzo-forte. The extremely long contact durations observed are probably seldom approached in normal playing.

When transformed to a log-log plot, the dependence of dynamic level was found to closely follow a straight line ($r=0.99$), reflecting a power-law relation between the force and the compression of the hammer felt (Hall & Askenfelt, 1988).

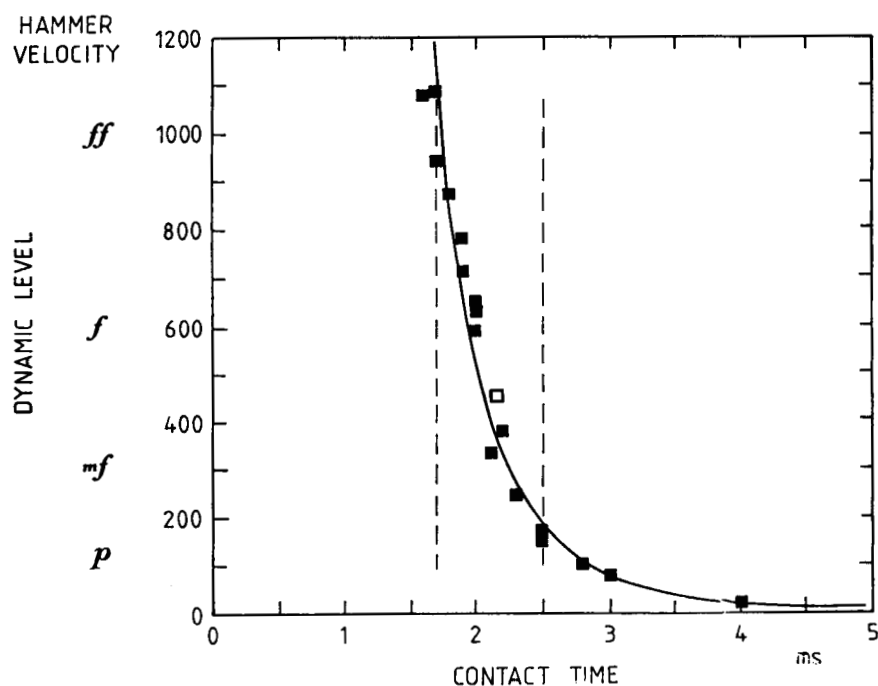
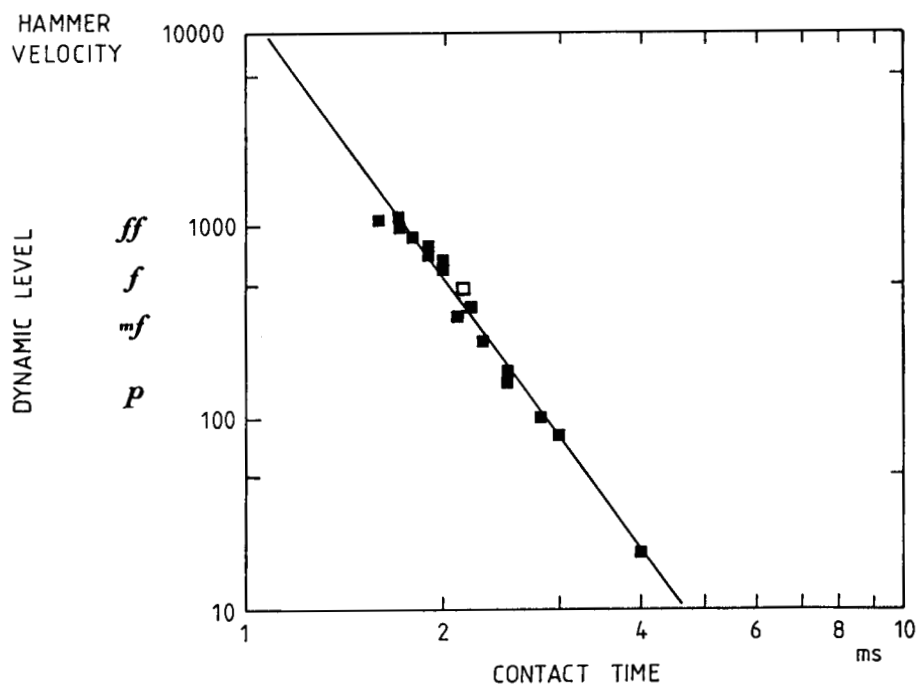


Fig. 10. Influence of dynamic level on string contact duration (C_4). The final hammer velocity was estimated by the velocity amplitude of the first pulse on the string (arbitrary units). A touch by a pendulum (*mf*) is represented by an unfilled square.

a. Linear plot. The dashed lines represent the range in contact duration covered in a comfortable dynamic span *p* to *ff*.



b. Log-log plot, showing a power-law relation ($r = .99$).

E. Conclusions

The timing in the piano action was found to be dependent on both regulation and dynamic level. Changing the hammer-string distance affected mainly the timing relation between the key bottom contact and the hammer-string contact. Changes in the setting of the let-off distance affected mainly the interval during which the hammer swings freely before the string contact. An increase in the dynamic level decreased the free time for the hammer, but with a normal regulation of the let-off distance, the force transmitting contact between the action and the hammer was always found to be interrupted before the hammer-string contact.

The timing relation between hammer-string contact and key bottom contact showed large changes with dynamic level. The overall variation between ff and pp was on the order of 30 ms. It could be assumed that the skilled pianist is familiar with this characteristic shift in timing with level and takes it into account when playing. The changes in the timing relation introduced by offsets in the adjustment of the action were much smaller than the overall variation with level, but they were probably large enough to influence the pianist's way of playing.

The hammer-string contact duration varied over the compass of the instrument. Expressed in absolute values, the contact durations were considerably longer in the bass (4 ms) than in the treble (0.5 ms). However, in comparison with the fundamental period of the string, the contact durations were short in the bass and long in the treble, which could be assumed to give characteristic differences in the string spectra.

The dynamic level affected the contact durations, a consequence of a nonlinear hammer compliance. The dependence was found to follow a powerlaw relationship. Typical variation in the contact duration between piano and fortissimo in the middle register was on the order of +20%, as compared to mezzo-forte.

PART II. THE MOTION OF THE KEY AND HAMMER

A. Properties of the action

The key is a two-armed lever, pivoted approximately at its mid-point. The lever arms are surprisingly long, approximately 270 mm (white keys). The stroke of the key in the playing end ("touch depth") is 9.5 mm (Dietz, 1968). The total levering ratio between key and hammer is slightly less than 1:5, making the hammer head rise approximately 45 mm for a full key stroke. However, the levering is divided into several steps, including the key, lever body, and hammer, which all act as levers. The motion at the contact point between the jack and the hammer roller is reduced by a factor 1.6:1 compared to the playing end of the key, which increases the force in the jack correspondingly.

The action is counterbalanced with pieces of lead in the key. Typically, the static force needed to barely hold the key pressed down (without damper) is adjusted to 0.5 N by the weighing of the key. Further considerations on the dynamical properties of the action are given in several works (Dijksterhuis, 1965; Junghanns, 1984; Lieber, 1985; Pfeiffer, 1967; 1979).

From the viewpoint of the pianist, the action serves as an interface between fingers and strings. As the key is depressed, the hammer is lifted and accelerated towards the string. Shortly before the hammer makes contact with the string, the hammer is disconnected from key control ("let-off"). Compared to other struck string instruments, e.g., the clavichord or the dulcimer, the pianist is in a sense given only indirect control of the string excitation, which could be assumed to be set by the final velocity of the hammer. However, the technique for depressing a single key ("touch") is given much attention by pianists, who often claim that important shadings in tone quality at a given dynamic level can be achieved by different types of touch. Pianists and experimenters have had conflicting standpoints regarding this question for a long time (Hart, Fuller, & Lusby, 1934).

B. Scope of measurements

The measurements aimed at a description of typical key and hammer motions at different dynamic levels. Also, different types of touch, as selected and performed by a professional pianist, were included in order to elucidate the general effects of touch on key and hammer motions. The accelerating force on the hammer up to hammer release ("let-off"), and the resulting hammer motion, attracted special interest. These measurements offered a possibility of studying the influence of touch on the hammer's free motion immediately before the string contact. A perceptual evaluation of the influence of touch was, however, not considered in this study. As strong oscillations in the hammer were

observed both before and after the string contact, the eigenmodes of the hammer were also examined.

C. Measurement methods

1. Key and hammer position

The key and the hammer motions were registered using an optical method. A position sensitive photodetector* was utilized, in which the position of a light spot on the detector area is converted into a voltage. Two light sources consisting of super-intense light emitting diodes (LED) were attached to the key and hammer, respectively. On the key, the LED was mounted in a plastic support (mass 3 g), fastened to the key with synthetic wax. On the hammer, the LED was mounted on top of a light wooden stick (length 160 mm, mass 1.5 g), glued to the side of the hammer head, and running between the string triplets. A direct mounting of the LED on the hammer head was not possible, as the iron frame and strings would hide most of its path. The increase in moving mass of the key and hammer respectively, due to the addition of the LEDs and mounting accessories, was relatively small, approximately 15%.

The length of the detector area was 10 mm. The motions of the key and the hammer were reproduced within this range by mounting the detectors in the film plane of two reflex cameras. This arrangement had the advantage of an easy focussing of the path of the LED on the detector, simply by looking in the view-finder. The key and the hammer motions were registered as viewed from the side of the piano, perpendicular to the direction of the strings.

Experiments were also made with a fixed light source and a piece of reflecting tape attached to the moving part. Unfortunately, this arrangement was not successful. The amount of reflected light was too low to make the received signal visible above the detector noise.

The linearity of the position measuring systems was readily checked by lowering the key in well-controlled steps with the aid of a micrometer screw. The position of the hammer was read on the stick on a level with the strings. The deviation from linearity was less than 5% for both key and hammer positions.

In order to achieve optimum linearity, the total amount of light on the detector area should be kept constant. This condition can be difficult to secure for large variations in the position of the light source as in these experiments. However, if necessary, the output signal can be normalized with respect to variations in the illumination of the detector area by dividing the position signal with a signal

* SiTek 1L10, SiTek Laboratories, P.O. Box 261, S-433 25 Partille, Sweden.

proportional to the total amount of received light. Preliminary experiments indicated that the improvement in linearity by this technique was rather small, indicating that total amount of light on the detector was approximately constant. As a consequence, the normalization could be omitted, and the supporting electronics was reduced to a minimum during the measurements.

A distortion error was introduced in the measurements as the playing end of the key and the hammer head both followed circular arcs, while the detectors registered the motion along a straight line between the end points of these arcs. However, due to the relatively small angles of the key and the hammer motions, this error was small. The differences between the distances actually travelled and those measured, were less than 0.5%.

2. Key and hammer velocity

The velocities of the key and the hammer were obtained by differentiating the position signals in RC-networks with a time constant of 4 ms, corresponding to an upper frequency limit for the differentiation of approximately 40 Hz.

The optical measuring system described above was compared with an electrodynamic method applied in earlier studies (Askenfelt & Jansson, 1982; Jansson, 1978). The electrodynamic method measured the key velocity with the aid of a flat coil mounted on the key. The coil moved in a magnetic field in the air-gap of an iron-armature resting on the neighboring keys. The induced voltage was used as an estimate of the key velocity*. Comparisons with the optical method revealed that the methods sometimes gave differing results, in particular at the extreme positions of the key travel. Probably, the discrepancies were caused by an inhomogeneous magnetic field. The electrodynamic method of measuring key velocity was not used for any measurements in this investigation.

Experiments with the electrodynamic method were also made in order to measure the hammer velocity in the immediate vicinity of the string. A thin copper wire was attached around the periphery of the hammer head in parallel with the strings, making sure that the wire passed between the strings at the string contact. During the last millimeters of the hammer's path before striking the string, the part of the wire on top of the hammer head passed a magnetic field which induced a voltage, proportional to the hammer velocity. The magnet field was supplied by two magnets mounted in a horseshoe armature, which rested on the adjacent strings. The same magnet was also used in measuring the string velocity (see Part III).

* The same principle has also been used by N. Fletcher & S. Thwaites (1981).

A comparison with the optical method of measuring the hammer motion showed a close agreement (see Part III). The electrodynamic method offers a practical alternative to the optical method, if only the final hammer velocity is to be measured. However, as only the very last part of the hammer motion can be observed with this method, it was not used for any measurements in this study*.

3. Key and hammer acceleration

The acceleration at different positions on the key and the hammer was measured by means of a miniature accelerometer (B&K 4374). The increase in mass due to the accelerometer was small (0.65 g), in the middle register (C_4) approximately 8% of the effective hammer mass.

4. Contact force

The compression force in the jack was measured by means of a piezoelectric film**. A piezoelectric film is a thin plastic foil which is sensitive to an applied mechanical strain. The resulting deformation causes a change in the surface charge density which gives rise to a voltage between the surfaces. The film can be cut to any desired shape and connected by the copper foil with conductive adhesive. The surface charges will decay with a time constant determined by the capacitance of the film and the input resistance of the connected electronics. Typically, a rather short time constant of the magnitude 1-4 ms was achieved, depending on the size of the piece of foil used.

A narrow hole was drilled from the top of the jack, and the jack was cut at about 5 mm from the top. A piece of piezoelectric film was put in the joint before the jack was glued together and reinforced with a piece of piano wire through the holes. The contact force between the jack and roller was also measured indirectly by monitoring the bending of the jack. Due to the friction in the sliding motion between jack and

* An even simpler method of measuring the final hammer velocity is used commercially (Bösendorfer 290-SE). A slotted vane is mounted on the hammer shank, close to a hammer head. When the vane passes an infrared light ray of a photodetector, immediately before striking a string, a pulse sequence is generated which indicates the final hammer velocity. A similar method was used by Hart & al. (1934) more than 50 years ago, however, in that study the entire hammer travel was displayed on a photographic film.

**KYNAR Piezo film, thickness 28 μm , Pennwalt Corp., 900 First Avenue, King of Prussia, PA 19406, USA.

roller, a high contact force was assumed to be reflected as an increase the bending of the jack. The bending was detected by strain gauges glued to the front and back sides of the jack.

D. Measurements and results

The presentation of the measurements begins with registrations of typical motions of the key and the hammer at three dynamic levels (p-m-f). Then, the influence of touch on the key and hammer motions is examined. Thereafter, the accelerating force on the hammer will be considered, followed by registrations of the vibrations in the hammer shank during the hammer's travel towards the string. The section closes with modal analysis of a piano hammer.

1. Key and hammer motion.

a. Influence of dynamic level

The motion of the key was different depending on the dynamic level (see Fig. 11). A touch at soft dynamics, requiring a slow key motion, could be characterized as a "pressing" of the key, while the motion at loud dynamics is better described as a "blow", with strong oscillations normally superimposed on the key motion.

The depressing of the key from rest to bottom position (9.5 mm) typically lasted 160 ms at the piano level, at mezzo-forte 80 ms, and at forte 25 ms. The key accelerated to a final velocity of 0.1, 0.4, and 0.6 m/s, respectively. The shortest key down-travelling time observed in the experiments was 16 ms, in which a maximum key velocity of 1 m/s was reached.

The motion of the hammer (hammer-string distance, 47 mm) lasted approximately the same time as the motion of the key (cf. Part I). Due to the levering in the action, the hammer velocities are much higher than the key velocities (see Fig. 12). Typical final hammer velocities at piano, mezzo-forte, and forte were approximately 1, 2, and 5 m/s, respectively, corresponding to a span of 14 dB. Upon return, the hammer was checked at different levels depending on its initial velocity.

b. Influence of touch.

Characteristic differences in the motions of the hammer and key due to principally different types of touch could be observed (see Fig. 13a-d).

- (a) A "staccato-touch", with a slightly bent finger starting some distance above the key, always gave a key velocity curve with two maxima separated by a retardation. This slow oscillation at about 40 - 50 Hz could be attributed to a resonance in the system ham-

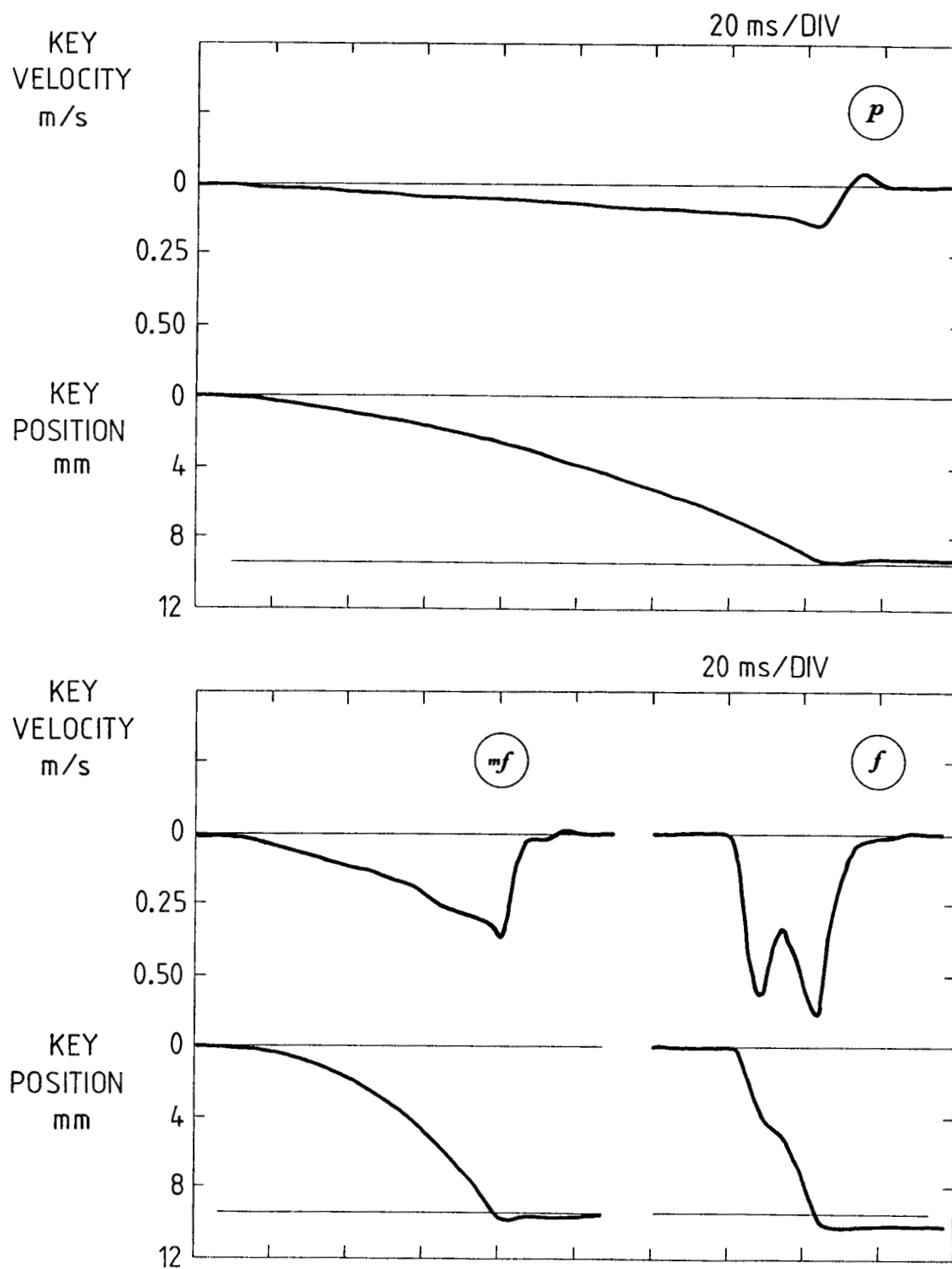


Fig. 11. Typical registrations of key position and velocity at three dynamics (piano, mezzo-forte, forte), C_4 . The horizontal line indicates the bottom position of the key*.

* All presented curves in this study are traced from the original charts.

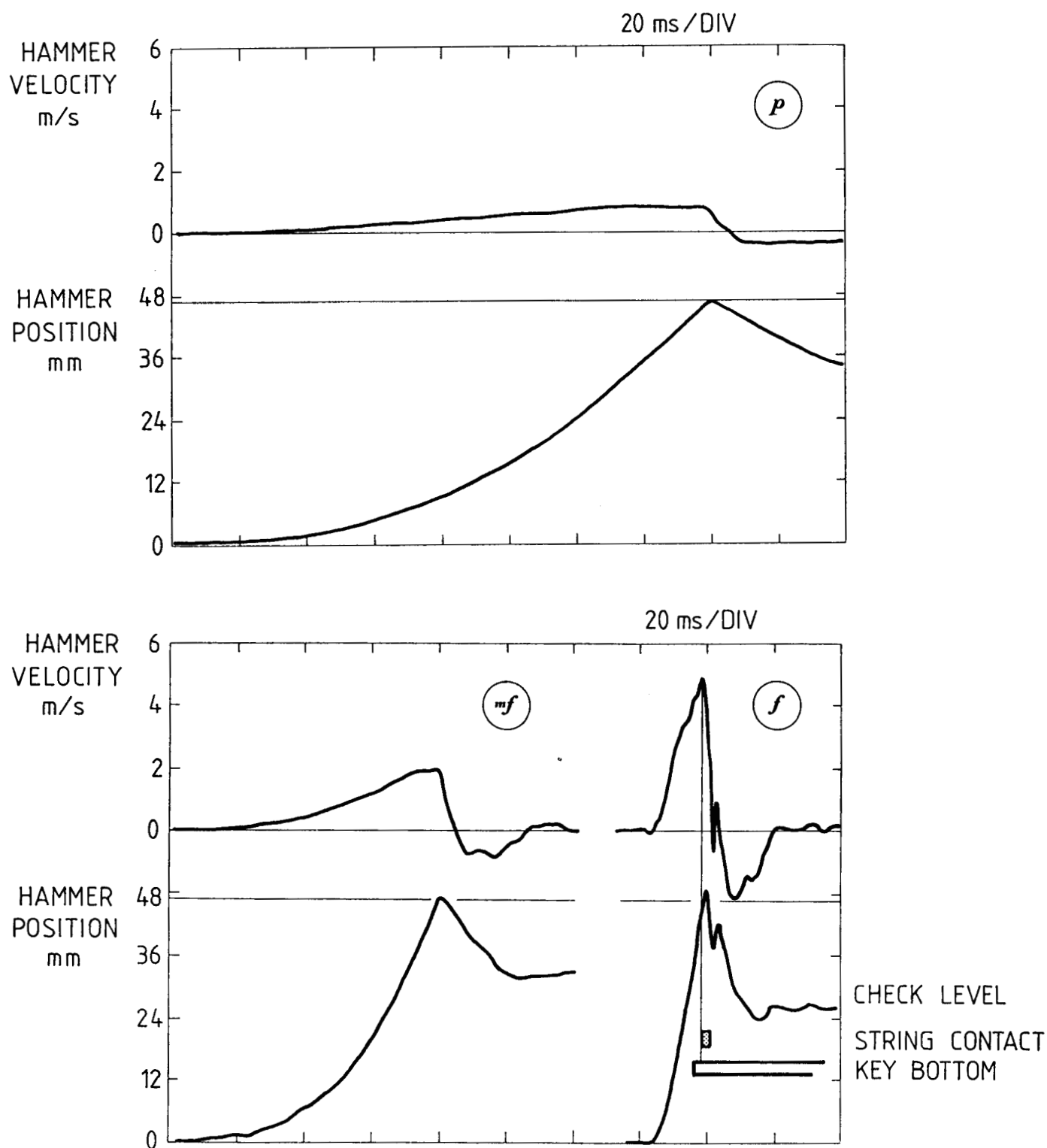


Fig. 12. Typical registrations of hammer position and velocity at three dynamics (piano, mezzo-forte, forte), C_4 . The horizontal line indicates the level of the string. Hammer-string and key bottom contact signals are included for reference in the forte-example. The registrations do not correspond to the key registrations in Fig. 11.

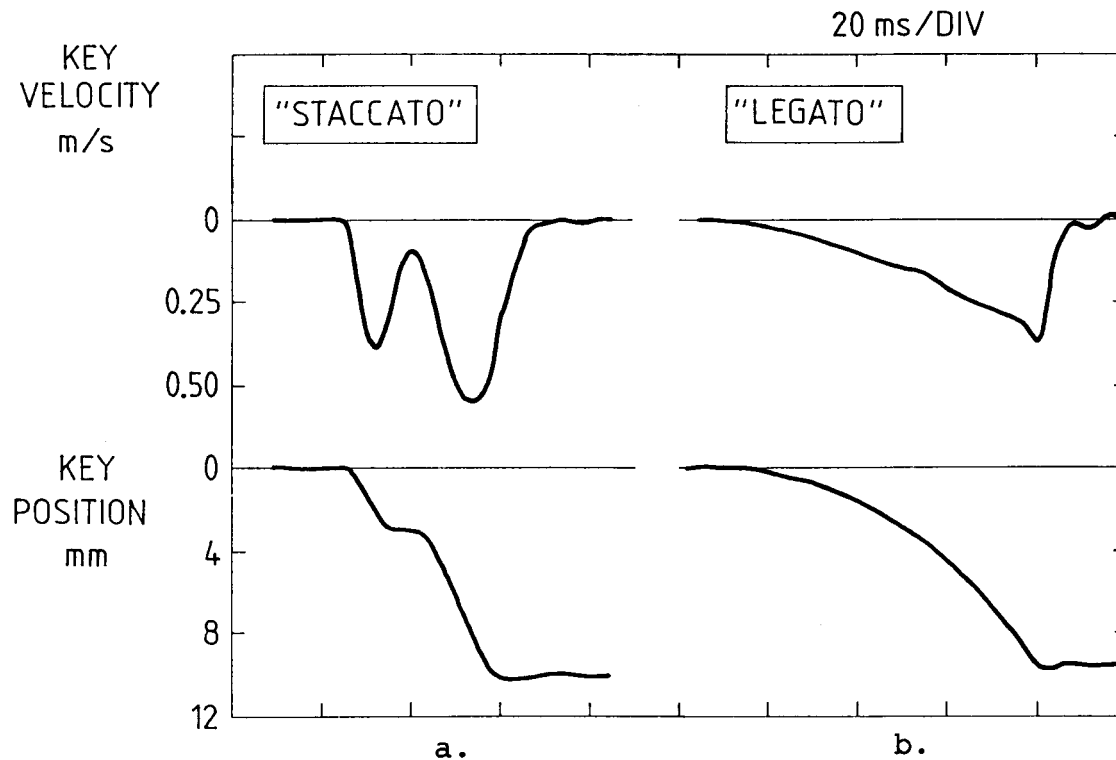
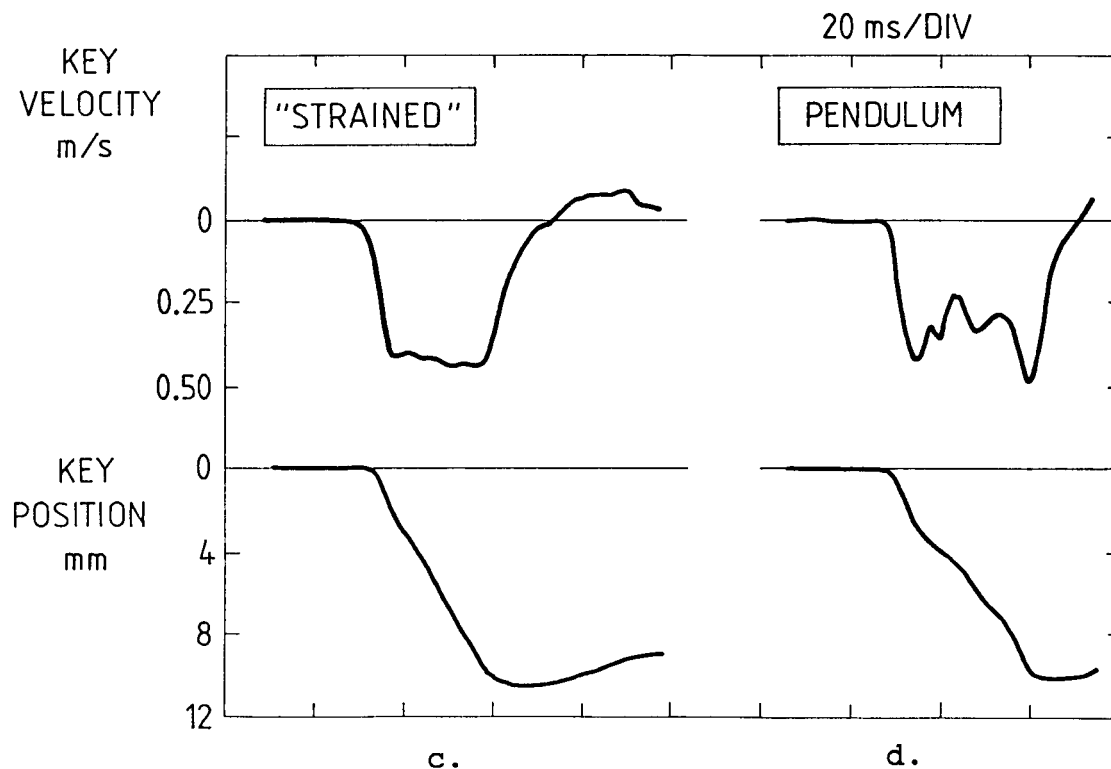


Fig. 13. Influence of different types of touch on the key motion, mezzo-forte, C_4 . The four cases represent:

- a. a "staccato-touch" with the finger starting its motion some distance above the key.
- b. a "legato-touch" with the finger initially resting on the key.



- c. an unnaturally strained touch with the finger and hand strained.
- d. a blow with a pendulum.

mer-action-hand, probably supported by a flexing mode of the hammer shank, discussed later in this section.

- (b) A "legato-touch" with the finger resting on the key from the beginning displayed a smooth motion with steadily increasing key velocity.
- (c) An unnatural touch with the finger and hand strained in a straight line gave a velocity curve divided in two parts. In this touch, an initial steep increase up to maximum velocity was followed by a constant velocity throughout the remaining part of the travel.
- (d) A touch by a pendulum performed somewhere in between the normally relaxed and the strained touch. The pendulum was used as a reference touch with regard to dynamic level in several of the experiments.

Two examples of simultaneous registrations of key and hammer velocities are shown in Fig. 14. In the "staccato-touch", the initial peak in key velocity seemed to correspond to a compression of the felt parts and probably also of the finger tip. An initial bending of the hammer shank is probably also introduced, as will be discussed later. The main acceleration of the hammer did not begin until the second peak in the key velocity developed. The key velocity curve showed a strange course during its final part, possibly caused by a sudden straining of the player's finger.

In the "legato"-touch, both the key and the hammer velocity increased continuously. A slower increase in the hammer velocity than in the key velocity during the initial part of the blow could be observed, probably also caused by a compression of the felt. The moment of retardation corresponding to the key bottom contact was reached before the estimated string contact, in accordance with the results discussed in Part I.

The acceleration of the hammer showed interesting differences depending on the type of touch. In particular, it was observed that the pianist could set the final velocity of the hammer by delivering the main acceleration during different parts of the key travel. Three examples of such "touch-profiles", resulting in approximately the same final hammer velocities, are displayed in Fig. 15a-15c. The following descriptions of each type of touch are compiled from the pianist's own descriptions of what he was doing, the experimenter's observations, and the measurements in Fig. 15.

- (a) A touch in which only the middle finger was involved, while the hand and the forearm were held still. The acceleration was smoothly

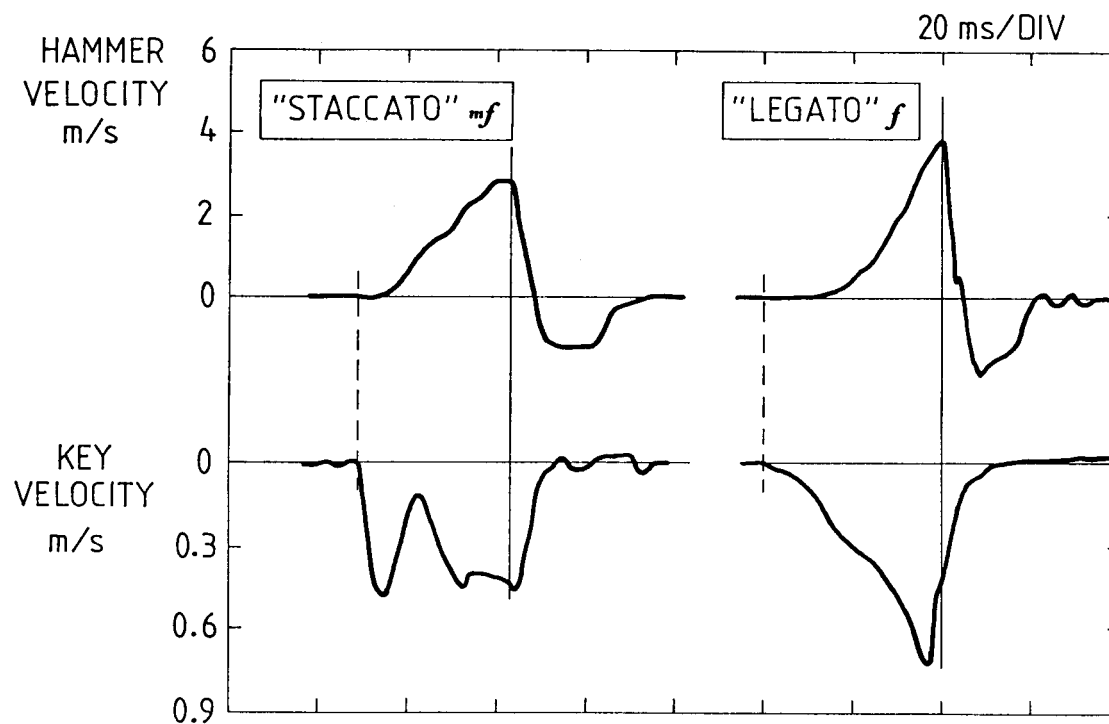


Fig. 14. Simultaneous registrations of key and hammer velocities for two types of touch ("staccato" mezzo-forte, "legato" forte), C_4 . The solid vertical line indicates the estimated moment of string contact, and the dashed line indicates the moment when the key starts to move.

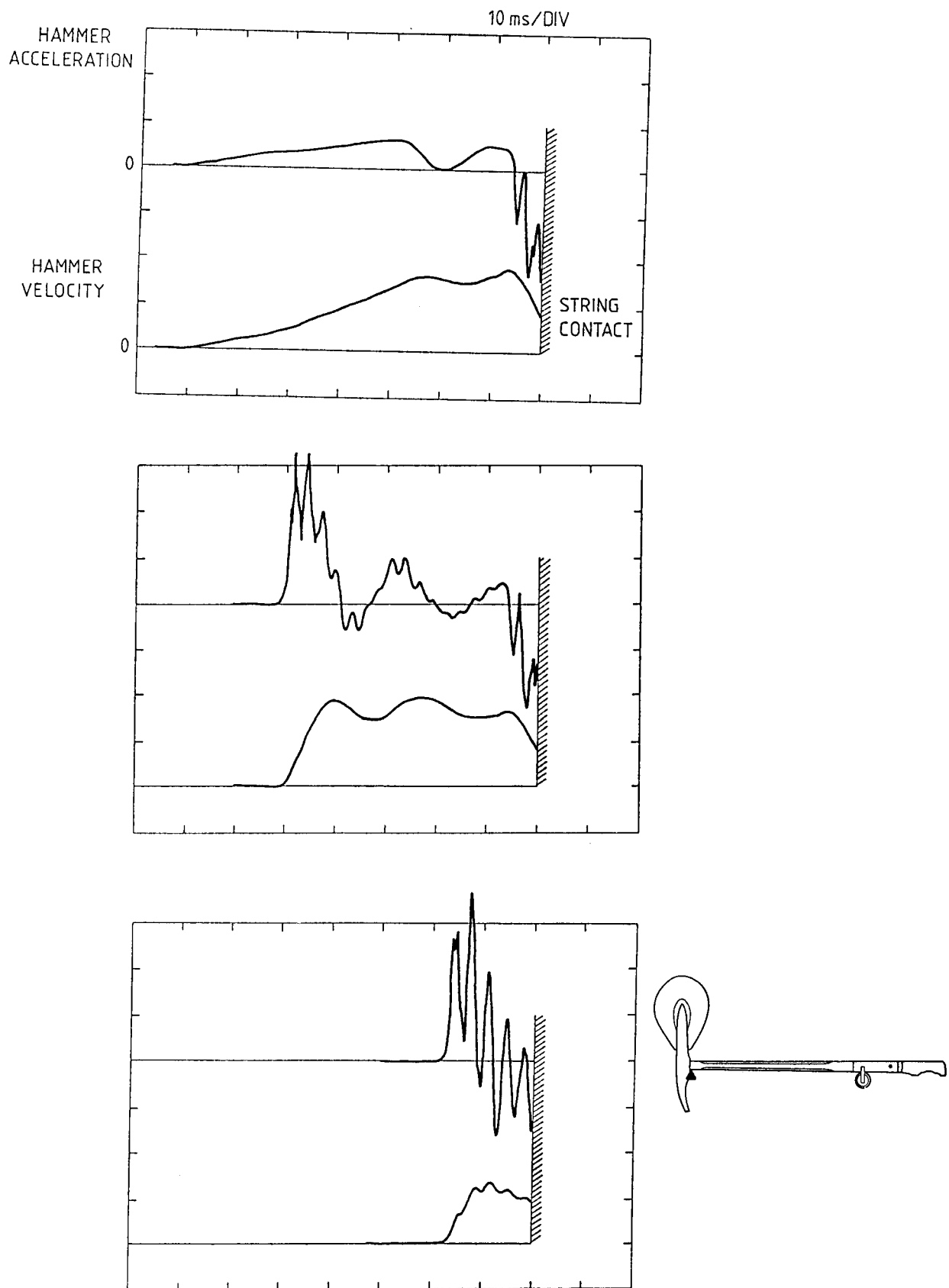


Fig. 15. Influence of different types of touch on hammer acceleration and velocity, mezzo forte, B_3 (arbitrary units). The time constant in integration was approximately 80 ms. The position of the accelerometer is marked with a triangle.

increased during more than half of the travel time, followed by a short relaxation and a final "let-go". The hammer velocity increased smoothly up to the relaxation, after which it remained approximately constant.

- (b) A touch performed by letting the arm fall while keeping the finger relaxed. A strong impulse was given at the beginning of the touch. The hammer velocity increased rapidly up to a high value which remained roughly constant until the string contact. A slow oscillation was superimposed on the hammer motion.
- (c) A fast touch in which the forearm and hand gave a rapid initial impulse, after which the finger was withdrawn ("surface character", "bite"). The entire touch lasted only 20 ms. The hammer was set in rapid, strong oscillations, resulting in a fast acceleration to maximum velocity. The rapid oscillation was superimposed on the velocity curve.

The pianist who performed the notes characterized the differences in touch as large, which also was verified by the measurements of the hammer motion (see Fig. 15). However, in an informal comparison by the experimenter, the perceived differences between the sounding notes were subtle. The small differences which seemed to be present may well be attributed to slight level differences between the notes.

2. Force transmission

The force transmission from key to hammer is interrupted at some point during the hammer motion due to the retreat of the jack at let-off. The timing pattern for the mechanical on/off contact between jack and roller has been discussed above (see Part I), but these measurements give little information on the history of the transmitted force. The moment when the accelerating force effectively ceases was estimated in an experiment, in which the acceleration of the hammer and the compression force in the jack were measured simultaneously. An example of a registration of these signals, together with contact signals in the action during a "legato-touch" with the finger initially resting on the key, is shown in Fig. 16a.

The figure shows that the hammer acceleration increased smoothly up to about 5 ms before the string contact, after which the acceleration rapidly dropped and changed into a retardation. A comparison with the contact signals shows that the acceleration started to drop as soon as the tail end of the jack reached the escapement dolly, and the jack started to retreat from the roller. This point could, consequently, be characterized as a "turning-off" point for the effective force transmission from key to hammer for this type of touch, although the jack

normally remains in mechanical contact with the roller up to about 1 ms before the string contact.

The compression force in the jack, measured by an inserted piece of piezo foil, showed low values during the acceleration part, probably due to a short-time constant compared to the slow changes in force. As the jack reached the escapement dolly, the force increased during a short interval, after which it suddenly dropped and was set in vibration. The initial small peak in force probably corresponded to the moment at which the edge of the jack slipped past the roller. It seems that this peak should not be interpreted as a "final kick" on the hammer roller, as the acceleration decreased continuously during this interval. Probably, the peak represented a temporary increase in pressure on a concentrated part of the foil, caused by the changing angle of the jack.

During a "staccato-touch", Fig. 16b, with the key being struck from some distance above, a definite "turning-off" point in force was harder to identify. The excitation was concentrated to a strong impulse in the beginning which set the hammer in slow oscillations, which were reflected in the compression force in the jack. A rapid oscillation was also observed, superimposed on the slow hammer oscillation. When the contact between jack and escapement dolly was reached, approximately 5 ms before the string contact, the slow oscillations decayed and were replaced by a fast retardation.

Interestingly, the details of the hammer acceleration during the interval between jack-dolly contact and hammer-string contact could be changed, depending on the touch and the regulation of the let-off distance. During this interval, the superimposed rapid oscillations seemed to be decisive for the hammer motion. Three such examples can be compared in Fig. 16b.

Attempts were also made to clarify the details in the force transmission by measuring the bending of the jack. Due to the friction in the sliding motion between jack and roller, a high contact force could possibly be reflected as an increase of the bending of the jack. However, no further information besides the results described above was obtained.

3. Hammer modes

a. General observations

Several of the previous registrations of the hammer motion (cf. Figs. 15 and 16b) suggest that eigenmodes of the hammer are excited during the acceleration before the string contact. A comparison of hammer motions during a "staccato-touch" for the same note on two different instruments showed that both hammers exhibited a slow oscilla-

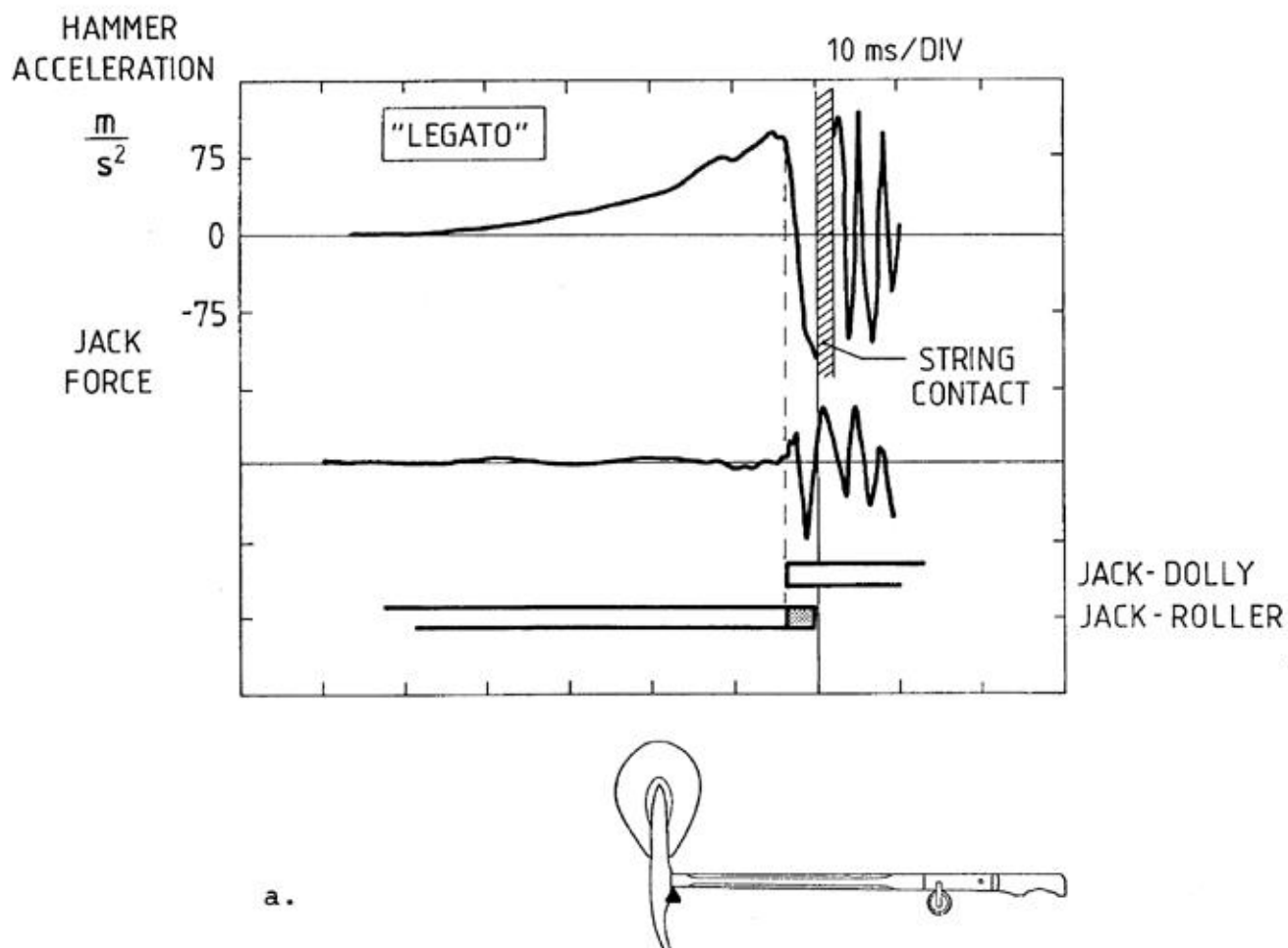
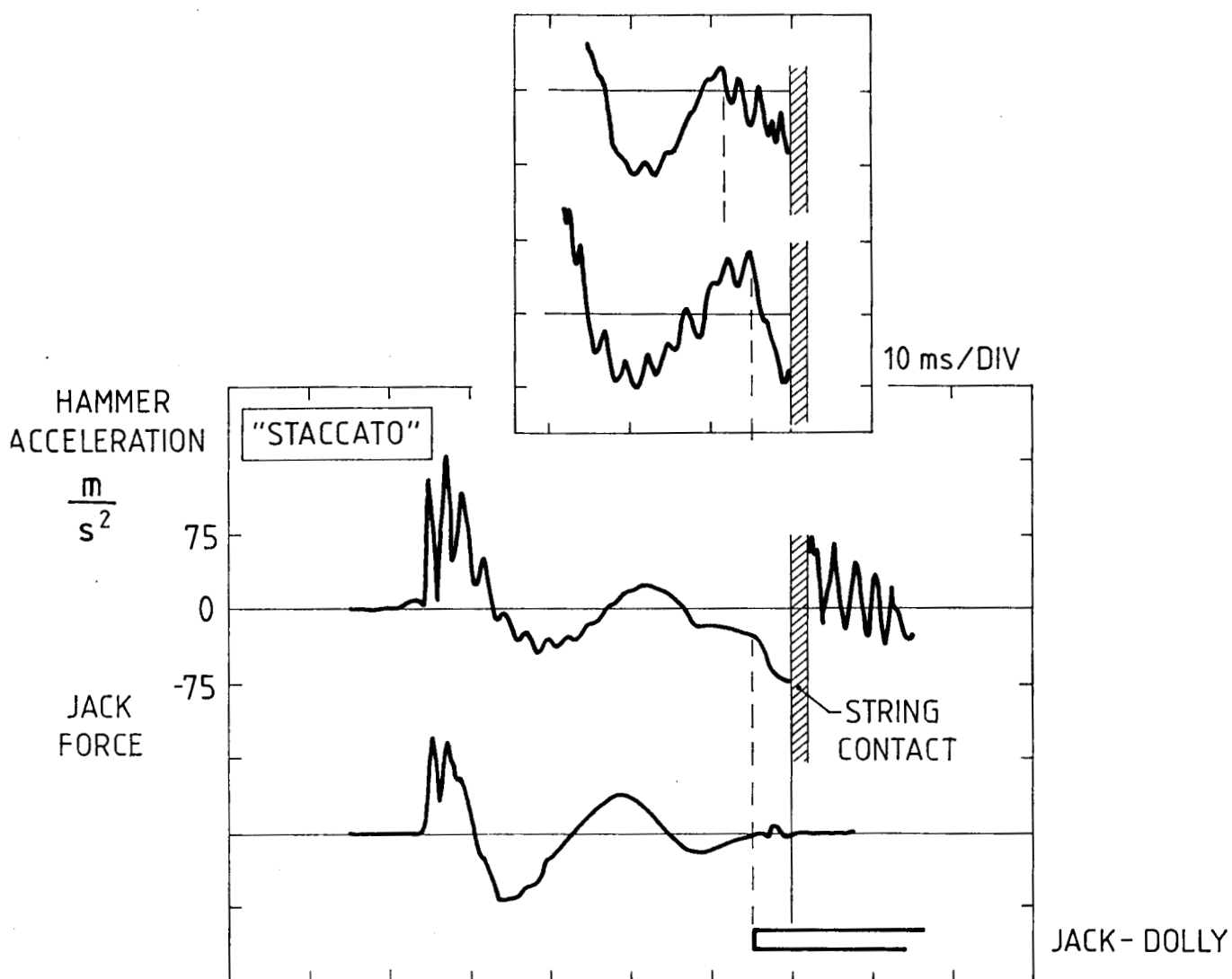


Fig. 16. Registrations of acceleration in the hammer head and compression force in the jack (arbitrary units), C_4 (Grotrian-Steinweg grand piano, Model 200 A 125, 52314, Braunschweig, 1926). The force was measured by means of a piezo foil in the jack. The position of the accelerometer is marked with a triangle.

a. "Legato-touch", forte.



b.

Fig. 16. b. "Staccato-touch", mezzo-forte. The upper two acceleration curves are examples obtained with slightly different types of touch. Also, for the curve at the top, the let-off was adjusted to a long condition (cf. Fig. 6). The dashed lines indicate the moment of contact between jack and escapement dolly.

tion at about 40-50 Hz, as well as a more rapid oscillation ("ripple") in the frequency range 300-400 Hz* (see Fig. 17.)

A few experiments were conducted on normal and prepared piano hammers, respectively, in order to study the cause of these oscillations (cf. Fig. 18).

One of the hammers was perturbed with the hammer in position in the piano. Adding a weight (5 g) to the hammer head (8.5 g) lowered the frequency of the slow vibration (-20%) without changing the frequency of the ripple. The rapid oscillations were strongly influenced by a small weight on the hammer shank at about one third of the distance from the hammer head to the roller. By using two perpendicularly mounted accelerometers on the hammer head (see Fig. 19), it was observed that the ripple mainly involved a horizontal oscillation of the hammer head in the string direction, whereas the slow oscillation occurred mainly in the vertical direction.

A very rigid hammer shank was prepared by gluing an aluminium strip in a groove made in the shank. The moving hammer mass was increased only 3% by this preparation. With this hammer, the ripple was no longer observed, while the slow oscillations remained unchanged.

In another experiment, one hammer was removed from the piano and the hammer flange was rigidly clamped in a vice. The hammer roller was supported on a rigid surface, while a finger rested lightly on top of the base of the shank. This arrangement was used as a rough approximation of the conditions during the hammer acceleration. In this experiment, an oscillation frequency of approximately 50 Hz was observed, both for the normal hammer and the hammer with the rigid shank. When the roller was removed, and the hammer instead was supported on a hard wedge, the same oscillating frequency was observed, but the oscillations were much more pronounced. When the hammer shank was rigidly clamped behind the roller, the oscillating frequency rose to approximately 70 Hz.

This series of observations suggest that the slow oscillations resembled a vibration of the hammer shank as a rigid bar with a mass load at the free end, while the rapid oscillations ("ripple") would correspond to a higher mode bending of the shank. The flexing in the low-frequency mode occurred mainly at the junction between the slim part of the shank and the stiff part at the base. This part is also weakened by the groove in which the wooden roller core is glued. In several cases it was observed that the glue joint was cracked on the side towards the hammer head.

* A similar motion has been observed in an upright piano (Boutillon, 1988).

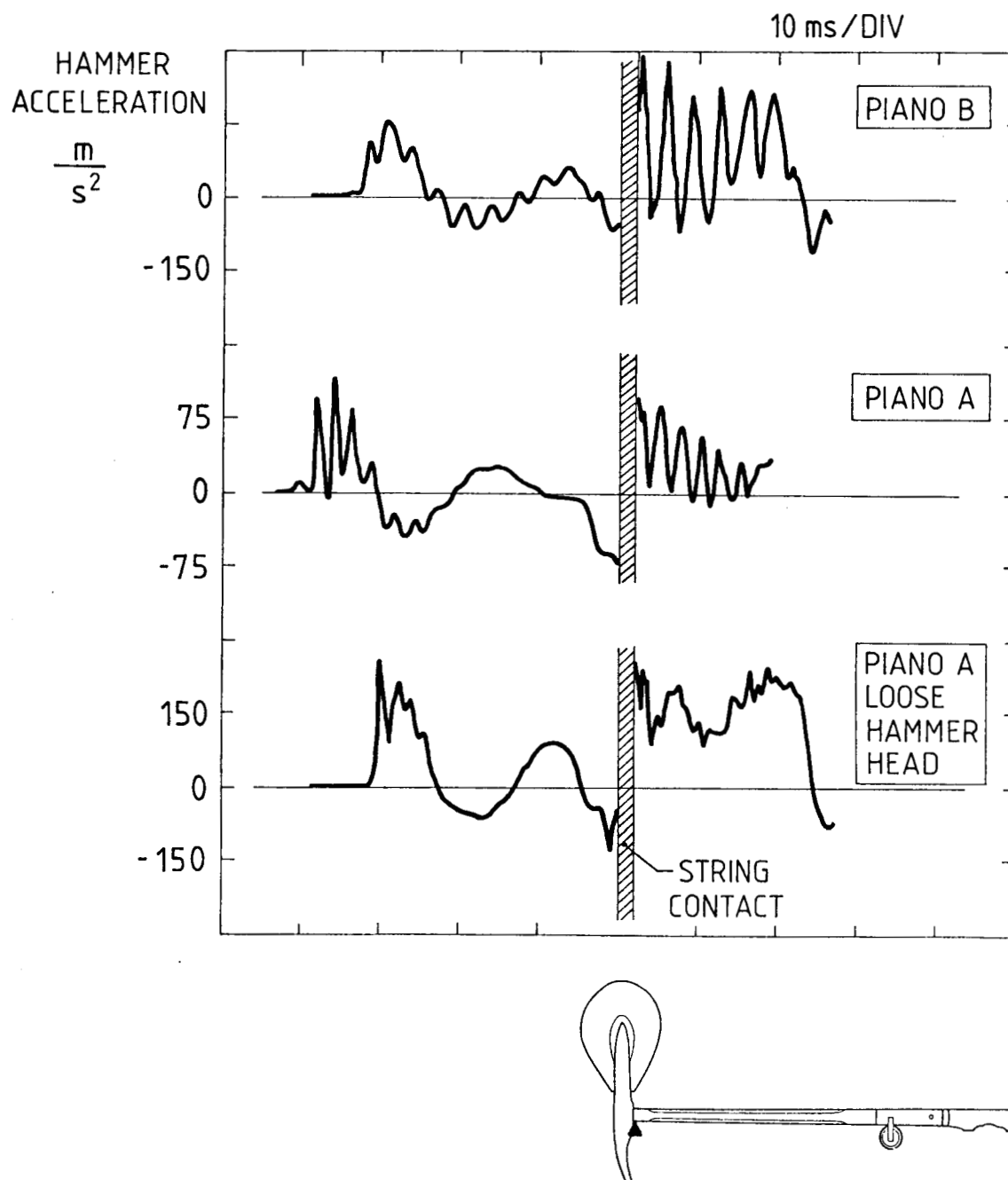


Fig. 17. Comparison of the acceleration at the hammer head for two grand pianos (staccato, forte) (Steinway & Sons, Model B 443001, Hamburg, 1975 and Grotrian-Steinweg grand piano, Model 200 A 125, 52314, Braunschweig, 1926). The position of the accelerometer is marked with a triangle. The curve at the bottom corresponds to a hammer with a slightly loose hammer head.

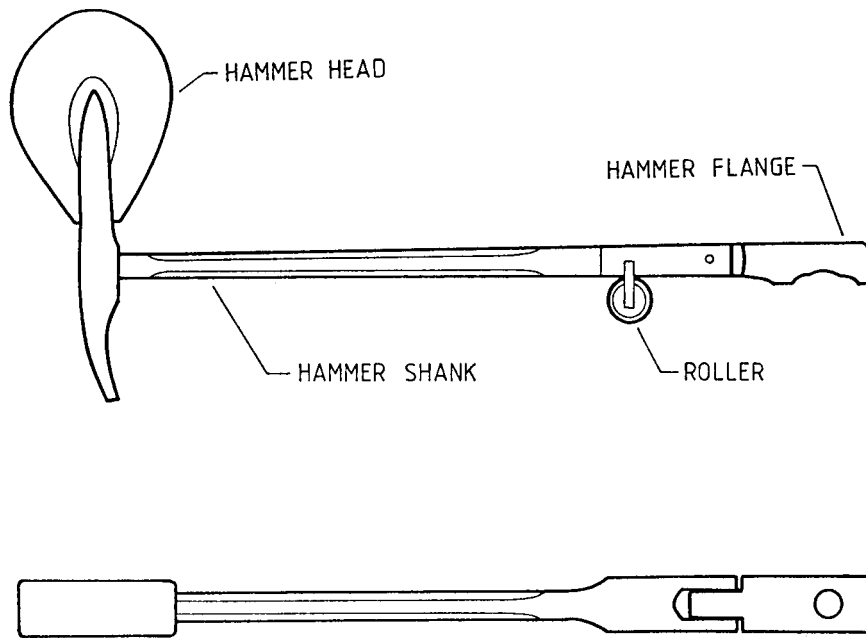


Fig. 18. Side and top views of a piano hammer, C_4 .

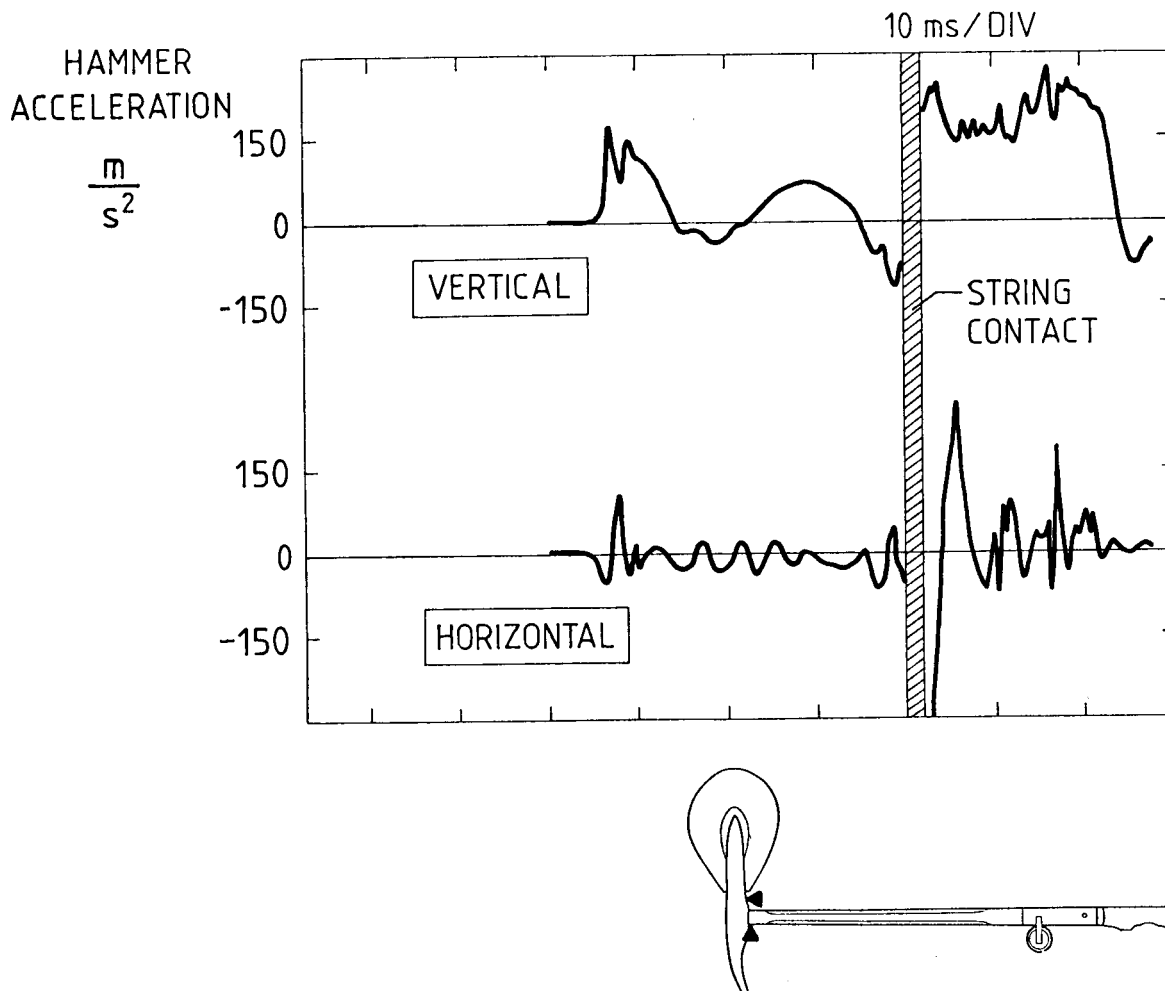


Fig. 19. Acceleration in the hammer head measured in the vertical and horizontal directions ("staccato", mf C_4) (Grotrian-Steinweg grand piano, Model 200 A 125, Braunschweig). The positions of the accelerometers are marked with triangles.

The two oscillating components were always observed in the motion of the hammer head for touches with a sudden attack. If the finger was relaxed, both components were also observed in the key motion, in particular the slow oscillation.

b. "Ripple" mode

The boundary conditions for the bending of the shank were a little complicated as the shank could rotate around the axis in the hammer flange, but it was disturbed in its downward motion at the roller by the jack. However, it could be assumed that a major part of the flexing occurred in the long, slim part of the shank between the roller and the hammer head with a roughly circular cross section (diameter 6 mm). The boundary conditions for this mode could thus be considered as approximately clamped at the base of the shank and supported at the outer end due to the concentrated mass of the hammer head. The predicted frequency for the lowest mode with these boundary conditions is approximately 380 Hz, which gives support to the above assumptions.

The low-frequency oscillations of the hammers in Fig. 17 were rather close in frequency, whereas the frequencies of the "ripple"-modes differed by about 20%. The hammer with the lower frequency of the "ripple"-mode belonged to the piano that was rated as far superior of the two. Interestingly, the ripple frequency of this hammer (piano B, 310 Hz) was closer to the fundamental of the string (C_4 , 262 Hz) than the ripple frequency for the other hammer (piano A, 380 Hz). Although more strongly excited at the beginning of the acceleration, the ripple decayed before string contact in this latter instrument.

Traditionally, the piano maker sorts the hammer shanks according to their tap tones before gluing the hammer heads to the shanks. Shanks with high tap tones are used in the treble, while shanks with low tap tones are used in the bass. Shanks with very low tap tones are discarded. According to skilled piano technicians, the quality of especially the treble notes is significantly changed depending on the properties of the hammer shank (personal communication, Norén & Carlsson).

This rule of sorting the hammer shanks was verified for a few sampled hammers of one of the pianos. A bass hammer (C_2 , piano B) exhibited a slightly lower "ripple"-frequency (-15%) compared to the C_4 -hammer, while a treble hammer (C_8 , piano B) showed a much higher ripple frequency (+80%).

The influence of a loose hammer head is also illustrated in Fig. 17. Such a defect is recognized among piano technicians as a notorious cause of poor tone quality. A comparison with the correctly glued hammer head shows that the "ripple" did not develop, either before or after string contact, if the hammer head was not rigidly joined with the shank.

c. Modal analysis

The properties of a piano hammer were further studied by modal analysis. The hammer flange was clamped in a vice and the shank was held in horizontal position by a rigid support under the roller, in accordance with the experiments described above. As mentioned, these arrangements could be assumed to correspond rather closely to the conditions during the upward acceleration of the hammer before release of the jack.

The shank was driven by an electrodynamic system, with a small magnet fastened to the shank and a coil in a fixed support. The vibrations at different points on the hammer were measured by an accelerometer. The hammer felt was removed and replaced by a piece of copper plate with the same mass as the felt (4.5 g) in order to make it possible to fasten the accelerometer on the upper part of the hammer head. The plate was folded and glued to the wooden hammer core at a position which gave the same point of gravity as with the felt.

It turned out to be difficult to obtain clear modes in the analysis, probably due to low Q-factors and the loading of the hammer shank by the accelerometer. However, the analysis showed modes at the following approximate frequencies (see Fig. 20):

400 Hz	bending of shank and hammer head
1000 Hz	bending of shank, hammer head not moving
1600 Hz	vibration of the check tail

The modal analysis supported the results of the earlier experiments with the hammer. The mode at 1000 Hz corresponds rather closely in frequency to a predicted second mode with clamped-supported boundary conditions. The low-frequency mode at approximately 50 Hz discussed previously was not included in the frequency range of the modal analysis. The bouncing frequency of the hammer, when it was dropped on the roller against a rigid support, was much lower than the lowest mode frequency, which indicates that the hammer was measured under approximately "free" conditions.

d. Influence of touch

An influence of the hammer modes on the string excitation can not be excluded. As illustrated in the registrations, hammer modes could be excited during the acceleration for certain types of touch, and always at the string contact. A typical example is given in Fig. 16b, where the slow oscillation can be observed during the entire acceleration, along with the high-frequency component. The registrations in Figs. 15 and 16b also suggest that the specific moment of hammer-string contact relative to the cycles of hammer oscillations could be influenced by touch.

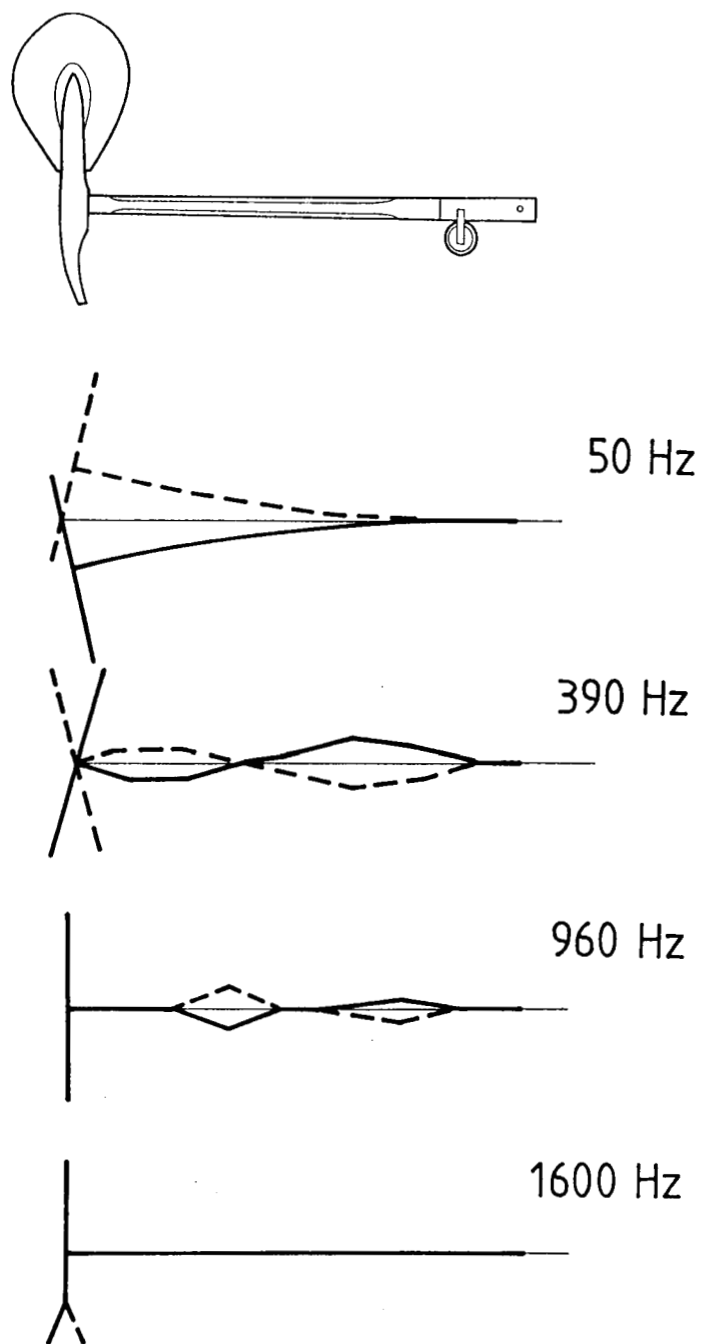


Fig. 20. Approximate hammer mode shapes and frequencies. The lowest mode was not included in the modal analysis, but inferred from perturbation experiments.

According to a theoretical modelling (Suzuki, 1937b), the time history of the contact force between the hammer-string changes significantly if an initial deformation of the hammer shank is introduced. This may imply a possibility for the pianist to influence the excitation of the string through touch, even though the mechanical contact between the key and the hammer is broken before string contact. Further, the mode frequencies, except the lowest, are high enough to allow one or several vibration periods during the string contact, which also may have an influence on the hammer-string interaction.

Interestingly, the manufacturer of the instrument used in the experiments emphasizes that the voicing of the hammer (setting the stiffness of the hammer felt by needling), should be made such that a hard, drop-shaped core is preserved in the upper, striking part of the hammer, surrounded by sectors with less resiliency (Dietz, 1968b). Possibly, this empirically motivated method of optimizing the hammer properties may have some connection with the oscillations in the hammer at the string contact. It is more plausible, however, that the recommended method of voicing has more relevance for obtaining the desired nonlinear characteristic of the hammer compliance (discussed in Part III).

Although it may seem that the oscillations of the hammer offer a possibility for the player to influence the string vibrations, and, hence, the tone quality by "touch", the question is by no means yet answered. In a classical study (Hart & al., 1934), it was shown that the sound pressure waveform of notes played by a concert pianist could not be distinguished from the waveform produced by a touch with a mechanical pendulum, provided that the final hammer velocities were identical. Unfortunately, the hammer shank was fitted with a rather large piece of stiff cardboard "firmly attached to the hammer shank", which may have eliminated any influence from the hammer shank oscillations.

Probably, the significance of "touch" involves several factors, of which the initial "thump" in the key bed and the iron frame could be assumed to be very important (Reinholdt, Jansson, & Askenfelt, 1987; Conklin, 1987). A recognized piano manufacturer pays great attention to the resonances in the key bed, which undoubtedly are excited differently depending on the motion of the key.

The expression "touch" is probably also often used as a general term for several qualities in playing besides of characterizing a single note at a given dynamic level. Other qualities probably covered by the term "touch" are the timing relation between a melody line and accompaniment, the overlap of notes, and the relative strength and timing between the notes in a chord, factors which have been shown to separate the skilled pianist from the amateur (Vernon, 1937; Palmer, 1937).

E. Conclusions

The experiments showed that the position and the velocity of the key and the hammer can be successfully registered during their entire travels by an optical method, by the use of simple components. The motions changed significantly depending on the dynamic level. Typical final key velocities ranged from 0.1 m/s at piano to 0.6 m/s at forte. The observed final hammer velocities ranged typically between 1 and 5 m/s.

The accelerating force on the hammer was found to be interrupted at the moment when the tail of the jack reached the escapement dolly and its top started to retreat from the roller. This moment could be considered as the effective point of release of the hammer.

The key motion showed typical differences between two principally different types of touch. A "legato-touch", with the finger initially resting on the key, gave a smooth key motion with continuously increasing key velocity. A "staccato-touch", with the finger striking the key from above, resulted in an oscillating key motion at a low frequency, approximately 50 Hz. This oscillation was assumed to be supported by a flexing mode of the hammer shank.

The hammer exhibited several resonances. In particular, a low-frequency mode at approximately 50 Hz, in which the hammer was flexing like a bar with a mass load at the free end, and a second mode at approximately 400 Hz, involving a higher mode bending of the shank, were observed. These modes were found to be excited by certain types of touch during the acceleration of the hammer before the string contact. Also the details of the motion of the hammer immediately before the string contact were observed to be influenced by touch. It cannot be excluded that the oscillations of the hammer may have an influence on the string excitation by affecting the time history of the interacting force during the string contact.

PART III. STRING VIBRATIONS

A. Pulse propagation on a string

The observed motion of a piano string depends on the excitation function, the point of excitation, and the point of observation, cf. Fig. 21. Therefore, this section is started with a theoretical overview of pulse propagation on strings. An understanding of pulse propagation is necessary to be able to interpret the measurement results, i.e., to be able to separate the excitation function from the reflected pulses and from the influence of excitation and observation points. In the following, the fundamentals of pulse propagation is discussed for an ideal string, stretched between two supports (bridge and agraffe) and excited by a hammer during a finite contact duration.

An ideal string, struck at a point (the excitation point), is displaced to a constant value bracketed by two slopes, $y_1(ct-x)$ moving in the $+x$ -direction and $y_2(ct+x)$ in the $-x$ -direction (cf. Kinsler, Frey, Coppens, & Sanders, 1982; cf. Morse, 1948). The slopes are spreading apart with the same propagation velocity c and undisturbed waveforms. The two slopes are mirror-symmetric (in the y -axis) and thus $y_2(ct+x)=y_1(ct+x)$ and, shortly after the blow, the transversal string displacement y in position x at time t can be written as:

$$y(x,t) = y_1(ct-x) + y_2(ct+x) = y_1(ct-x) + y_1(ct+x) \quad (\text{Eq. 1})$$

The transversal string velocity equals the time derivative of the displacement, i.e.,

$$dy/dt = d(y_1(ct-x) + y_2(ct+x))/dt = v_1(ct-x) + v_1(ct+x) \quad (\text{Eq. 2})$$

It consists of the two velocity pulses $v_1(ct-x)$ and $v_1(ct+x)$ spreading apart from the excitation point with the propagation velocity c , cf. Fig. 21.

After some delay, $\Delta t = x_B/c$, the pulse $v_1(ct-x)$ passes the observation point (B), and a detector can record the undisturbed waveform.

The pulse propagates to the support (the bridge), where it is reflected. At a perfectly rigid support there is no motion (vibration displacement or velocity) and, thus, the sum of incoming and reflected pulses must be zero. This means that the reflected pulse must be a perfect copy of the incoming wave but with change of sign, i.e., $-v_1(ct+x)$. The reflected pulse propagates in the $-x$ -direction, passes the observation point again with a further delay, $\Delta t = 2(L-x_B)/c$. At the second support (the agraffe), the pulse is reflected once more. If perfectly rigid, as the first support, the reflected pulse is inverted once more but otherwise undisturbed, i.e., the original pulse $v_1(ct-x)$

is obtained. The pulse propagates past the observation point, at which the original undisturbed pulse is detected a second time. The time between the repetitions, the period time, is $T=2L/c$.

The initial velocity pulse $v_1(ct+x)$ propagates in the $-x$ -direction and is reflected in a similar manner at the agraffe, the bridge, and so on. When the two pulses meet, they are superimposed and form a complex waveform. The timetable for pulses passing the observation point is given by the propagation distance divided by the propagation velocity.

The impedance of the piano bridge is high compared to the characteristic impedance of a string (Wogram, 1981), and the bridge can be regarded as rigid. For the rigid bridge, the small string vibrations give a dynamical force F_{bridge} acting on the bridge:

$$F_{\text{bridge}} = -T \frac{\partial^2 y_1}{\partial x^2} = 2(T/c) \frac{\partial y_1}{\partial t} = 2(T/c) v_1(ct-x) \quad (\text{Eq. 3})$$

where T denotes the string tension. Eq. 3 is valid in general ($y_1(ct-x)$ can represent any wave shape). Thus, it is shown that the dynamical force exerted by the string on the bridge equals $2T/c$ (twice the characteristic impedance) multiplied by the velocity pulse arriving to the bridge. As a measurement the velocity at a point of the string gives direct information on the dynamic forces acting on the bridge, an informative parameter for the radiated sound.

The hammer-string contact duration and contact length are not infinitely short. Therefore, it should in principle be expected that when half a period time or half a wavelength of the string partials are shorter than the contact duration and contact length, respectively, then this partial and higher are weakly excited (Hall, 1987a; Benade, 1976).

If the contact duration exceeds the round trip time from the hammer to the agraffe, the returning pulse will see the hammer as a nonrigid support. The incoming pulse is partly propagating past the hammer, partly transmitted to the hammer, and partly reflected. The amplitudes and phases between the four pulses are set by the properties of the hammer and the string. The incoming pulse will try to throw the hammer off the string. The reflected pulse returns after a new round trip and the procedure is repeated, as suggested by Benade (1976).

The vibrations of a string can be represented by a combination of its modes. At a node, a string mode can neither be driven nor observed, i.e., we should expect the corresponding partials to be weak or non-existent. The partial number N of the nonexcited string resonance, and the corresponding partial frequency f_N can easily be calculated from the nodal position by Eq. 4:

$$N = (l/a) \quad \text{and} \quad f_N = (1/a)(c/2L), \quad (\text{Eq. 4})$$

where $a=(l_s/L)$, l_s is the distance from support to excitation and obser-

vation points, respectively, and L is the total string length. The same conditions predict that certain resonances cannot be observed at a specific observation point. Note, that the resonances have more than one node where excitation and observation are not possible. The envelope of the partial amplitude A_n for the ideal string varies as:

$$A_k = A_1 (\sin n\pi l)/L \quad (\text{Eq. 5})$$

along the string, where n is the partial number. Eq. 5 can be used to correct the spectrum for the influence of the observation point. The ideal string struck with a Dirac-pulse obtains a velocity spectrum of partials with constant amplitudes (neglecting the influence by excitation position and observation position).

In conclusion, the following can be said. The timetable for pulses passing the observation point is given by the propagation distance divided by the propagation velocity. To study an excitation pulse as undisturbed as possible, i.e., with no interference by reflected pulses, the excitation and the observation points should be as far away as possible from the end supports. In the piano, the excitation point is fixed (approximately $L/8$ from the agraffe). This distance sets the minimum observation time for the excitation waveform with no disturbance of agraffe pulses. The transmission past the hammer should be possible to see at an observation point close to the hammer at the bridge side. The hammer-string contact parameters should be best observed close to the hammer on the agraffe side. The hammer-string contact duration and contact length should set the upper limit for strongly excited partials. The excitation force of the bridge should be best estimated by recording the velocity of the motion at an observation point between the hammer and the bridge. The observed spectra depend on excitation and observation points. A "correct" spectrum with no influence of the observation point can be obtained by means of the inverse of Eq. 5.

B. Scope of measurements

The measurements aimed at registrations of typical initial string vibrations and their relations to the excitation and observation points. The measured string vibrations were compared with the timetable of events to see that the vibrations are understood. Furthermore, the measurements aimed at registering the influence on string vibrations and string spectra by differences in playing, in different hammer properties and hammer adjustments (voicing). Finally, interaction between hammer and string, and between damper and string should be studied.

C. Measurement method

The string motion was detected by applying a concentrated magnetic field (two Samarium-Cobalt magnets, Recoma, dia. 7 mm, giving a field

0.1 Vs/m²) at a specific observation position along the string. The string motion in the magnetic field induced a voltage over the string proportional to the string velocity at the position of the magnet. High amplification was needed, since the induced voltage was much reduced (peak values of microvolts) by the almost perfect short-circuiting of the iron frame in the piano. The string displacement was obtained by integrating the velocity signal in an RC-network (time constant 4 ms, corresponding to a low frequency limit of 40 Hz). For the notes fitted with string triplets, the middle string was used for the measurements and the two outer strings were left free to vibrate. Experiments showed no influence from the vibrations of the outer strings on the velocity signal of the middle string.

Comparison of vibration detection with the optical system earlier described, Part II, showed minor differences to the magnetic method. The magnetic detection was simpler to use and was therefore used in the experiments. However, the width of the magnetic field caused an averaging of the velocity over the string segment covered by the field. This averaging should give a minimum in the string spectrum for high partials with a wavelength equal to the field width.

For the measurements of string waveforms and spectra, an FFT-analyzer was used (HP 3562 Dynamic Signal Analyser). Spectra were plotted for averages of 0-80 ms and 35-115 ms. The spectra showed minor differences at highest partials. Therefore, it was concluded that the spectrum envelope as function of time is slowly changing and that spectra of the first 80 ms are representative and insensitive to unavoidable small time shifts in the analysis.

D. Measurements and results

1. Typical string waveforms

Typical string waveforms for a note in the middle register (C4) are shown in Fig. 22. The displacement waveform as measured at the bridge side, shows a positive initial slope, a broad maximum, a negative slope, a constant level, a negative slope, and a minimum; thereafter, a minimum, a maximum, a minimum, and a constant level follow. The velocity waveform shows a steep initial positive slope, a maximum followed by a less steep negative slope, a broad minimum, and a constant zero level.

The displacement waveform measured on the agraffe side shows in large the same course as that measured on the bridge side. Only the positive displacement pulse looks quite different. It contains several minor wiggles. The velocity waveform shows initially several peaks and dips followed by a constant zero level; thereafter a sharp minimum, and a broad maximum with wiggles, and a constant zero level.

The hammer-string contact duration extends from the start of the initial slope to the first almost constant zero level for all four

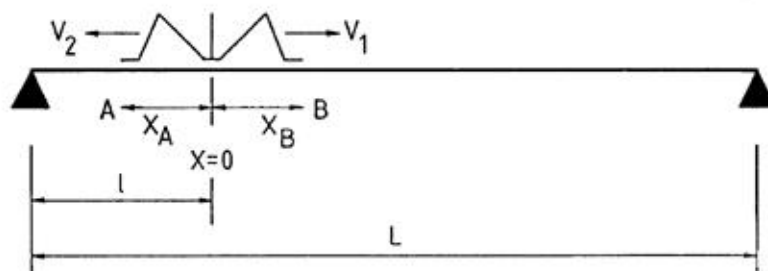


Fig. 21. The piano string with supports (bridge and agraffe), excitation point (hammer at $x=0$), observation point on the agraffe side (A), observation point on the bridge side (B), and velocity pulses (v_1 and v_2) shortly after the blow.

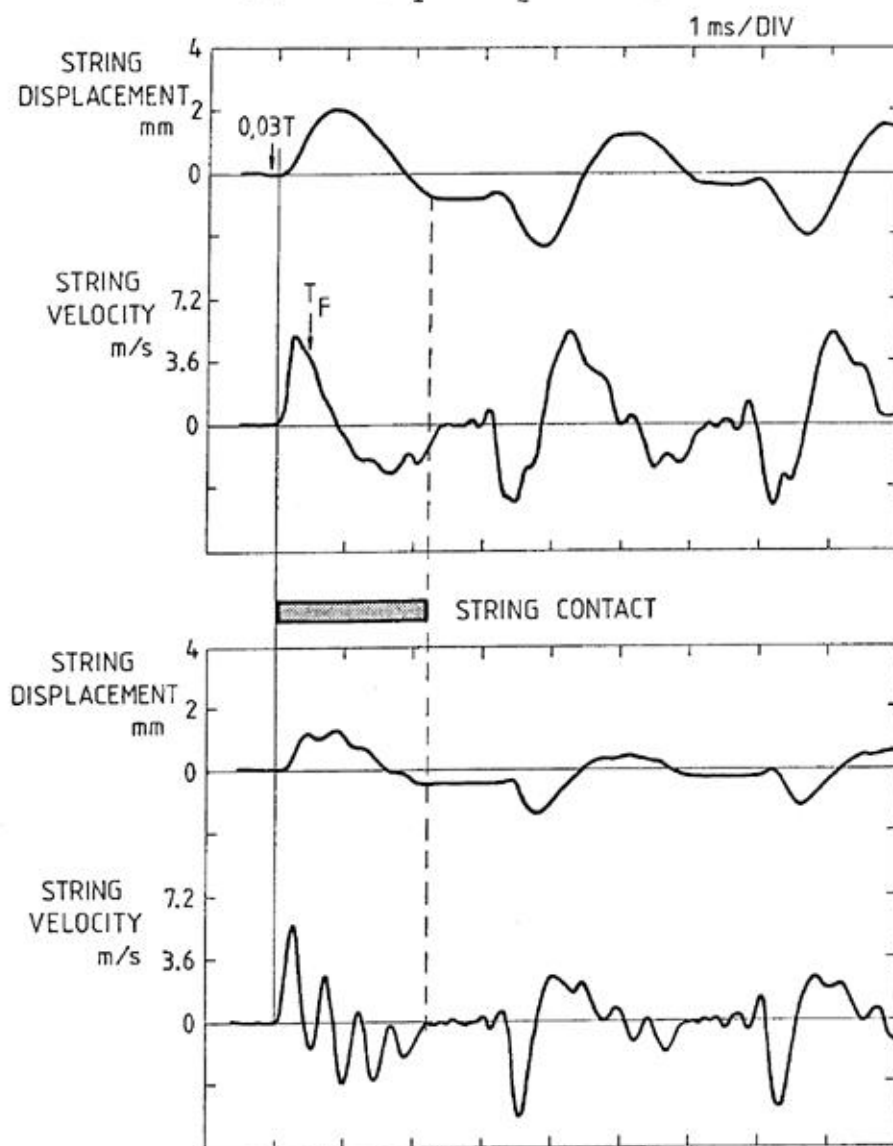


Fig. 22. String displacements and velocities (hammer-string contact durations are included for reference) on the bridge and agraffe sides (upper and lower panels, respectively) of the hammer for note C4. T_F marks end of undisturbed initial pulse. String length 664 mm, agraffe-hammer distance 82 mm and observation points 40 mm from the hammer (Grotrian Steinweg 200 A 125).

curves presented. After the contact has ceased, the waveforms repeat themselves periodically. The hammer-string contact is considerably longer than the round trip time hammer-agraffe.

The string displacement amplitude of C4 is typically 0.2, 0.8, and 2 mm at p, mf, and f, respectively. The corresponding velocities are 0.3, 1.5 and 5 m/s.

a. Detailed analysis of the string waveforms

The analysis in the preceding paragraph indicated that the velocity curves are the most suitable for inspection of the timing of the events, as the events are most clearly shown here. By comparing the events with the theoretical timetable, it is controlled that the events are correctly understood and the contained information is pointed out.

The period time is easily read in velocity waveforms as the distance between the second and third maxima or the preceding zero crossings. This gives an accurate calibration of the diagram in period time T (thus measured T predicts the measured fundamental frequency within 1%).

The distance from hammer to either of the observation points (40 mm) gives a delay relative to the hammer-string contact of $40 \cdot T / (2 \cdot 664) = 0.030T$ (0.03T marked in the Fig. 22).

During the hammer-string, contact the string is temporarily divided into two parts, one short (hammer-agraffe) and one long (hammer-bridge). Looking at the curve recorded at the agraffe side, the following can be observed. The first pulse passing the observation point is reflected at the agraffe and should pass the observation point once more at $2(82-40)T / (2 \cdot 664) = 0.063T$, which is close to the maximum at $0.07T$ in the figure. Thereafter, a pulse train follows with 3.5 clear periods and an increase to the zero level. The frequency of the 3.5 periods is $8.1/T$, which agrees with the predicted $2 \cdot 664 / (2 \cdot 82 \cdot T) = 8.10/T$. The average level during the pulse train is not constant. The negative peaks gave four pushes to the hammer during the contact time. The sinusoidal vibrations are heavily damped after the hammer-string contact ceases. There is a leakage of the agraffe-hammer pulses in the end of the contact time. In the following periods the pulses spaced $T/8.1$ apart can also be found.

Looking at the recorded velocity curve at the bridge side the following is found. The hammer may not act as a rigid support. Therefore, the first pulse from the hammer reflected at the agraffe and partly passing the hammer will be superimposed on the first pulse coming directly from the hammer. The time delay between the two pulses limits the guaranteed, undisturbed duration of the excitation pulse. This duration T_p equals $2 \cdot 82 \cdot T / 664 = 0.123T$ and has been plotted in Fig. 22. The "undisturbed" velocity waveform on the bridge side shows a steep uphill slope and a marked maximum followed by a fairly steep downhill

slope. Thereafter, traces of superimposed "agraffe pulses" can be found. There are tendencies to dents at the first and second possible superpositions, while the third and fourth ones are clearly seen as maxima. After another minimum, the curve increases to the constant zero-level. The first maximum is at $0.06T$, i.e., at approximately the same time as the maximum observed at the agraffe side, and it is of approximately the same amplitude. The negative slope follows approximately the average level of the pulse train.

The theoretical timetable predicts that the first pulse reflected at the bridge should return to the observation point at the bridge side after $T \cdot (2.664 - 2.122) / (2.664) = 0.816T$. The duration from the start of the first positive pulse to the start of the negative pulse after the zero level is closely the same ($0.81T$) and, thus, it can be concluded that the position of the observation point can be read off the recorded waveform. The start of the positive pulse (the zero crossing) is thereafter delayed a period time T compared to the first pulse.

The shift in zero level for the train of velocity pulses on the agraffe side can be explained by the velocity of the hammer. As the velocity of the hammer is in the same direction as the incoming pulse, the velocity of the reflected pulse will be reduced.

The first displacement pulse (both for the bridge and the agraffe side recordings) passes through a maximum and reaches during the contact time a negative value and thereafter an approximately constant negative level is kept. The curves follow then the same course as previously described. The negative level can partly be explained by a too short time constant of integration, but the same negative level was also obtained with capacitive and optical measurements implying that the negative level is real. The negative level is possible to obtain if the hammer pushes the string in addition to the blow (Benade makes comparisons with "bow and arrow") (Benade, 1976).

2. String contact duration

The fast repetitions on the short string part during the hammer-string contact suggest an indirect way of measuring the contact duration. A comparison with the directly measured contact duration (by means of a thin wire on top of the hammer head, cf. Part I) and the velocity waveform on the agraffe side of the hammer shows that the fast train of the velocity pulses ends at the end of the contact time, i.e., where the velocity curve flattens out (error < 0.1 ms). This can clearly be demonstrated by applying a DC-voltage to the wire of the hammer, which cuts out the pulse train during the hammer-string contact.

It can be concluded that the contact time can be determined within $T/16$ (half a period of the fast pulse train) by looking at the fast pulse train end at the agraffe side close to the hammer. The method is restricted to dynamic levels above mf.

3. Initial pulse and final hammer velocity

Earlier, it was shown that the height of the initial velocity pulse is not disturbed by interference of other pulses. It can be expected that the height of the initial velocity pulse should be proportional to the final hammer velocity (Hall, 1987b). Therefore, the final hammer velocity (cf. Part II) and the initial pulse heights were measured for different dynamic levels, which showed a close proportionality between the "optical" and "electrodynamic" measures and the initial pulse height, see Fig. 23. The amplitude of the initial velocity pulse can thus be used as an estimate of the final hammer velocity. This method was easier to use than the optical and electrodynamic methods. The optical method turned out to be a little awkward, needing a troublesome fitting of a stick to the hammer (working with the hammer in position under the strings), repeated calibration, and dimmed light. The electrodynamic method demands preparation of the hammer.

4. String waveform and spectra

a. Different strings with original hammers

The string waveforms and the corresponding spectra exhibit characteristic differences over the compass of the piano, due to differences in the excitation. Factors influencing the spectrum envelope are the ratio hammer to string mass, the hammer compliance, the string stiffness, the hammer-string contact duration, and the hammer width (Hall & Askenfelt, 1988). Typical examples of the waveforms and spectra in different ranges (C2, C4, and C7) are illustrated in Fig. 24.

In the bass (C2), the hammer-string contact duration is short compared to half a fundamental period ($T/5$ in the Fig. 24, cf. Part I). In the middle register (C4), it is approximately half a period and in the treble range (C7), it may be several periods. As a consequence, this gives a waveform in the bass, in which the individual outgoing and reflected pulses can be clearly seen. In the middle register, the pulses could still be identified, whereas in the treble, the waveform resembles of a standing wave.

The C4-note shows the waveform previously discussed. The waveform of the bass note (C2) displays a first positive pulse, which includes most of the full contact time. It is followed by a broad negative pulse, a zero level, and the reflected pulses. In the negative slope and in the negative pulse, there are two local maxima. The reflected pulse from the bridge is preceded by a high-frequency component with increasing amplitude. This is the result of dispersion, i.e., the higher frequencies propagate with the higher velocity caused by the stiffness of the string (Podlesack & Lee, 1988). The measured waveforms showed that the observation point was $1/3$ of the string length from the end of the string. Measured agraffe-hammer distance was $1/8.6$ of the string length.

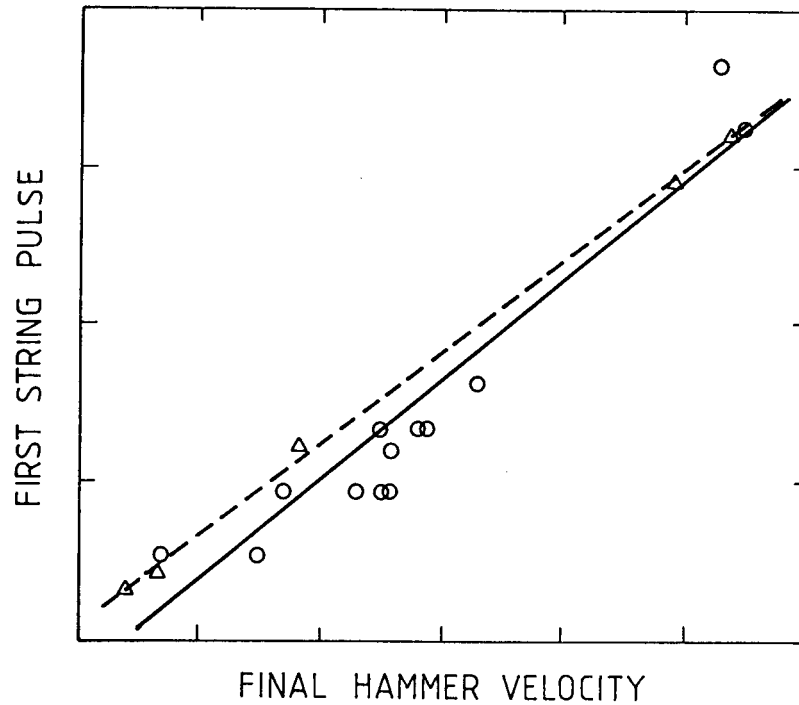


Fig. 23. Final hammer velocities as measured by the optical method (circles and full line adapted by the least mean square method), and the electrodynamic (triangles and broken line) method vs first string pulse amplitude (arbitrary units).

For a treble tone C7 (2093 Hz), a quite different waveform is found. The waveform is sinusoidal and the zero crossings are at approximately similar distances. The first pulse is the widest (more than half a period, cf. Part I) and with the lowest amplitude. The long hammer-string contact-time means that the string vibrations are initially damped by the hammer. After the first crest follows higher and equally high pulses. The distance from hammer to capo dastro bar is small.

The spectra show slowly descending envelopes with regularly spaced minima reflecting the points of excitation (the hammer) and observation (the magnet). For C4, the first minimum at the 4th partial corresponds to the observation point at $1/4$ string length from the agraffe (estimated from the velocity wave give 0.24). The second minimum at the 8th to 9th partial corresponds to the hammer-agraffe distance ($635/80=7.9$) and to that of the observation point. For higher frequencies, minima are found at multiples of four times the fundamental. The maxima form an envelope dropping approximately 40 dB to 6 kHz (at the 23rd partial).

The spectrum of C2 gives an envelope decreasing 50 dB to 5 kHz (approximately the 80th partial). A minimum is found at every 3rd partial, which corresponds to the distance from agraffe to observation point. In addition, the agraffe-hammer distance ($L/8.6$) corresponds to minima every 9th partial.

For C7, the spectrum envelope has decreased approximately 35 dB at 9 kHz (the 4th partial). Only this of the three notes has the fundamental as the strongest partial. The weak 3rd partial implies that the observation point was at a third of the string length.

The envelope decreases (neglecting the zeros from the excitation and observation points) with about 15 dB for the first octave of C7. For C4, there is a 15 dB decrease for the first three octaves and for C2, the 15 dB decrease is found between four and five octaves. The decreases indicate substantial influence from another excitation mechanism than that of the ideally struck string (cf. Hall & Askenfelt, 1988).

As mentioned earlier, the width of the magnetic field predicts a minimum in the observed spectra. However, this minimum occurs at higher frequencies (for C2, C4, and C7 at 10.7, 24.5, and 28.5 kHz, respectively) than the upper frequency limits for the spectra given above.

The spectral properties can be summarized in the following way. The upper frequency for energy in the spectra increases from bass to treble. Assuming an "audible" range of 50-60 dB, the spectrum of the bass note (C2) is limited to 5 kHz (about 60 partials), the middle register note (C4) to 8 kHz (about 15 partials), and the treble note (C7) spectrum reaches above 10 kHz. The spectrum of C7 exceeds the frequency range of analysis (10 kHz). The envelope has decreased approximately 35 dB at 9 kHz (at the 4th partial). All examples refer to mezzo-forte levels.

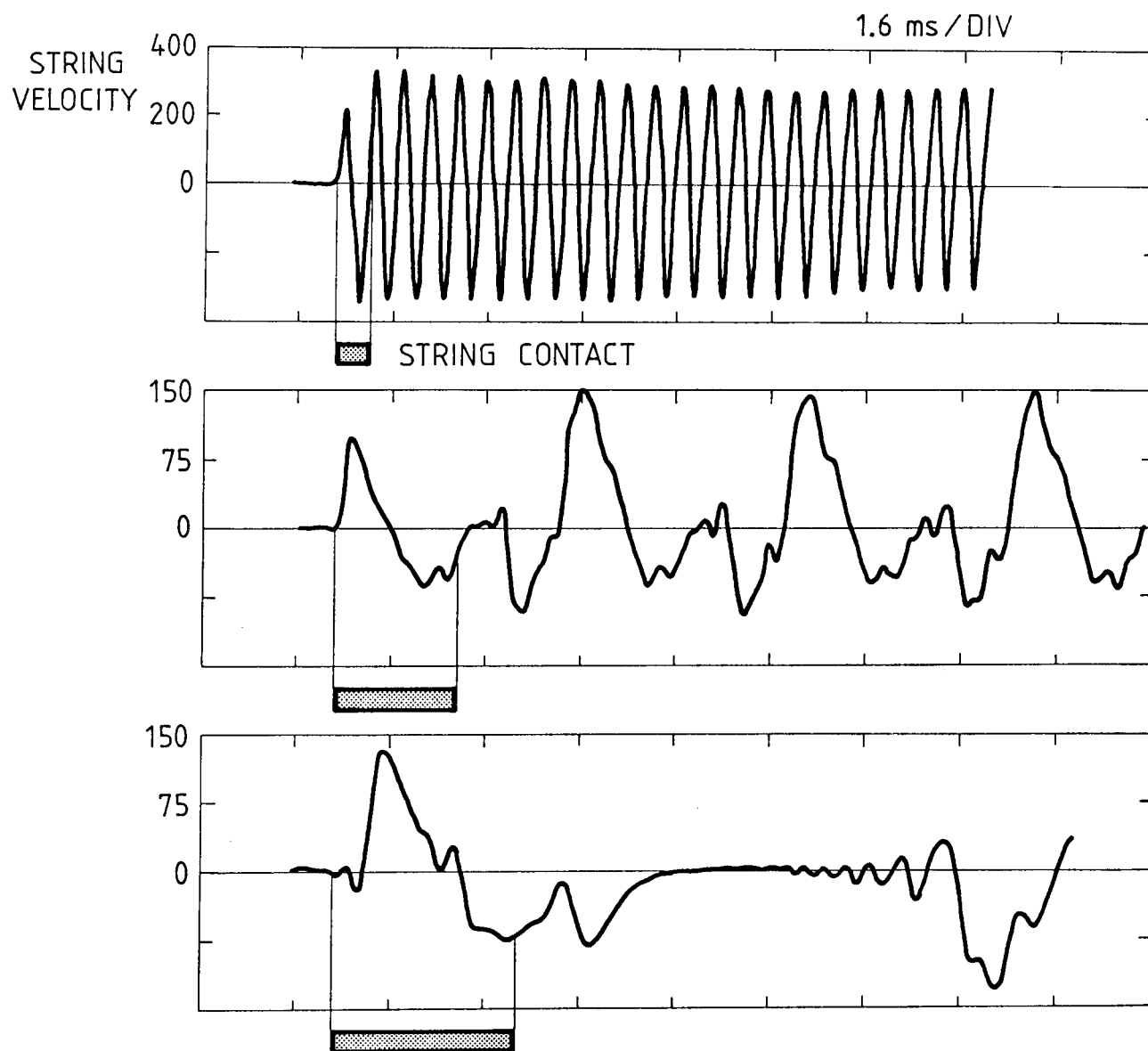


Fig. 24. String velocities (arbitrary units), hammer-string contact durations, and spectra for tones C7 (2093 Hz), C4 (262 Hz), and C2 (65.4 Hz) played mf by pendulum, observation point on the bridge side (B).

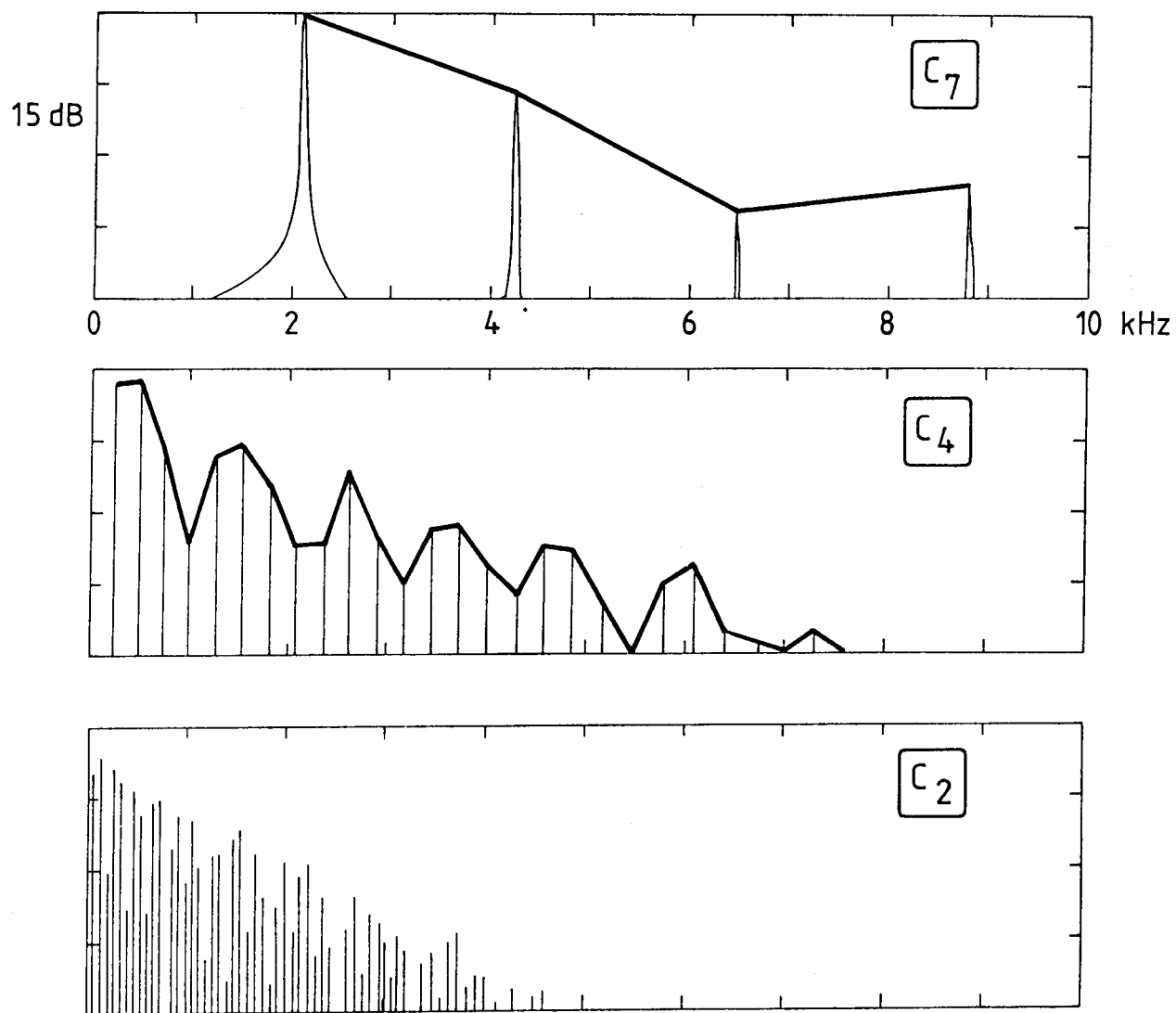


Fig. 24.

b. Different hammers on the same string

The influence of the hammer properties (mass and compliance) on the spectral characteristics of a note was studied by exchanging hammers. The C4-key was fitted with a C7-, a C4-, and a C2-hammer, in succession, and the hammer-string contact durations, waveforms, and spectra were measured, see Fig. 25. The pointed, harder treble hammer gave a shorter contact duration, a waveform with a steeper initial slope, an extended zero level, and a much richer spectrum compared to that of the C4-hammer. The broad, soft bass hammer gave a longer contact duration, a smoother waveform with less steep initial slope, a slightly shortened zero level, and a spectrum with weaker high-frequency partials. Note, that the hammer-string contact durations vary little compared to those measured with the hammers on their original strings, cf. Fig. 24. The fundamental levels are approximately the same (5 dB lower for the C7-hammer). The partial level at 3 kHz is approximately 15 dB lower with the C2-hammer, and the C7-hammer gave approximately 10 dB higher level above 4 kHz. The high-frequency limits are as the combinations for the hammers and original strings, i.e., 5 kHz for the C2-hammer and a 30 dB drop from 2 to 10 kHz for the C7-hammer (cf. Figs 24 and 25).

The spectral envelopes seem to a first approximation be set by the hammer properties. The hammer-string contact durations are set by the hammer and string properties in combination and were at the most a minor factor in shaping the spectra in this experiment. This supports the conclusion that a nonlinearity is one of the main factors of the hammer-string interaction (cf. Hall & Askenfelt, 1988). Apparently, the piano manufacturers' scaling of hammer properties is of critical importance to achieve the right piano sound over the entire compass of the instrument.

c. Influence of hammer voicing

The spectral content of a note can be changed by adjusting the compliance of the hammer. The piano technician thereby uses needles and softens the hammer felt, he "voices" the hammer. The voicing is a delicate adjustment which involves an adjustment of the effective compliance within 5 to 10% of the desired value (Hall & Askenfelt, 1988). An example of the effects of voicing a hammer is given in Fig. 26. A C4-hammer was voiced in several steps from "a little too hard" to "much too soft" ("ruined").

In short, the initial pulse of the waveform was lowered and its slope less steep with the softening of the hammer. In the spectra, the partials above 2 kHz become considerably weaker. The difference between "normal" and "a little too hard" or "much too soft" is approximately 5 dB above 2 kHz, which is a rather small level shift. Nonetheless, this shift is perceived as a large change in tonal quality.

d. Dynamic level and spectrum

The pianist's variation of the dynamic level also changes the partial content of the spectrum. Due to the nonlinear properties of the hammer, the pianists have at their control a hammer which changes from hard to soft depending on the dynamic level (see Fig. 27) (Hall & Askenfelt, 1988). This property of the hammer is reflected in the waveforms for the three dynamics. With a harder touch, the first pulse becomes not only higher but also more peaked and compressed in time. The changes are consequences of the higher peak force and the shorter hammer-string contact duration, induced by the progressively decreasing hammer compliance.

By comparing the waveforms in Fig. 27 it is easily seen that the forte waveform is not only the amplified mezzo-forte version. In the spectra, the differences are quite clear. Calculated total power for the three cases gives the relative levels 0 dB (forte), -10 dB (mezzo-forte), and -16 dB (piano), which closely agree to the relative levels of the fundamental. The spectral power can also be estimated by means of the amplitude of the first velocity pulse. Calculations show that the shift in power is overestimated by this method from piano to mezzo-forte with 1 dB and from piano to forte with 3 dB.

At 3 kHz, the level in mezzo-forte has dropped approximately 15 dB compared to forte and in piano an additional 15 dB. The upper frequency limits for level shifts to 60 dB compared to the fundamental can be estimated to 3, 6, and 10 kHz, respectively. The number of prominent partials increases from 9 at piano, to about 20 at mezzo-forte and more than 30 at forte.

It is interesting to note that the spectral changes, controlled by the player via the dynamic level, are about three times larger than the changes caused by voicing (15 dB compared to 5 dB). The spectral changes are even larger than for the shifts of hammers (Fig. 25). This coupling between "gain" and "treble" controls are not a feature unique to the piano. It is a characteristic shared between almost all traditional instruments.

e. String and bridge spectra

The string vibrations give the driving force of the soundboard. The motion of the bridge is determined by the terminating impedance that the string "sees" at the bridge. The soundboard impedance is much higher than the characteristic impedance of the string, typically 8 kg/s for the string triplet compared to 1000 kg/s for the bridge with sound board (Wogram, 1981). Consequently, the bridge can be regarded as an almost rigid support and the string pulses as undisturbed after the reflexion.

Measured spectra of the string velocity pulses and bridge velocity are closely the same, which further supports the informative value of the string velocity (see Fig. 28). There is, however, a tendency that an

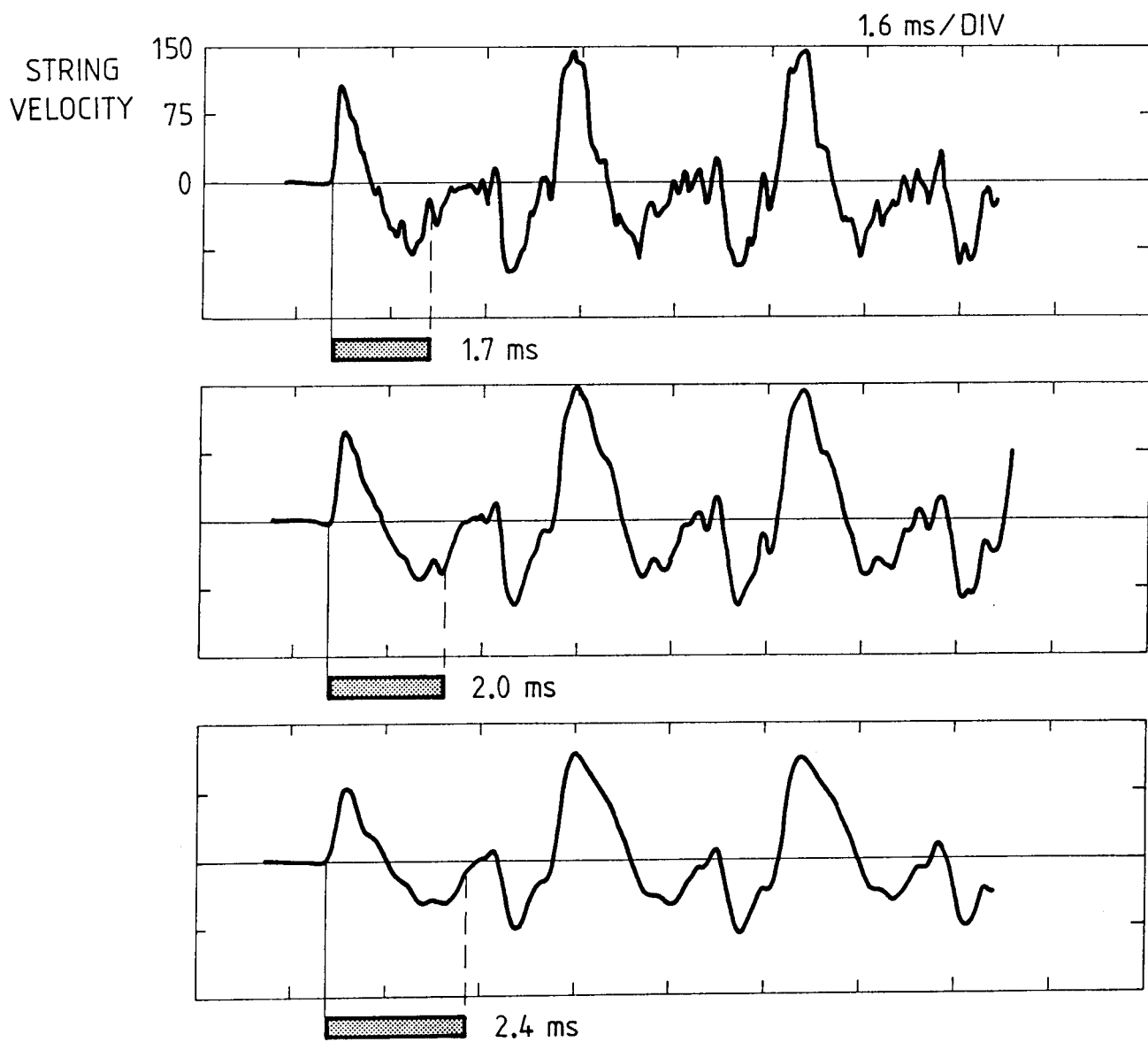


Fig. 25. String velocities (arbitrary units), hammer-string contact durations, and spectra for tone C4 played mf with C7, C4, and C2 hammers by the pendulum.

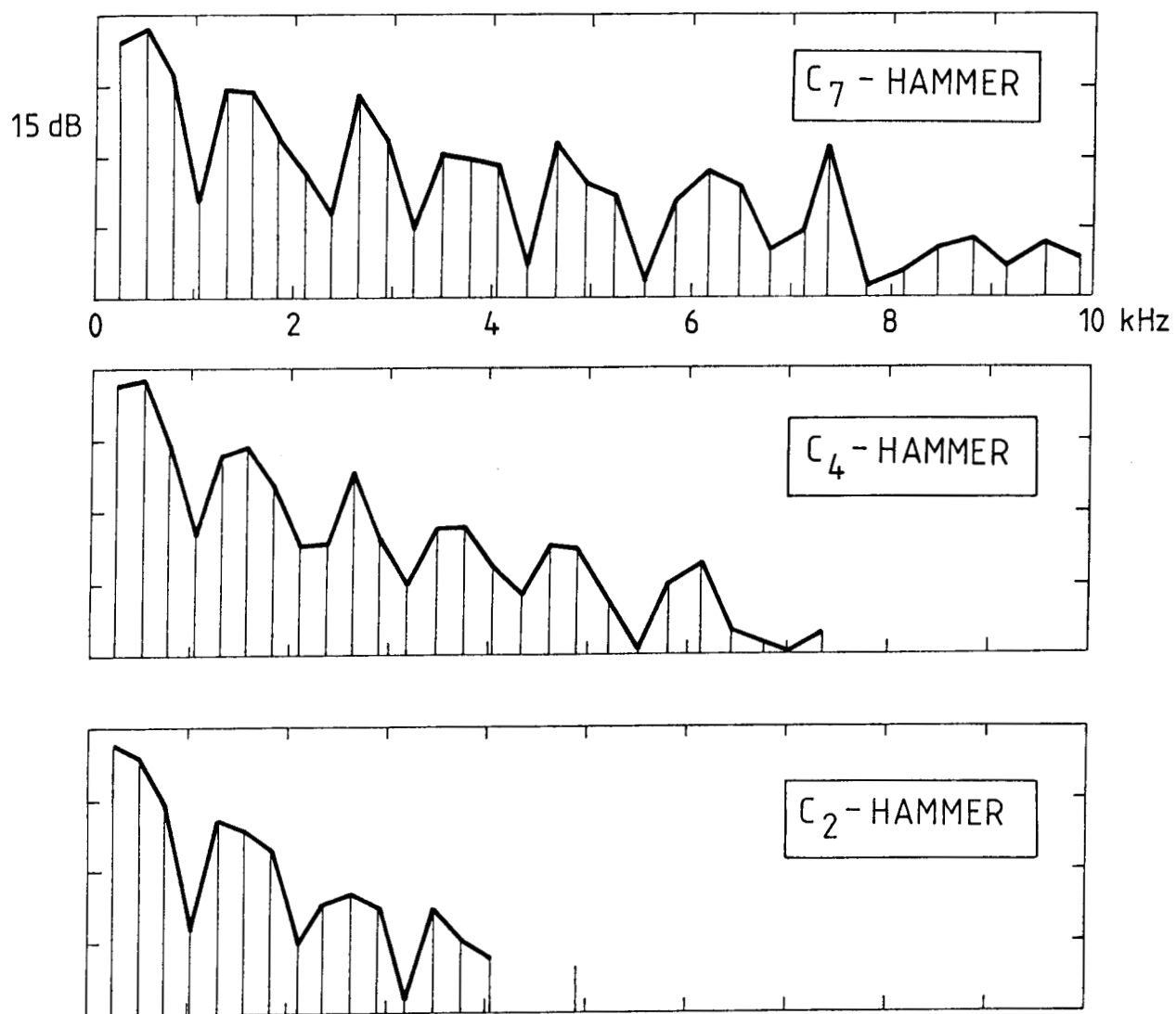


Fig. 25.

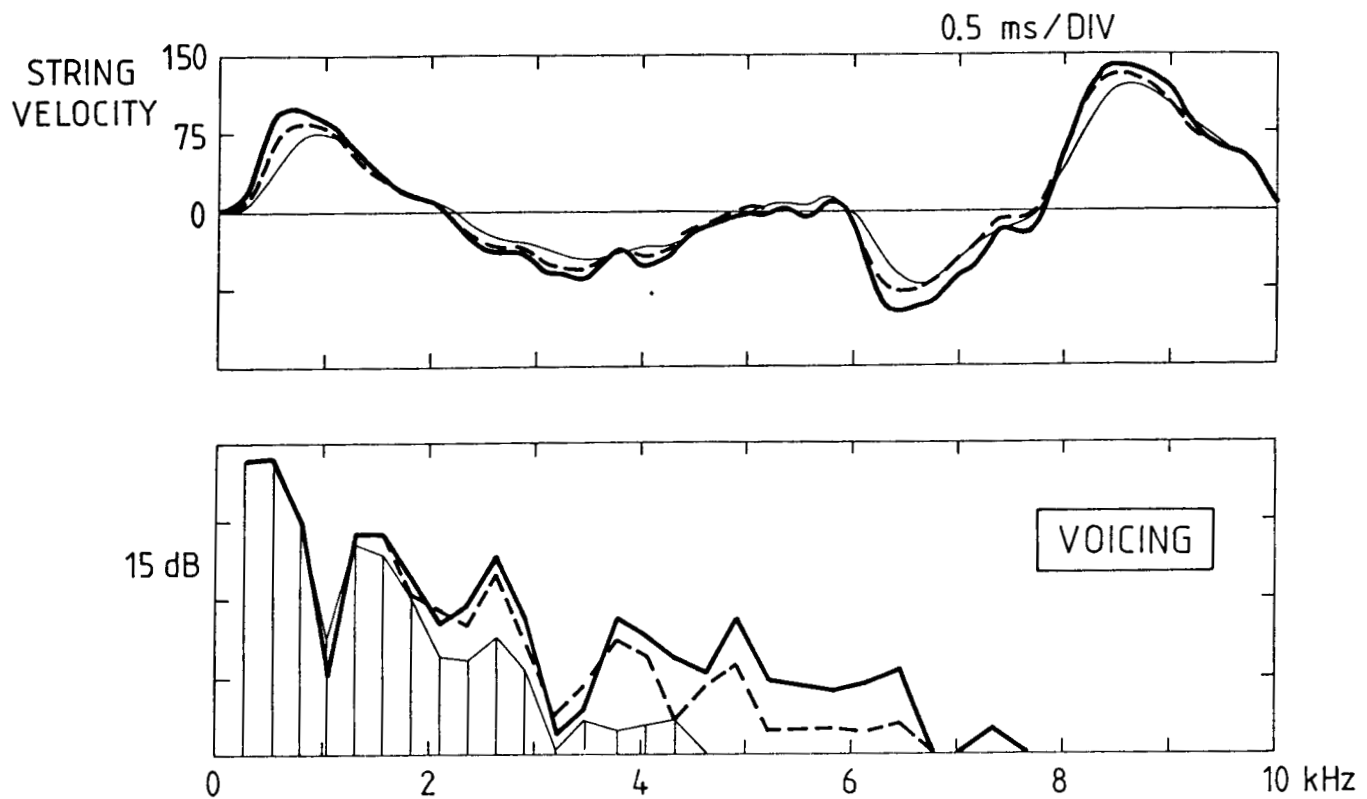


Fig. 26. String velocities and spectra for tone C4 played mf with C4 hammer in different stages of voicing (full line - "too hard", broken line - "normal", and thin line - "ruined") played by a pendulum (mf).

increase of the dynamic level increases the high-frequency content of in the bridge spectrum slightly more than the string spectrum. Possibly, this could be caused by a nonlinear effect due to the slightly curved shape of the bridge side towards the vibrating length of the string.

5. Hammer-string contact

a. Agraffe pulses and hammer retardation

In the middle and treble ranges, the contact durations were about half a period or more. The distance hammer-agraffe gives a much shorter round trip time (approximately $1/8$) than half a period. Thus, several agraffe pulses will be reflected by the hammer in contact with the string (cf. Fig. 22). These repeated pulses on the short string part between hammer and agraffe try to push away the hammer. The energy transfer between hammer and string is not instantaneous. The hammer moves with the string and energy is transferred first from hammer to string and in the end, some energy is used to throw the hammer off the string (cf. Boutillon, 1988; Hall & Askenfelt, 1988; Suzuki, 1987). An example of this process is given in Fig. 29. The hammer typically receives four such "push-pulses" before it is released from the string. A doubling of the hammer mass by a small weight extended the contact duration and increased the number of push pulses to about six.

b. Multiple contacts

In the bass, the contact duration becomes relatively short compared to the period time, and the hammer may loose contact with the string without the help from returning pulses on the string. It has been predicted that this can occur at soft blows in the bass (Hall, 1987b). In such a case, it may occur that a returning pulse will "catch up" with the hammer and make renewed contact. This phenomenon was observed in the low bass (C1) above the pp-level, see Fig. 30. The velocity curve shows that as the first inverted pulse returns from the agraffe, the displacement of the string decreases faster than the hammer is moving down and this causes the multiple contact. This observation is in agreement with the predictions obtained by computer simulations (Suzuki, 1987).

6. String motion and dampers

When the key is released, the damper will fall down on the string and stop the string vibrations. As the damper has a limited mass, it cannot stop the vibrations momentarily. The damping of the strings were studied by measuring the string velocity and the damper motions (acceleration). The measurements were made for a bass note (C2), a middle register note C4, and a treble note (C6).

The measurements showed that the damping is fairly inefficient in the bass. The strings shake the dampers vividly and it takes approxima-

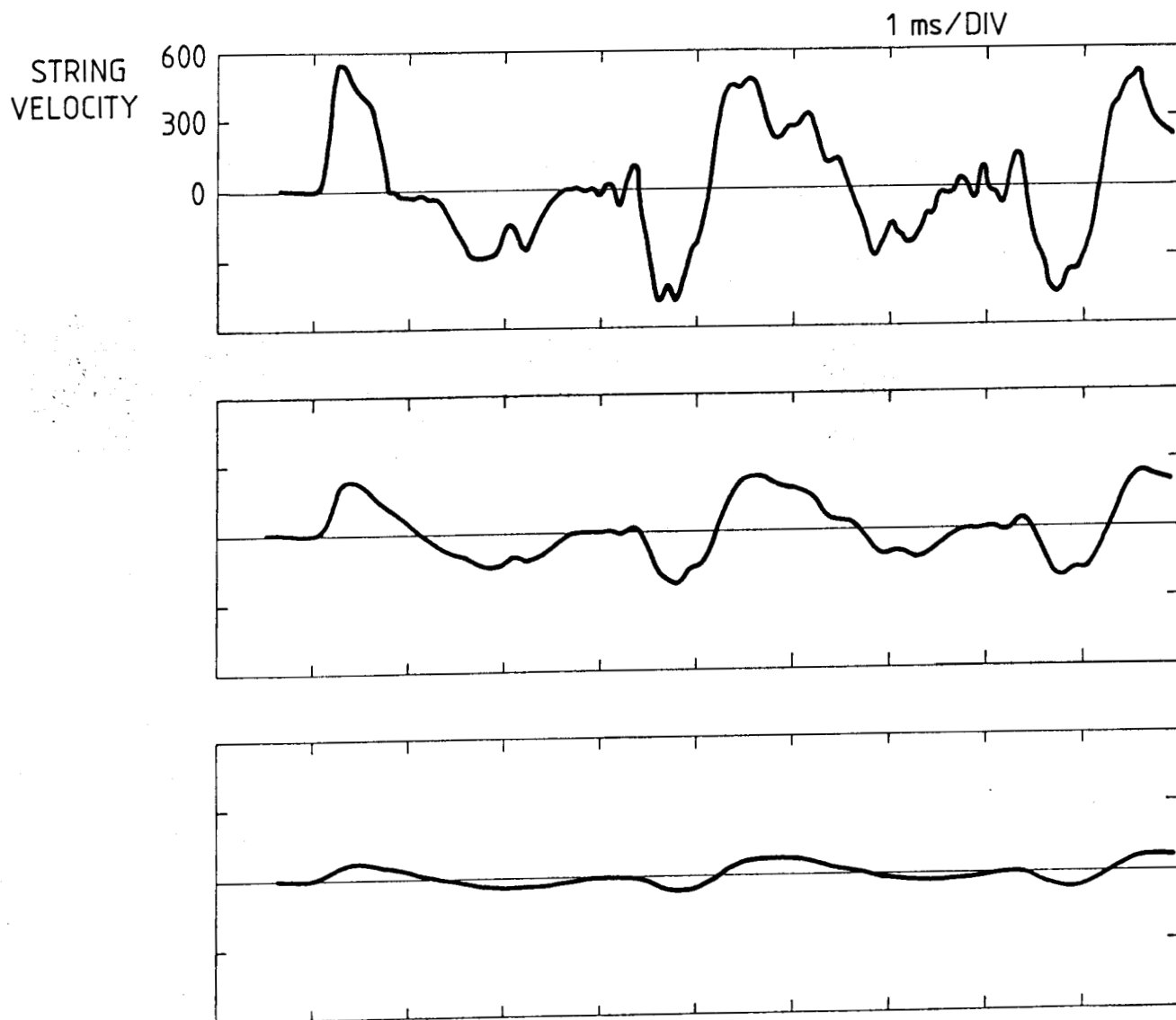


Fig. 27. String velocities (arbitrary units) and spectra for tone C4 played p, mf, and f with original C4 hammer.

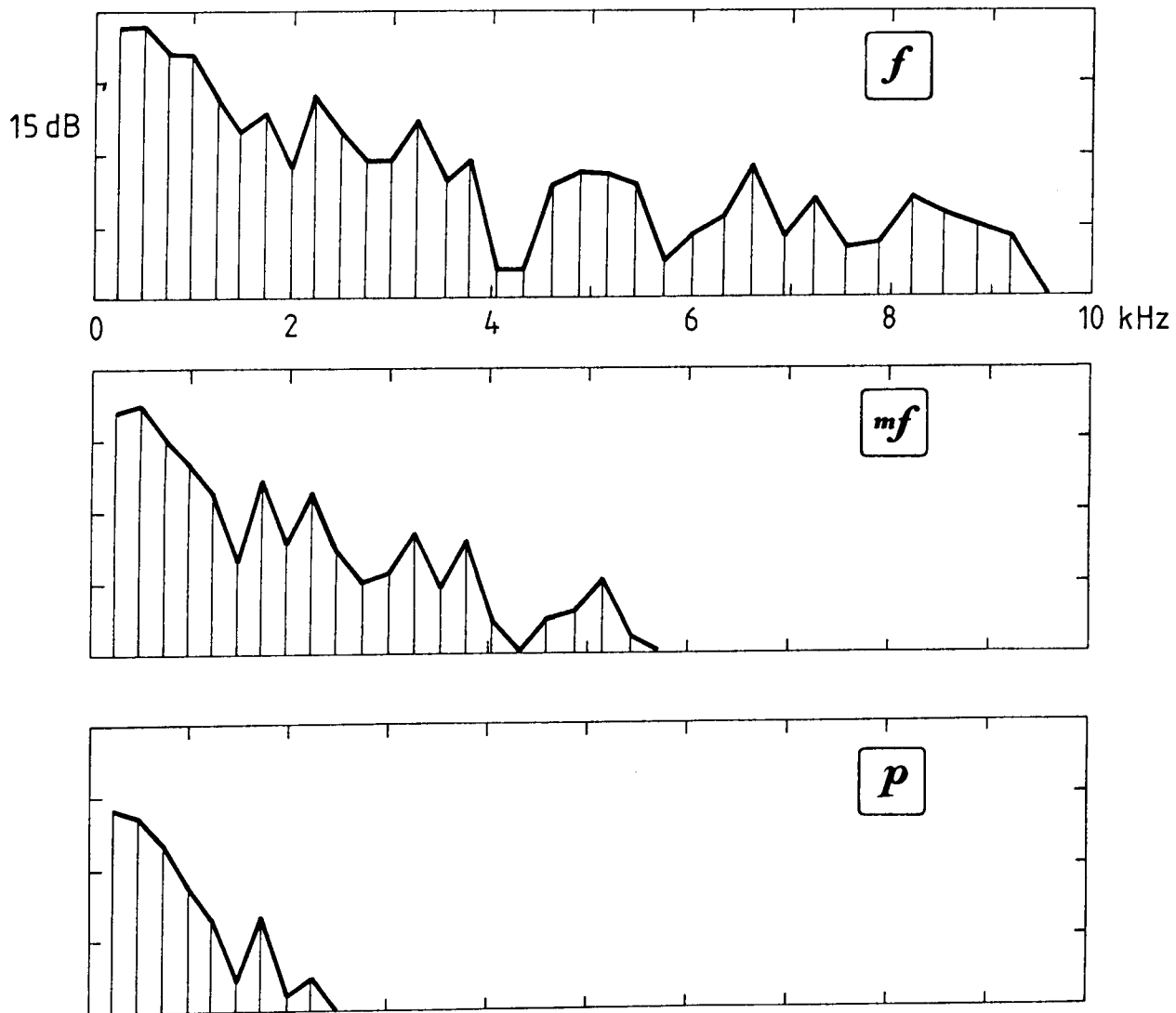


Fig. 27.

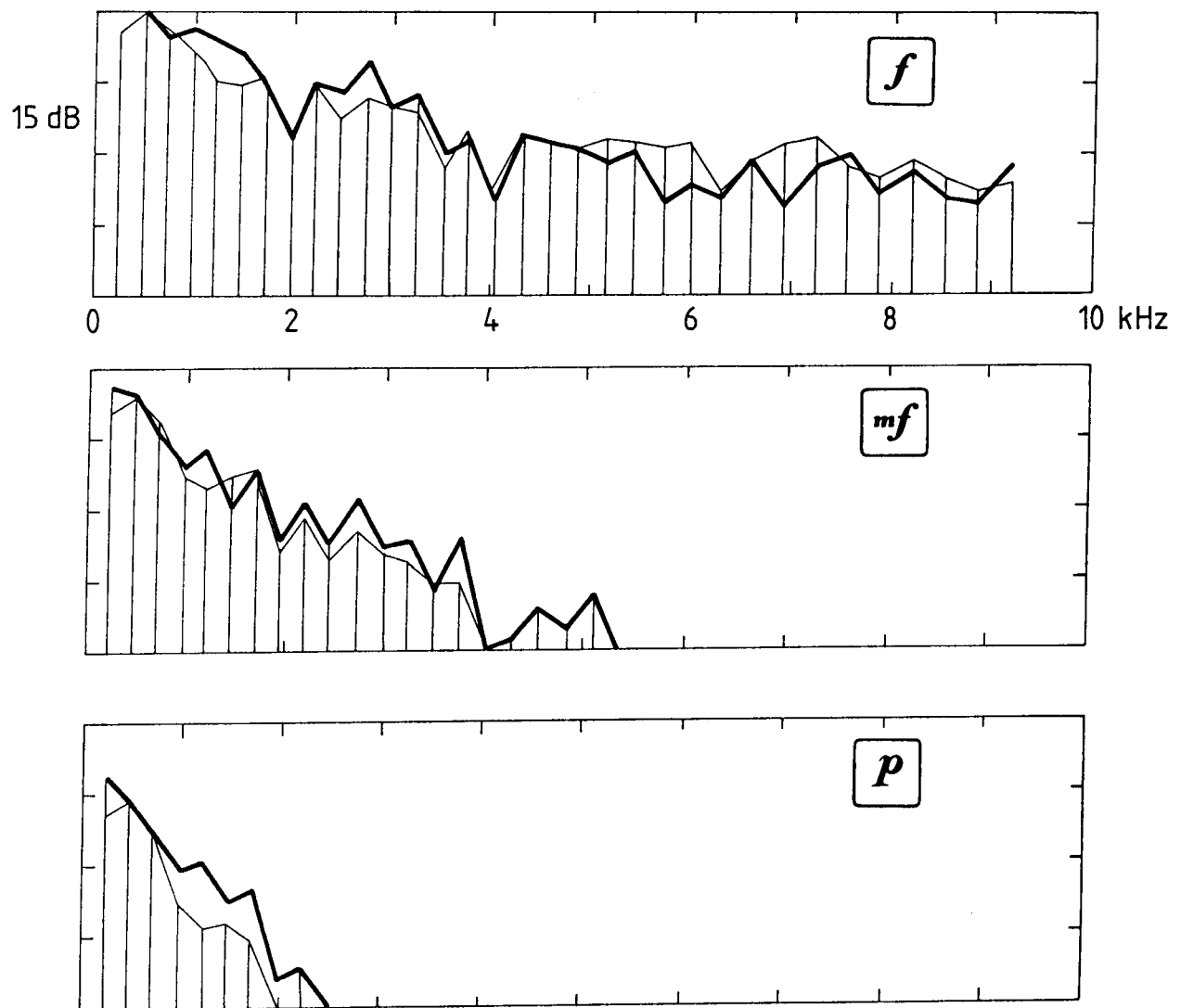


Fig. 28. Measured spectra of string velocity (broad lines) and bridge velocity (thin lines) at three dynamic levels (C_4). The spectra have been shifted to the same level of the second partials. The string spectra are corrected for the influence of detector position.

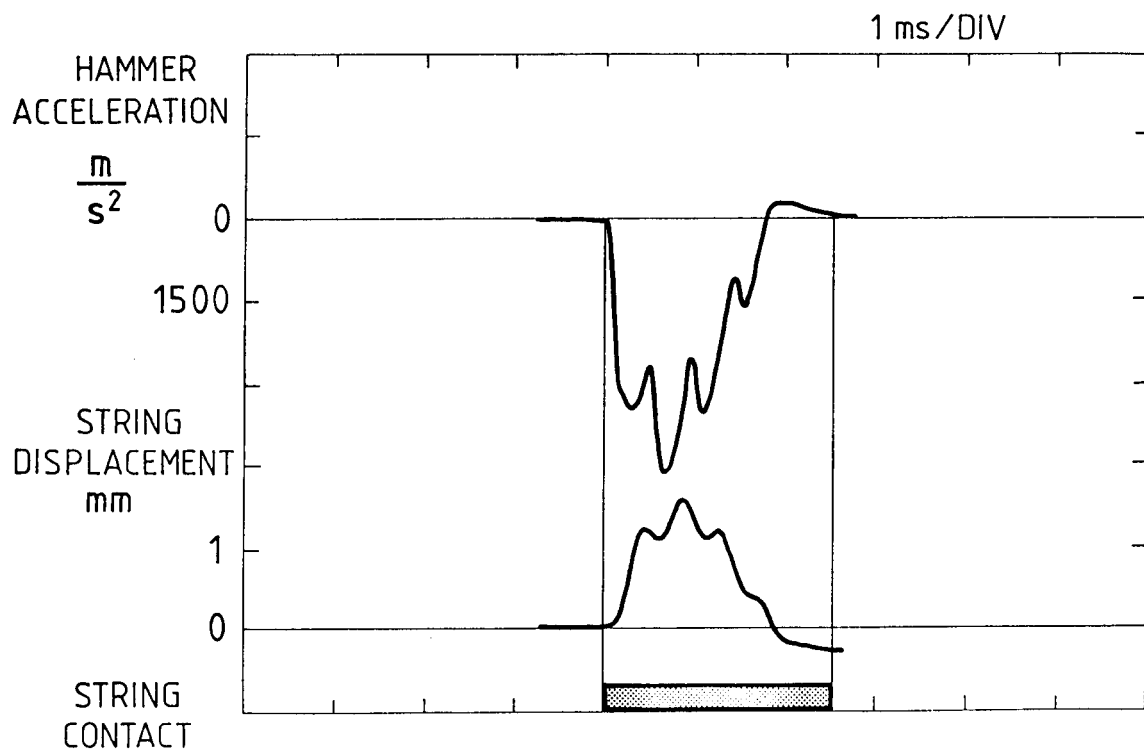


Fig. 29. Hammer-string interaction: hammer acceleration, string displacement and hammer-string contact duration (C4).

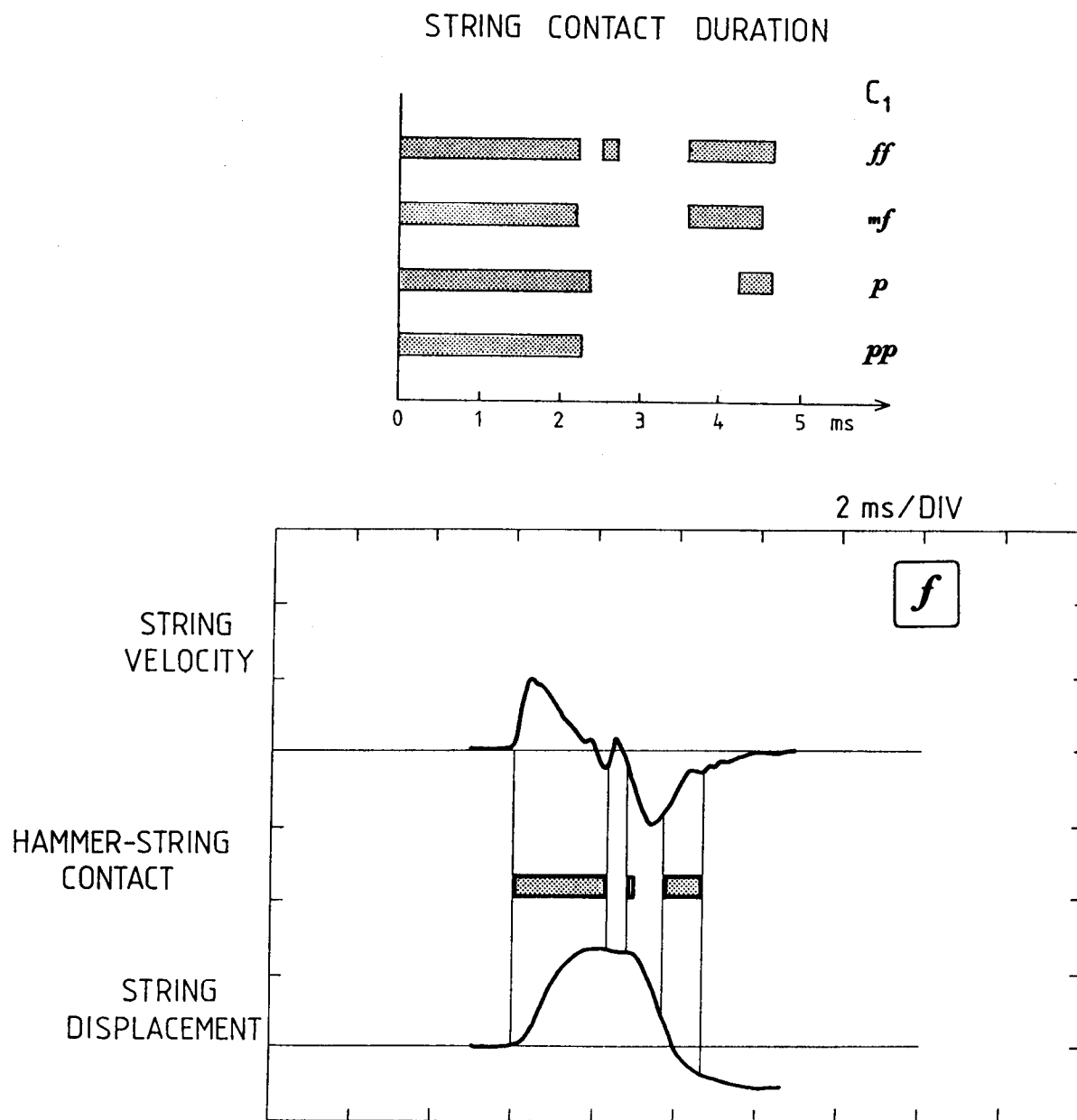


Fig. 30. Hammer-string interaction: hammer-string contact durations (C_1) played *pp* to *ff* (top), and hammer acceleration, string displacement (arbitrary units), and hammer-string contact time played *f*, observation point coinciding with the hammer-string contact point (bottom).

tely 200 ms before the string vibrations have settled. For the middle range note, the damping time is considerable, at least 100 ms. In the treble, the damper is more efficient; the damping time is about 40 ms only. In the highest treble, the string vibration decay is so fast that all manufacturers have found it unnecessary to provide the strings with dampers. The strings' own decay is fast enough. It is mainly determined by the admittance of the sound board (Hall & Askenfelt, 1983).

D. Conclusions

In this section, the relations between the hammer-string contact, the initial velocity pulse of transversal string vibrations, and the spectrum of string vibrations have been investigated. It is found that the contact duration diminishes, the initial pulse becomes higher and narrower, and the high-frequency components become stronger with harder touch. It was found that the dynamic level of playing gave the strongest influence, large shifts of hammer properties (by exchanging "normal" to bass and treble hammers) gave less, and the voicing of a hammer the least influence. The spectrum envelopes seem to be set primarily by the hammer properties, width, and/or compliance and not the contact duration. The initial velocity pulse amplitude gives also a measure of the final hammer velocity.

Summary

In this study, important properties of the initial stages of sound production in the grand piano have been investigated. In a first section, the timing in the piano action was studied. Important timing properties included were the relation between key-bottom contact and hammer-string contact and the "free" time of the hammer's motion before it strikes the string. Both these timing properties were found to be largely dependent on the regulation and the dynamic level. The hammer-string contact duration was nonuniformly scaled over the piano compass in relation to the fundamental period of the strings. The contact durations also changed significantly with dynamic level, an effect due to the nonlinear characteristics of the felt hammers.

In a second section, the motion of the key and the hammer was studied at different dynamics and using different types of "touch". Characteristic differences in key and hammer motion could be observed depending on the type of touch. In particular, a rapid oscillation of the hammer head superimposed on a low frequency oscillation could be observed during the acceleration of the hammer. These two components were attributed to resonances in the hammer. It was concluded that the oscillations offered a hypothetical possibility for the pianist to influence the string excitation by touch.

In a third section, string motion was analyzed. The string motion was found to be of a pulse character in the bass and middle register,

depending on short hammer-string contact durations in relation to the fundamental period of the string. During the hammer-string contact, a fast pulse train was observed on the short string segment between the hammer and the agraffe, which gave repeated impulses on the hammer, determining the moment of hammer release. Typical differences in the string waveform and spectra over the compass of the piano were presented showing an increase in excitation of high-frequency partials towards the treble. The influence of changing the hammer mass and adjusting the hammer compliance ("voicing") was investigated. It was shown that these spectral differences were smaller than the differences evoked by the pianist in changing the dynamic level.

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